Binary Classification

Alan Ritter

(many slides from Greg Durrett and Vivek Srikumar)

This Lecture

- Linear classification fundamentals
- Naive Bayes, maximum likelihood in generative models
- Three discriminative models: logistic regression, perceptron, SVM
  - Different motivations but very similar update rules / inference!

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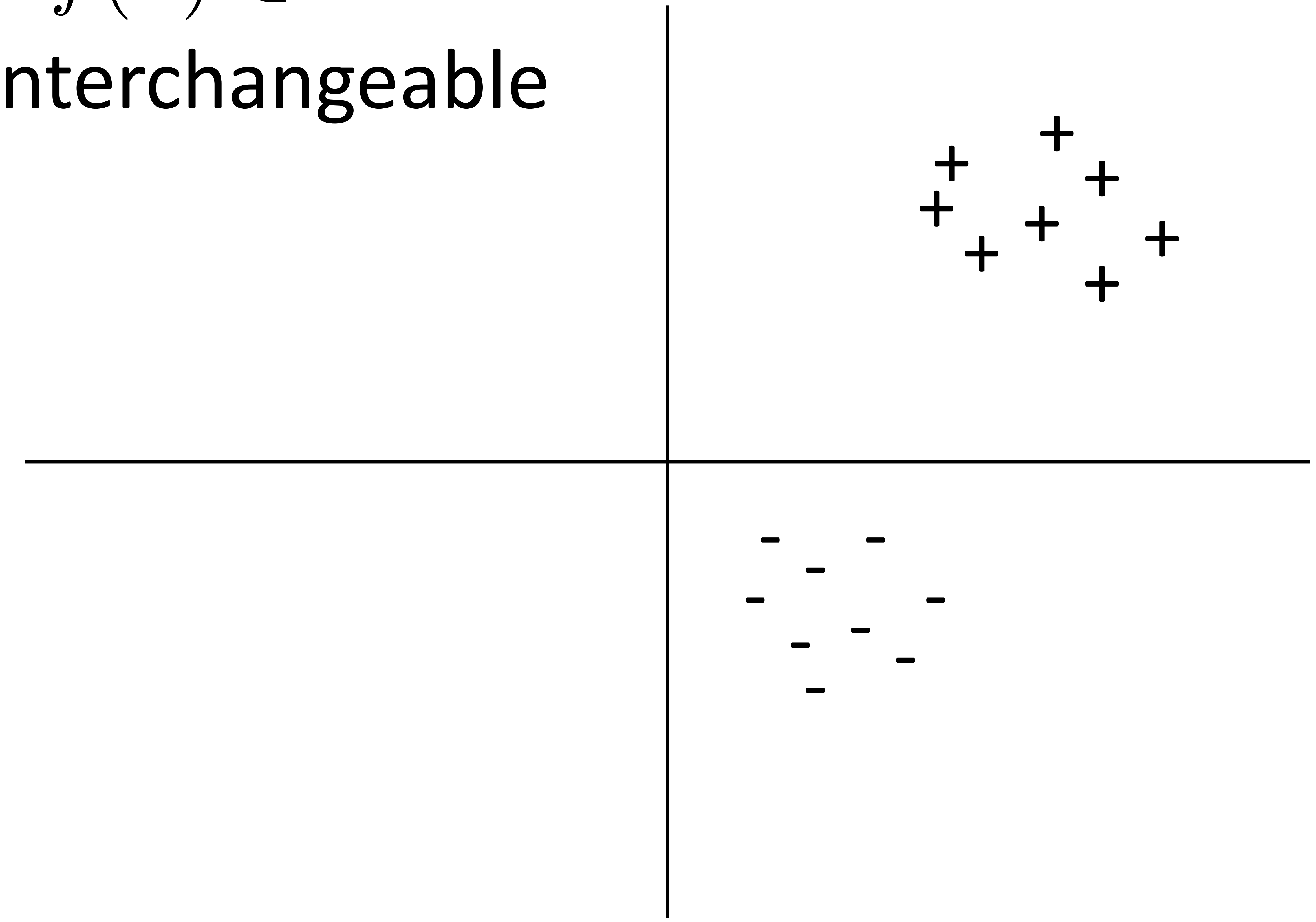

