CS 5522: Artificial Intelligence II

Bayes’ Nets: Independence

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[These slides were adapted from CS188 Intro to AI at UC Berkeley. All materials available at http://ai.berkeley.edu.]
Probability Recap

- **Conditional probability**
  \[ P(x|y) = \frac{P(x,y)}{P(y)} \]

- **Product rule**
  \[ P(x,y) = P(x|y)P(y) \]

- **Chain rule**
  \[ P(X_1, X_2, \ldots, X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2)\ldots \]
  \[ = \prod_{i=1}^{n} P(X_i|X_1, \ldots, X_{i-1}) \]

- **X, Y independent if and only if:**
  \[ \forall x, y : P(x,y) = P(x)P(y) \]

- **X and Y are conditionally independent given Z if and only if:**
  \[ X \perp Y | Z \]
  \[ \forall x, y, z : P(x, y|z) = P(x|z)P(y|z) \]
A Bayes’ net is an efficient encoding of a probabilistic model of a domain.

Questions we can ask:

- Inference: given a fixed BN, what is \( P(X \mid e) \)?
- Representation: given a BN graph, what kinds of distributions can it encode?
- Modeling: what BN is most appropriate for a given domain?
Bayes’ Net Semantics

- A directed, acyclic graph, one node per random variable
- A conditional probability table (CPT) for each node
  - A collection of distributions over $X$, one for each combination of parents’ values $P(X|a_1 \ldots a_n)$
- Bayes’ nets implicitly encode joint distributions
  - As a product of local conditional distributions
  - To see what probability a BN gives to a full assignment, multiply all the relevant conditionals together:

$$P(x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(x_i|\text{parents}(X_i))$$
Example: Alarm Network

\[ P(+b, -e, +a, -j, +m) = \]
Example: Alarm Network

\[
P(+b, -e, +a, -j, +m) =
\]
Example: Alarm Network

\[
P(+b, -e, +a, -j, +m) =
\]
\[
P(+b)P(-e)P(+a|+b, -e)P(-j|+a)P(+m|+a) =
\]
\[
0.001 \times 0.998 \times 0.94 \times 0.1 \times 0.7
\]
Example: Traffic

- Causal direction

\[
P(R) = \begin{array}{c|c}
+r & 1/4 \\
-r & 3/4 \\
\end{array}
\]

\[
P(T | R) = \begin{array}{c|c|c}
+r & +t & 3/4 \\
+r & -t & 1/4 \\
-r & +t & 1/2 \\
-r & -t & 1/2 \\
\end{array}
\]

\[
P(T, R) = \begin{array}{c|c|c}
+r & +t & 3/16 \\
+r & -t & 1/16 \\
-r & +t & 6/16 \\
-r & -t & 6/16 \\
\end{array}
\]
Example: Reverse Traffic

- Reverse causality?
Example: Reverse Traffic

- Reverse causality?
Example: Reverse Traffic

- Reverse causality?

\[
P(T) \begin{array}{c|c} +t & 9/16 \\ -t & 7/16 \end{array}
\]

\[
P(R|T) \begin{array}{c|c|c} +t & +r & 1/3 \\ & -r & 2/3 \\ -t & +r & 1/7 \\ & -r & 6/7 \end{array}
\]
Example: Reverse Traffic

- Reverse causality?

\[
P(T)
\]
\[
\begin{array}{c|c}
  & +t & 9/16 \\
+t & 9/16 & \\
-t & 7/16 \\
\end{array}
\]

\[
P(R|T)
\]
\[
\begin{array}{c|c|c}
  & +r & 1/3 \\
+t & +r & 1/3 \\
-t & -r & 2/3 \\
\end{array}
\]

\[
P(T, R)
\]
\[
\begin{array}{c|c|c}
  & +t & \\
+r & +t & 3/16 \\
+r & -t & 1/16 \\
-r & +t & 6/16 \\
-r & -t & 6/16 \\
\end{array}
\]
Causality?

