CSE 5523 Homework 1: Math Review

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Problem 1

Consider the following function:

$$f(x) = \frac{1}{1 + e^{-x}}$$

(a)

Draw a plot of $f(x)$ - what are the minimum and maximum values of $f(x)$? What values of x result in the largest or smallest values of $f(x)$?

(b)

Show that the derivative of $f(x)$ can be written simply in terms of the function's value like so:

$$\frac{df(x)}{dx} = f(x)(1 - f(x))$$

Hint: start by computing the derivative of $f(x)$ using the chain rule, then re-arrange terms to get the result into the form of the answer.

Problem 2

Assume the following joint distribution for $P(A, B)$:

$$\begin{aligned} P(A = 0, B = 0) &= 0.2 \\ P(A = 0, B = 1) &= 0.2 \\ P(A = 1, B = 0) &= 0.6 \\ P(A = 1, B = 1) &= 0.0 \end{aligned}$$

(a) What is the marginal probability of $P(B = 0)$?

(b) What is $P(A = 1|B = 0)$?

(c) What is $P(A = B)$?

Problem 3

Assume X is conditionally independent of Y given Z. Which of the following statements are always true?

(a) $P(X, Y) = P(X) + P(Y) - P(Z)$

(b) $P(X, Y, Z) = P(X) + P(Y) + P(Z)$

(c) $P(X, Y) = \sum_{c \in \mathcal{X}_Z} P(X, Y, Z = c)$

(d) $P(X, Y|Z) = P(X|Z)P(Y|Z)$

(e) $P(X, Y) = P(X)P(Y)$

Problem 4

Derive Bayes Rule in the form:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

from the definition of conditional probability:

$$P(A, B) = P(A|B)P(B)$$

Problem 5

Consider the following matrix, M and vector, v:

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 1 & 3 & 3 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

Compute the following matrix-vector and vector-vector products explaining how you arrived at each answer (show your work):

(a)

$M \cdot v =$

(b)

$v^T \cdot M =$

(c)

$v^T \cdot v =$

Problem 6

Install Python (`https://www.python.org/`), Numpy (`http://www.numpy.org/`) and Scipy (`https://www.scipy.org/`) on your computer (assuming you have not already done so). If you strongly prefer not to use Python for some reason you can use another programming language for example Matlab/Ocatave. Python is strongly recommended for the class, however - starter code will be provided for some homeworks in Python.

Write a short program that computes the answers to Problem 5 above. Turn in your code (can be hand-written).

Survey

(a) What is your favorite programming language?

(b) What programming languages do you feel comfortable using?

(c) Are there any specific topics you would like to see covered in the class?