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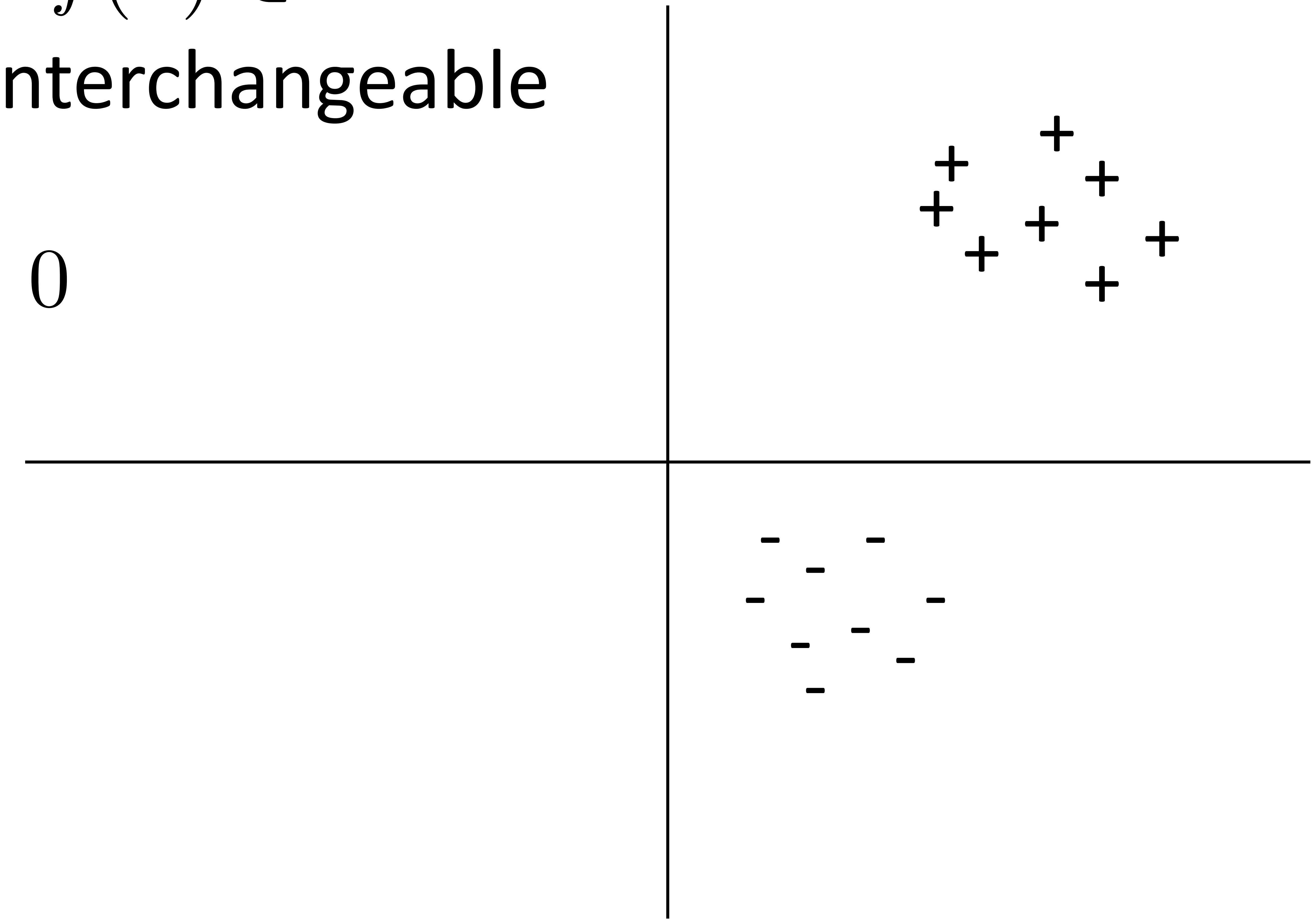

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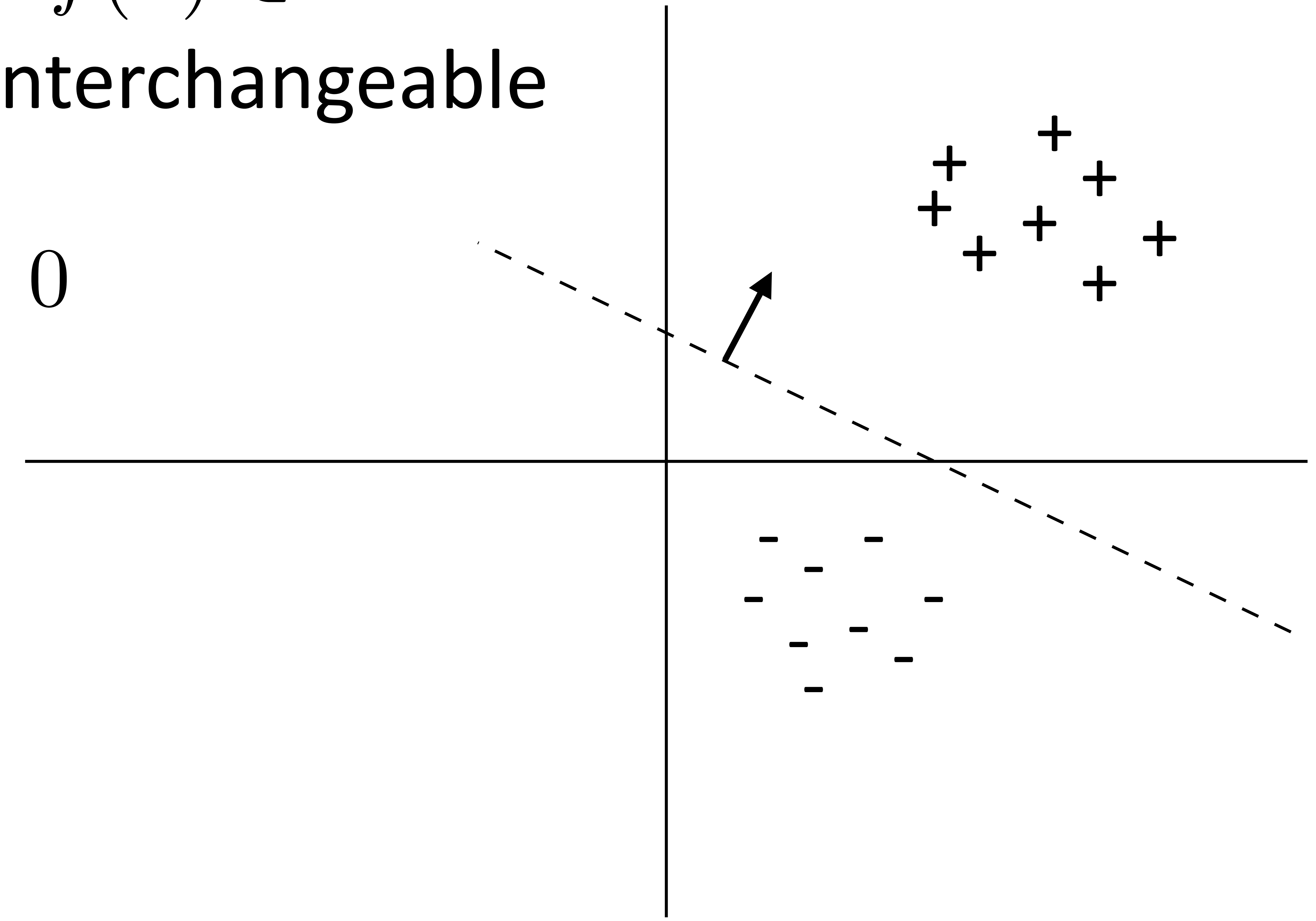