- When Bayes’ nets reflect the true causal patterns:
  - Often simpler (nodes have fewer parents)
  - Often easier to think about
  - Often easier to elicit from experts

- BNs need not actually be causal
  - Sometimes no causal net exists over the domain (especially if variables are missing)
  - E.g. consider the variables Traffic and Drips
  - End up with arrows that reflect correlation, not causation

- What do the arrows really mean?
  - Topology may happen to encode causal structure
  - Topology really encodes conditional independence

\[ P(x_i | x_1, \ldots, x_{i-1}) = P(x_i | \text{parents}(X_i)) \]
Size of a Bayes’ Net
Size of a Bayes’ Net

- How big is a joint distribution over N Boolean variables?
  \[ 2^N \]
Size of a Bayes’ Net

- How big is a joint distribution over N Boolean variables?
  \[ 2^N \]

- How big is an N-node net if nodes have up to k parents?
  \[ O(N \times 2^{k+1}) \]
Size of a Bayes’ Net

- How big is a joint distribution over N Boolean variables?
  \[2^N\]

- How big is an N-node net if nodes have up to k parents?
  \[O(N \times 2^{k+1})\]

- Both give you the power to calculate
  \[P(X_1, X_2, \ldots X_n)\]

- BNs: Huge space savings!

- Also easier to elicit local CPTs

- Also faster to answer queries (coming)
Bayes’ Nets

- Representation
  - Conditional Independences
  - Probabilistic Inference
  - Learning Bayes’ Nets from Data
Conditional Independence

- X and Y are independent if

\[ \forall x, y \quad P(x, y) = P(x)P(y) \quad \implies \quad X \perp Y \]

- X and Y are conditionally independent given Z

\[ \forall x, y, z \quad P(x, y|z) = P(x|z)P(y|z) \quad \implies \quad X \perp Y|Z \]

- (Conditional) independence is a property of a distribution

- Example:

  \[ \text{Alarm} \perp \text{Fire}|\text{Smoke} \]
Bayes Nets: Assumptions

- Assumptions we are required to make to define the Bayes net when given the graph:

\[ P(x_i|x_1 \cdots x_{i-1}) = P(x_i|\text{parents}(X_i)) \]

- Beyond above “chain rule → Bayes net” conditional independence assumptions
  - Often additional conditional independences
  - They can be read off the graph

- Important for modeling: understand assumptions made when choosing a Bayes net graph
Example

- Conditional independence assumptions directly from simplifications in chain rule:

- Additional implied conditional independence assumptions?
Independence in a BN

- Important question about a BN:
  - Are two nodes independent given certain evidence?
  - If yes, can prove using algebra (tedious in general)
  - If no, can prove with a counter example
Important question about a BN:
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Example:
Important question about a BN:
- Are two nodes independent given certain evidence?
- If yes, can prove using algebra (tedious in general)
- If no, can prove with a counter example
- Example:

```
X → Y → Z
```

- Question: are X and Z necessarily independent?
Important question about a BN:
- Are two nodes independent given certain evidence?
- If yes, can prove using algebra (tedious in general)
- If no, can prove with a counter example

Example:

Question: are X and Z necessarily independent?
- Answer: no. Example: low pressure causes rain, which causes traffic.
- X can influence Z, Z can influence X (via Y)
Important question about a BN:

- Are two nodes independent given certain evidence?
- If yes, can prove using algebra (tedious in general)
- If no, can prove with a counterexample

Example:

Question: are X and Z necessarily independent?

- Answer: no. Example: low pressure causes rain, which causes traffic.
- X can influence Z, Z can influence X (via Y)
- Addendum: they could be independent: how?
D-separation: Outline

- Study independence properties for triples
- Analyze complex cases in terms of member triples
- D-separation: a condition / algorithm for answering such queries
This configuration is a “causal chain”

\[ P(x, y, z) = P(x)P(y|x)P(z|y) \]
Causal Chains

- This configuration is a “causal chain”
- Guaranteed X independent of Z? No!

\[ P(x, y, z) = P(x) P(y|x) P(z|y) \]
Causal Chains

- This configuration is a “causal chain”
- Guaranteed $X$ independent of $Z$? No!
  - One example set of CPTs for which $X$ is not independent of $Z$ is sufficient to show this independence is not guaranteed.

$$P(x, y, z) = P(x)P(y|x)P(z|y)$$

$X$: Low pressure  \quad $Y$: Rain  \quad $Z$: Traffic
This configuration is a “causal chain”

Guaranteed $X$ independent of $Z$? **No!**

- One example set of CPTs for which $X$ is not independent of $Z$ is sufficient to show this independence is not guaranteed.