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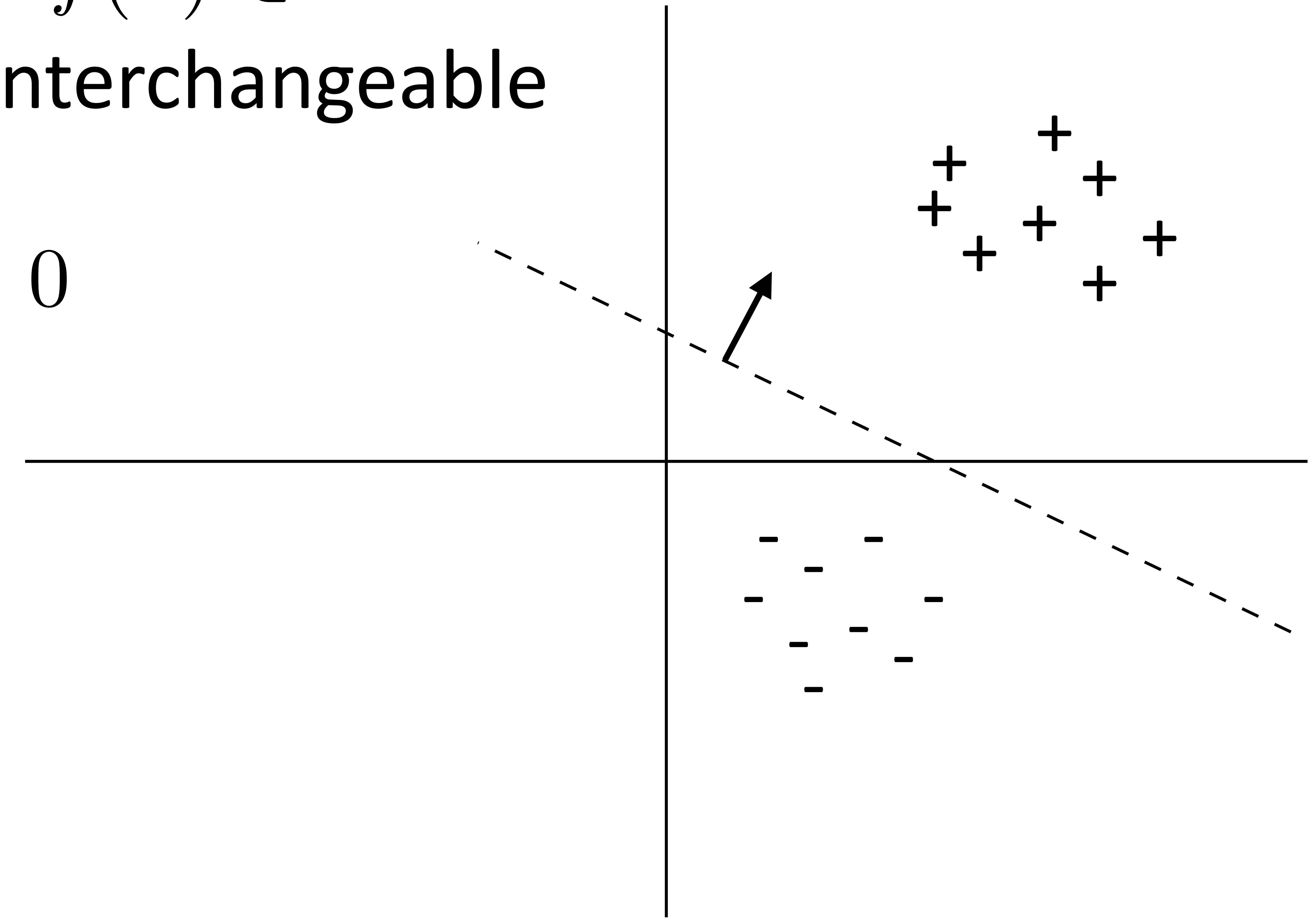