- Example:

$$P(x, y, z) = P(x)P(y|x)P(z|y)$$

X: Low pressure  Y: Rain  Z: Traffic
Causal Chains

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- Guaranteed $X$ independent of $Z$? No!
  - One example set of CPTs for which $X$ is not independent of $Z$ is sufficient to show this independence is not guaranteed.
  - Example:
    - Low pressure causes rain causes traffic, high pressure causes no rain causes no traffic

$$P(x, y, z) = P(x)P(y|x)P(z|y)$$

X: Low pressure  Y: Rain  Z: Traffic
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  - One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.
    - Example:
      - Low pressure causes rain causes traffic, high pressure causes no rain causes no traffic
      - In numbers:

\[ P(x, y, z) = P(x)P(y|x)P(z|y) \]
Causal Chains

- This configuration is a “causal chain”

- Guaranteed X independent of Z? **No!**
  - One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.
  - Example:
    - Low pressure causes rain causes traffic, high pressure causes no rain causes no traffic
  - In numbers:
    - $P(+y \mid +x) = 1$, $P(-y \mid -x) = 1$
    - $P(+z \mid +y) = 1$, $P(-z \mid -y) = 1$
Causal Chains

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This configuration is a “causal chain”

Guaranteed X independent of Z? No!

One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.

Example:

Low pressure causes rain causes traffic, high pressure causes no rain causes no traffic

In numbers:

\[ P( +y | +x ) = 1, \ P( -y | - x ) = 1, \ P( +z | +y ) = 1, \ P( -z | -y ) = 1 \]
This configuration is a “causal chain”

- Guaranteed X independent of Z? No!
  - One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.
  - Example:
    - Low pressure causes rain causes traffic, high pressure causes no rain causes no traffic
  - In numbers:
    \[ P( +y \mid +x ) = 1, \quad P( -y \mid -x ) = 1, \quad P( +z \mid +y ) = 1, \quad P( -z \mid -y ) = 1 \]
This configuration is a “causal chain”

Causal Chains

\[ P(x, y, z) = P(x)P(y|x)P(z|y) \]
Causal Chains

- This configuration is a “causal chain”
- Guaranteed $X$ independent of $Z$ given $Y$?

\[ P(x, y, z) = P(x)P(y|x)P(z|y) \]
Causal Chains

- This configuration is a “causal chain”

\[ P(z|x, y) = \frac{P(x, y, z)}{P(x, y)} \]

- Guaranteed X independent of Z given Y?

\[ P(x, y, z) = P(x)P(y|x)P(z|y) \]

X: Low pressure  Y: Rain  Z: Traffic
Causal Chains

- This configuration is a “causal chain”

\[ P(x, y, z) = P(x)P(y|x)P(z|y) \]

- Guaranteed X independent of Z given Y?

\[
P(z|x, y) = \frac{P(x, y, z)}{P(x, y)} = \frac{P(x)P(y|x)P(z|y)}{P(x)P(y|x)} = P(z|y) \]

Yes!
This configuration is a “causal chain”

Guaranteed X independent of Z given Y?

\[ P(z|x, y) = \frac{P(x, y, z)}{P(x, y)} \]
\[ = \frac{P(x)P(y|x)P(z|y)}{P(x)P(y|x)} \]
\[ = P(z|y) \]

Yes!

Evidence along the chain “blocks” the influence
This configuration is a “common cause”

- $Y$: Project due
- $X$: Forums busy
- $Z$: Lab full

$P(x, y, z) = P(y)P(x|y)P(z|y)$
This configuration is a “common cause”

Y: Project due

X: Forums busy

Z: Lab full

Guaranteed X independent of Z? No!

\[ P(x, y, z) = P(y)P(x|y)P(z|y) \]
This configuration is a “common cause”

\[ P(x, y, z) = P(y)P(x|y)P(z|y) \]

Guaranteed X independent of Z? No!