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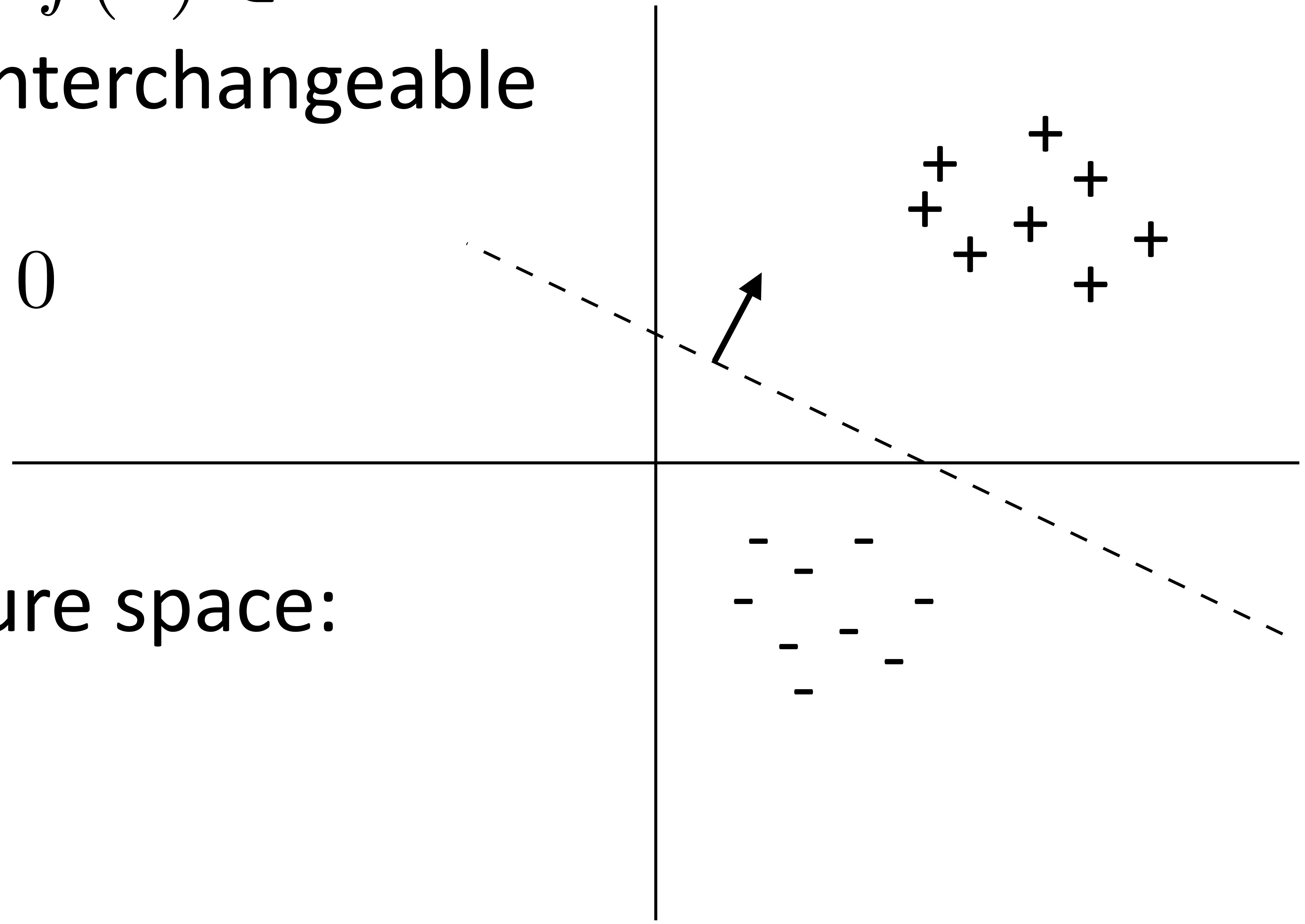
- Can delete bias if we augment feature space:

  $f(x)$ = [0.5, 1.6, 0.3]

  ↓

  [0.5, 1.6, 0.3, **1**]



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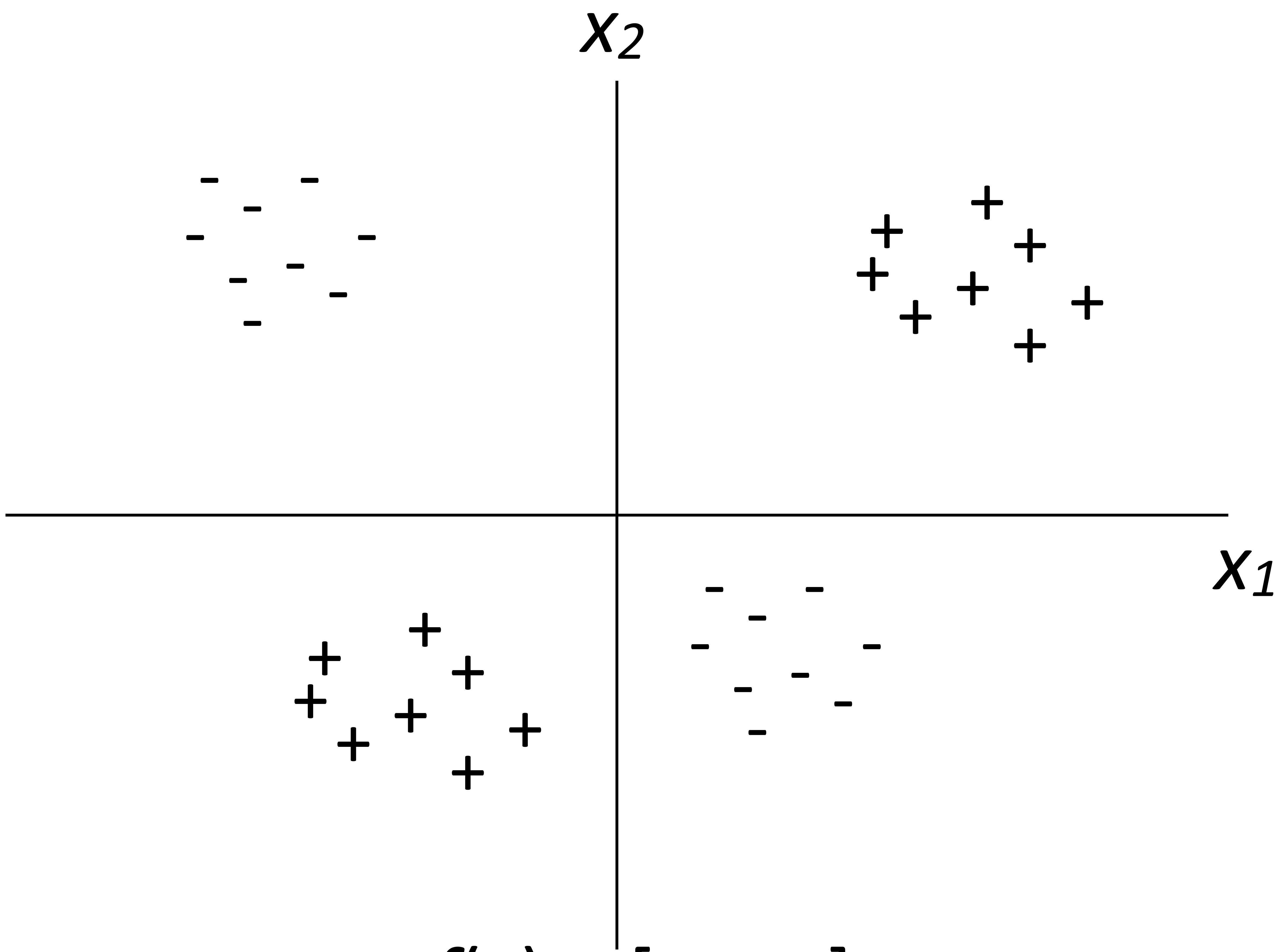

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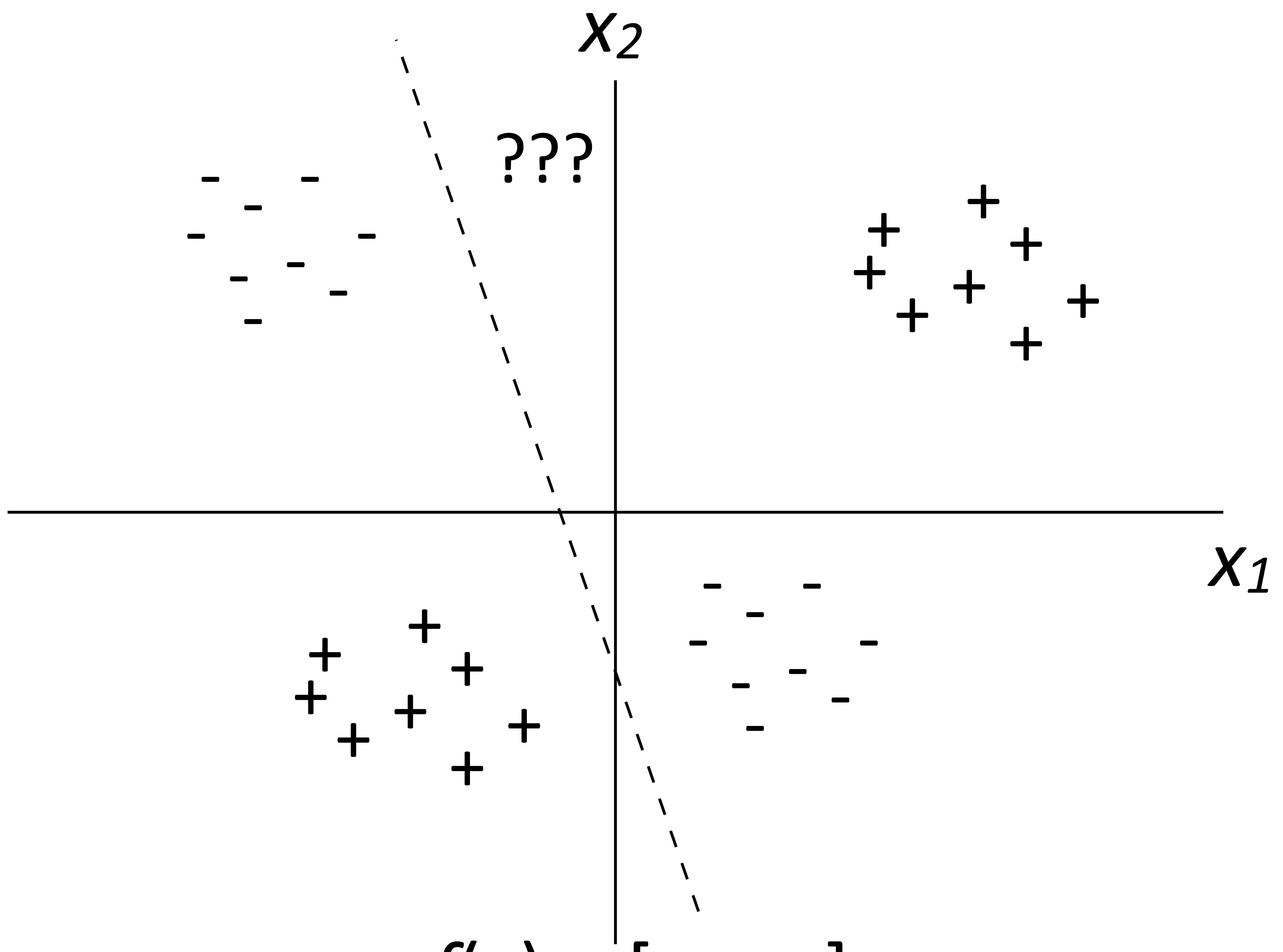