- One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.
This configuration is a “common cause”

- Guaranteed X independent of Z? No!
  - One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.
  - Example:

\[
P(x, y, z) = P(y)P(x|y)P(z|y)
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This configuration is a “common cause”

- Guaranteed $X$ independent of $Z$? No!
  - One example set of CPTs for which $X$ is not independent of $Z$ is sufficient to show this independence is not guaranteed.
  - Example:
    - Project due causes both forums busy and lab full

$$P(x, y, z) = P(y)P(x|y)P(z|y)$$
This configuration is a “common cause”

Guaranteed $X$ independent of $Z$?  No!

- One example set of CPTs for which $X$ is not independent of $Z$ is sufficient to show this independence is not guaranteed.

Example:

- Project due causes both forums busy and lab full

In numbers:

$$P(x, y, z) = P(y)P(x|y)P(z|y)$$
This configuration is a “common cause”

- Guaranteed $X$ independent of $Z$? No!
  - One example set of CPTs for which $X$ is not independent of $Z$ is sufficient to show this independence is not guaranteed.
  - Example:
    - Project due causes both forums busy and lab full
    - In numbers:
      \[
      P(+x \mid +y) = 1, \quad P(-x \mid -y) = 1, \\
      P(+z \mid +y) = 1, \quad P(-z \mid -y) = 1
      \]
This configuration is a “common cause”

Y: Project due

X: Forums busy

Z: Lab full

Guaranteed X independent of Z? **No!**

One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.

Example:

- Project due causes both forums busy and lab full

In numbers:

\[ P(+x \mid +y) = 1, \ P(-x \mid -y) = 1, \ P(+z \mid +y) = 1, \ P(-z \mid -y) = 1 \]
This configuration is a “common cause”

- **Y**: Project due
- **X**: Forums busy
- **Z**: Lab full

Guaranteed X independent of Z? **No!**

- One example set of CPTs for which X is not independent of Z is sufficient to show this independence is not guaranteed.

Example:

- Project due causes both forums busy and lab full

In numbers:

\[
P(+x \mid +y) = 1, \ P(-x \mid -y) = 1,
\]

\[
P(+z \mid +y) = 1, \ P(-z \mid -y) = 1
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This configuration is a “common cause”

Y: Project due

X: Forums busy

Z: Lab full

Guaranteed X independent of Z? No!

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Example:
- Project due causes both forums busy and lab full

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P(+z \mid +y) = 1, \ P(-z \mid -y) = 1
\]
This configuration is a “common cause”

- Y: Project due
- X: Forums busy
- Z: Lab full

\[ P(x, y, z) = P(y)P(x|y)P(z|y) \]
Common Cause

- This configuration is a “common cause”

Y: Project due

X: Forums busy

Z: Lab full

- Guaranteed X and Z independent given Y?

\[ P(z|x, y) = \frac{P(x, y, z)}{P(x, y)} \]

\[ P(x, y, z) = P(y)P(x|y)P(z|y) \]
Common Cause

- This configuration is a “common cause”
  - Y: Project due
  - X: Forums busy
  - Z: Lab full

\[ P(x, y, z) = P(y)P(x|y)P(z|y) \]

- Guaranteed X and Z independent given Y?
  \[ P(z|x, y) = \frac{P(x, y, z)}{P(x, y)} = \frac{P(y)P(x|y)P(z|y)}{P(y)P(x|y)} = P(z|y) \]
  Yes!

- Observing the cause blocks influence between effects.
This configuration is a “common cause”

Y: Project due

X: Forums busy

Z: Lab full

Guaranteed X and Z independent given Y?

\[
P(z|x, y) = \frac{P(x, y, z)}{P(x, y)}
\]

\[
= \frac{P(y)P(x|y)P(z|y)}{P(y)P(x|y)}
\]

\[
= P(z|y)
\]

Yes!