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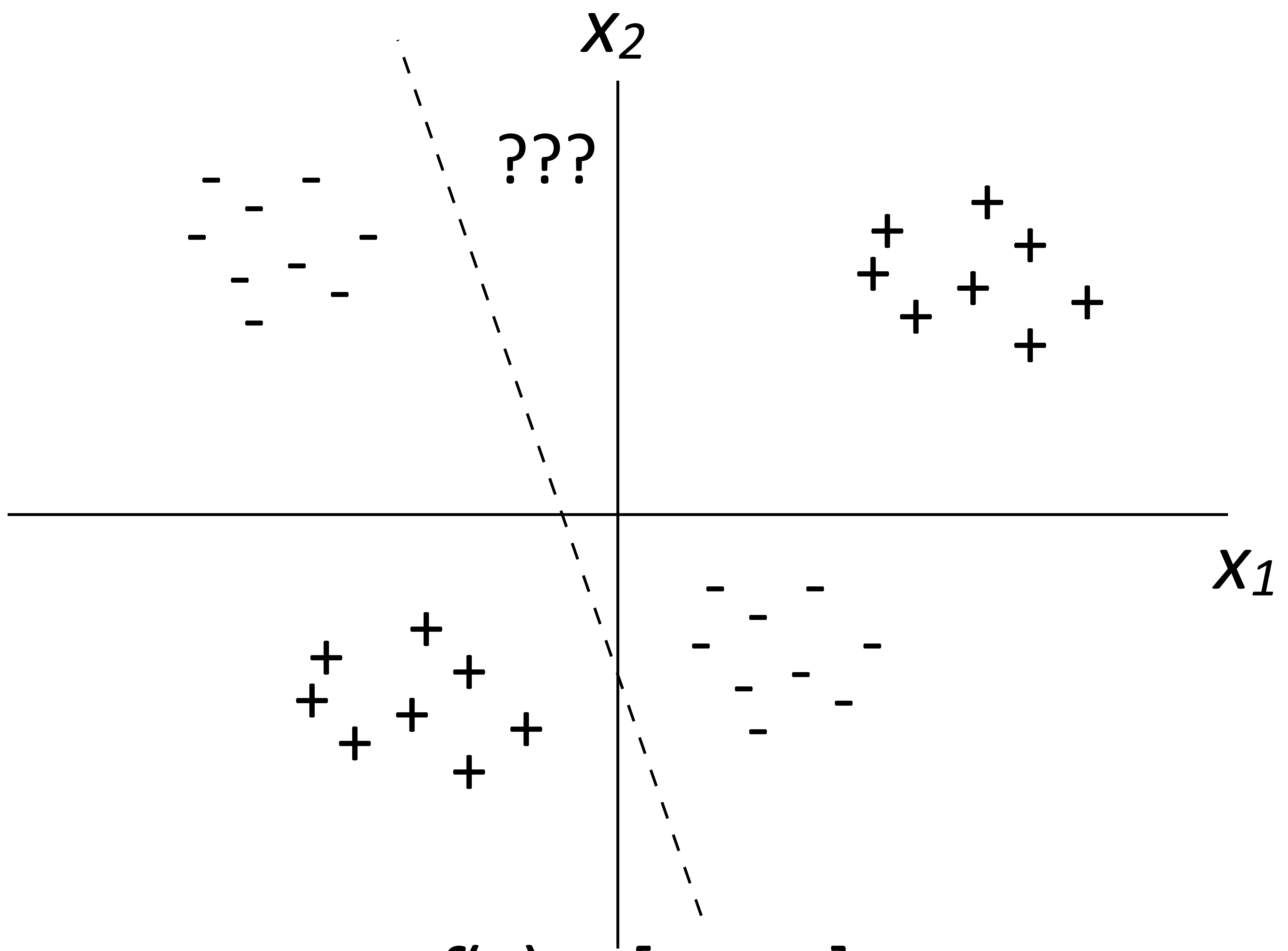

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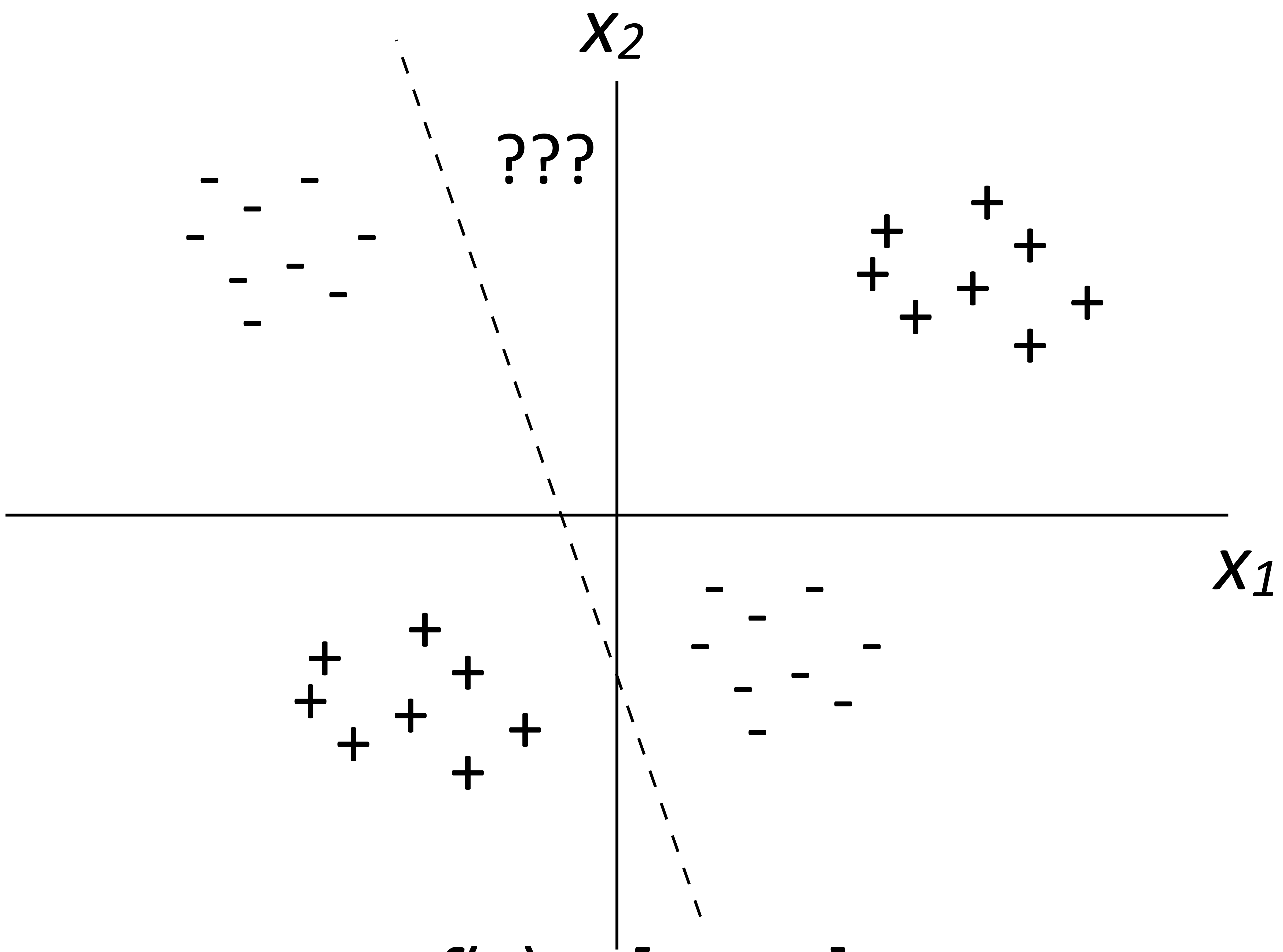

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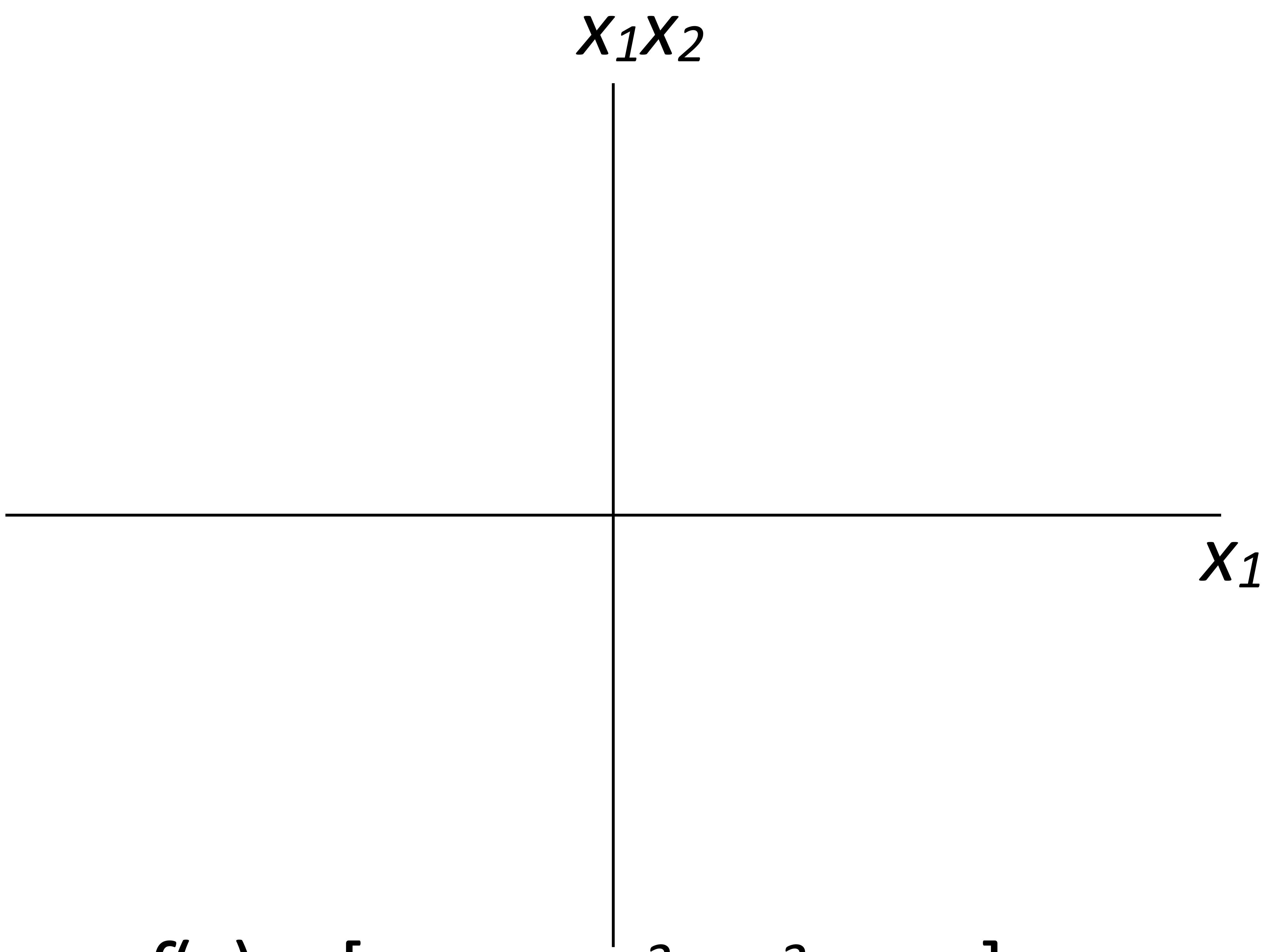