Observing the cause blocks influence between effects.
Common Effect

- Last configuration: two causes of one effect (v-structures)

X: Raining  Y: Ballgame

Z: Traffic
Common Effect

- Last configuration: two causes of one effect (v-structures)
- Are X and Y independent?

X: Raining  
Y: Ballgame  
Z: Traffic
Common Effect

- Last configuration: two causes of one effect (v-structures)

- Are X and Y independent?
  - Yes: the ballgame and the rain cause traffic, but they are not correlated
  - Still need to prove they must be (try it!)

X: Raining
Y: Ballgame
Z: Traffic
Common Effect

- Last configuration: two causes of one effect (v-structures)

- Are X and Y independent?
  - Yes: the ballgame and the rain cause traffic, but they are not correlated
  - Still need to prove they must be (try it!)

- Are X and Y independent given Z?
Common Effect

- Last configuration: two causes of one effect (v-structures)

- Are X and Y independent?
  - Yes: the ballgame and the rain cause traffic, but they are not correlated
  - Still need to prove they must be (try it!)

- Are X and Y independent given Z?
  - No: seeing traffic puts the rain and the ballgame in competition as explanation.
Common Effect

- Last configuration: two causes of one effect (v-structures)

- Are X and Y independent?
  - Yes: the ballgame and the rain cause traffic, but they are not correlated
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- This is backwards from the other cases
  - Observing an effect activates influence between possible causes.
Common Effect

- Last configuration: two causes of one effect (v-structures)

- Are X and Y independent?
  - Yes: the ballgame and the rain cause traffic, but they are not correlated
  - Still need to prove they must be (try it!)

- Are X and Y independent given Z?
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- This is backwards from the other cases
  - Observing an effect activates influence between possible causes.
The General Case

Conditional Independence

In 3 Easy Steps!
The General Case

General question: in a given BN, are two variables independent (given evidence)?

Solution: analyze the graph

Any complex example can be broken into repetitions of the three canonical c
Reachability

- **Recipe:** shade evidence nodes, look for paths in the resulting graph

- **Attempt 1:** if two nodes are connected by an undirected path not blocked by a shaded node, they are conditionally independent
Recipe: shade evidence nodes, look for paths in the resulting graph

Attempt 1: if two nodes are connected by an undirected path not blocked by a shaded node, they are conditionally independent

Almost works, but not quite
Reachability

- **Recipe:** shade evidence nodes, look for paths in the resulting graph

- **Attempt 1:** if two nodes are connected by an undirected path not blocked by a shaded node, they are conditionally independent

- **Almost works, but not quite**
  - Where does it break?
Reachability

- **Recipe:** shade evidence nodes, look for paths in the resulting graph

- **Attempt 1:** if two nodes are connected by an undirected path not blocked by a shaded node, they are conditionally independent

- **Almost works, but not quite**
  - Where does it break?
  - Answer: the v-structure at T doesn’t count as a link in a path unless “active”
Active / Inactive Paths

- **Question:** Are X and Y conditionally independent given evidence variables \{Z\}?
  - Yes, if X and Y “d-separated” by Z
  - Consider all (undirected) paths from X to Y
  - No active paths = independence!

- **A path is active if each triple is active:**
  - Causal chain \(A \rightarrow B \rightarrow C\) where B is unobserved (either direction)
  - Common cause \(A \leftarrow B \rightarrow C\) where B is unobserved
  - Common effect (aka v-structure) \(A \rightarrow B \leftarrow C\) where B or one of its descendants is observed

- All it takes to block a path is a single inactive segment
D-Separation

- Query: \( X_i \perp\!\!\!\perp X_j \mid \{X_{k_1}, \ldots, X_{k_n}\} \) ?

- Check all (undirected!) paths between \( X_i \) and \( X_j \)
  - If one or more active, then independence not guaranteed
    \[ X_i \perp\!\!\!\perp X_j \mid \{X_{k_1}, \ldots, X_{k_n}\} \]
  - Otherwise (i.e. if all paths are inactive), then independence is guaranteed
    \[ X_i \perp\!\!\!\perp X_j \mid \{X_{k_1}, \ldots, X_{k_n}\} \]
Example
Example

\[ R \perp B \]

Diagram:

- Nodes: \( R \), \( B \), \( T \), \( T' \)
- Edges: \( R \rightarrow T \), \( B \rightarrow T \), \( T \rightarrow T' \)
Example

\[ R \perp B \quad \text{Yes} \]

\[ \begin{array}{c}
R \\
\downarrow \\
T \\
\downarrow \\
T' \\
\end{array} \quad \begin{array}{c}
B \\
\downarrow \\
T \\
\downarrow \\
T' \\
\end{array} \]
Example

\[ R \perp B \quad \text{Yes} \]
\[ R \perp B | T \]
Example

\[ R \perp B \quad \text{Yes} \]
\[ R \perp B | T \]
\[ R \perp B | T' \]
Example
Example

\[ L \perp T' | T \]
Example

\[ L \perp T' | T \quad \text{Yes} \]
Example

$L \sqsubset T' | T$
$L \sqsubset B$

Yes
Example

\[ L \perp T' | T \quad \text{Yes} \]

\[ L \perp B \quad \text{Yes} \]
Example

$L \perp T' \mid T$

$L \perp B$

$L \perp B \mid T$

Yes

Yes
Example

$L \perp T'|T$  Yes
$L \perp B$  Yes
$L \perp B|T$
$L \perp B|T'$
Example

$L \perp T'|T$  
$L \perp B$  
$L \perp B|T$  
$L \perp B|T'$  
$L \perp B|T, R$  

Yes  
Yes
Example

\[ L \perp T'|T \quad \text{Yes} \]
\[ L \perp B \quad \text{Yes} \]
\[ L \perp B|T \]
\[ L \perp B|T' \]
\[ L \perp B|T, R \quad \text{Yes} \]
Example

![Diagram with nodes R, T, D, S connected by arrows]

1. R to D
2. D to T
3. T to S
4. S to R
Example

- Variables:
  - R: Raining
  - T: Traffic
  - D: Roof drips
  - S: I’m sad
- Questions:
Example

- **Variables:**
  - R: Raining
  - T: Traffic
  - D: Roof drips
  - S: I’m sad

- **Questions:**

\[ T \perp D \]
Example

- **Variables:**
  - R: Raining
  - T: Traffic
  - D: Roof drips
  - S: I’m sad

- **Questions:**

\[
T \perp D
\]
\[
T \perp D | R
\]
Example

- **Variables:**
  - R: Raining
  - T: Traffic
  - D: Roof drips
  - S: I’m sad

- **Questions:**

\[
T \perp D
\]

\[
T \perp D | R \quad \text{Yes}
\]
Example

- **Variables:**
  - R: Raining
  - T: Traffic
  - D: Roof drips
  - S: I’m sad

- **Questions:**

\[
T \perp D \\
T \perp D | R \quad \text{Yes} \\
T \perp D | R, S
\]
Given a Bayes net structure, can run d-separation algorithm to build a complete list of conditional independences that are necessarily true of the form

\[ X_i \perp\!\!\!\perp X_j \mid \{X_{k_1}, \ldots, X_{k_n}\} \]

This list determines the set of probability distributions that can be represented.
Computing All Independences

Compute **ALL THE INDEPENDENCES**!
Topology Limits Distributions

- Given some graph topology $G$, only certain joint distributions can be encoded.
- The graph structure guarantees certain (conditional) independences.
- (There might be more independence)
- Adding arcs increases the set of distributions, but has several costs.
- Full conditioning can encode any distribution.
Bayes Nets Representation Summary

- Bayes nets compactly encode joint distributions
- Guaranteed independencies of distributions can be deduced from BN graph structure
- D-separation gives precise conditional independence guarantees from graph alone
- A Bayes’ net’s joint distribution may have further (conditional) independence that is not detectable until you inspect its specific distribution
Bayes’ Nets

- Representation
- Conditional Independences
  - Probabilistic Inference
    - Enumeration (exact, exponential complexity)
    - Variable elimination (exact, worst-case exponential complexity, often better)
    - Probabilistic inference is NP-complete
    - Sampling (approximate)
- Learning Bayes’ Nets from Data