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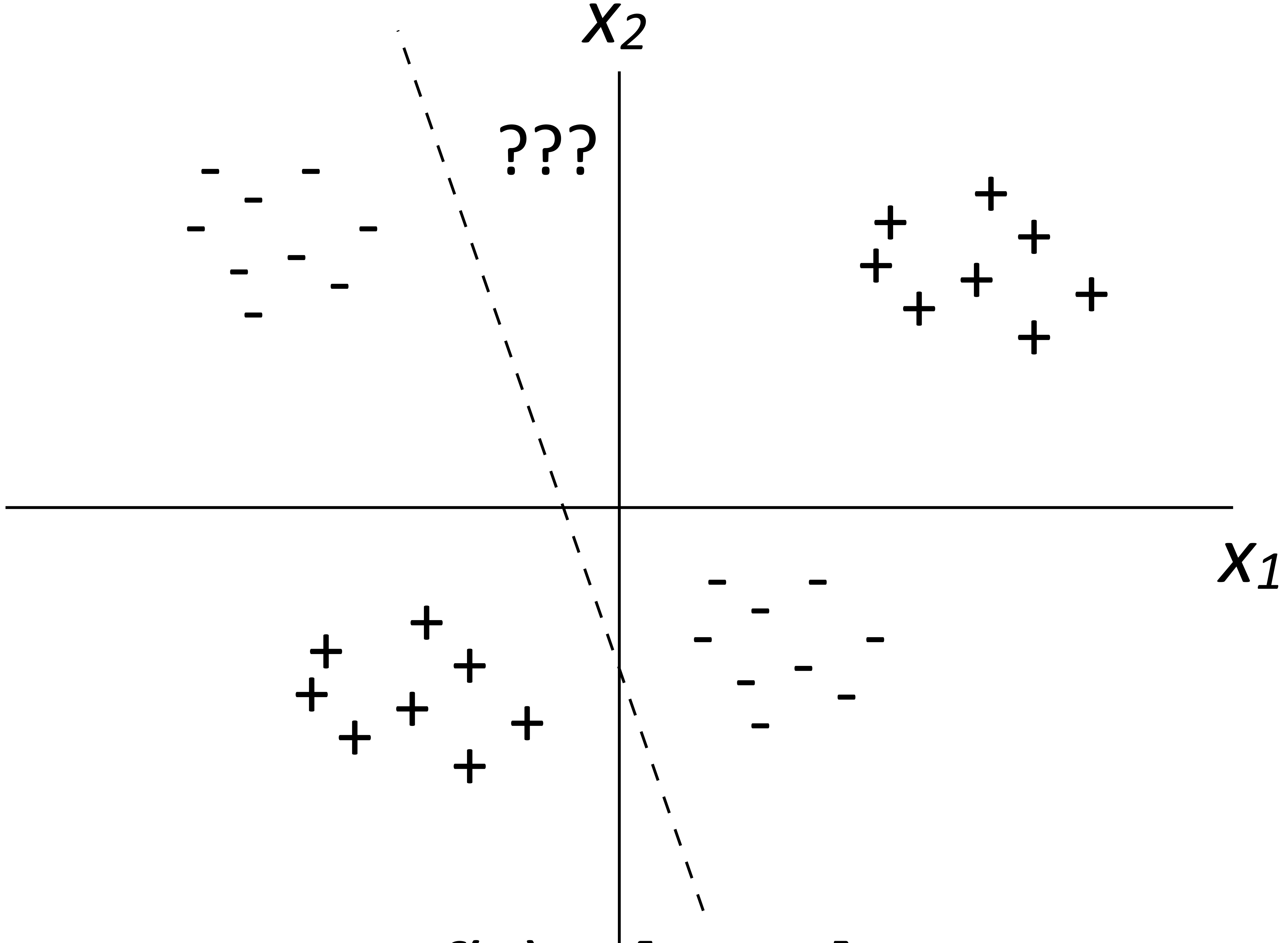

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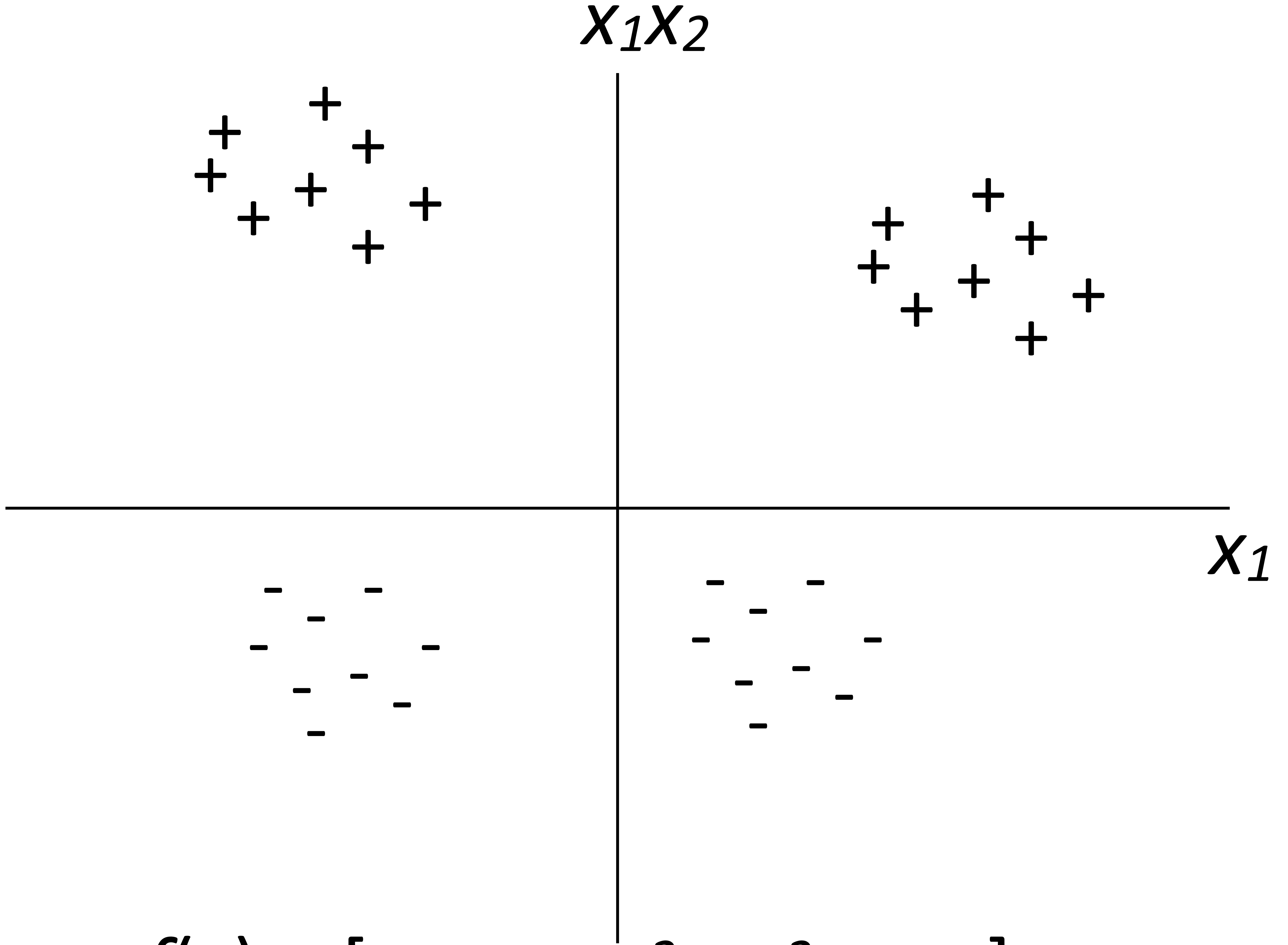

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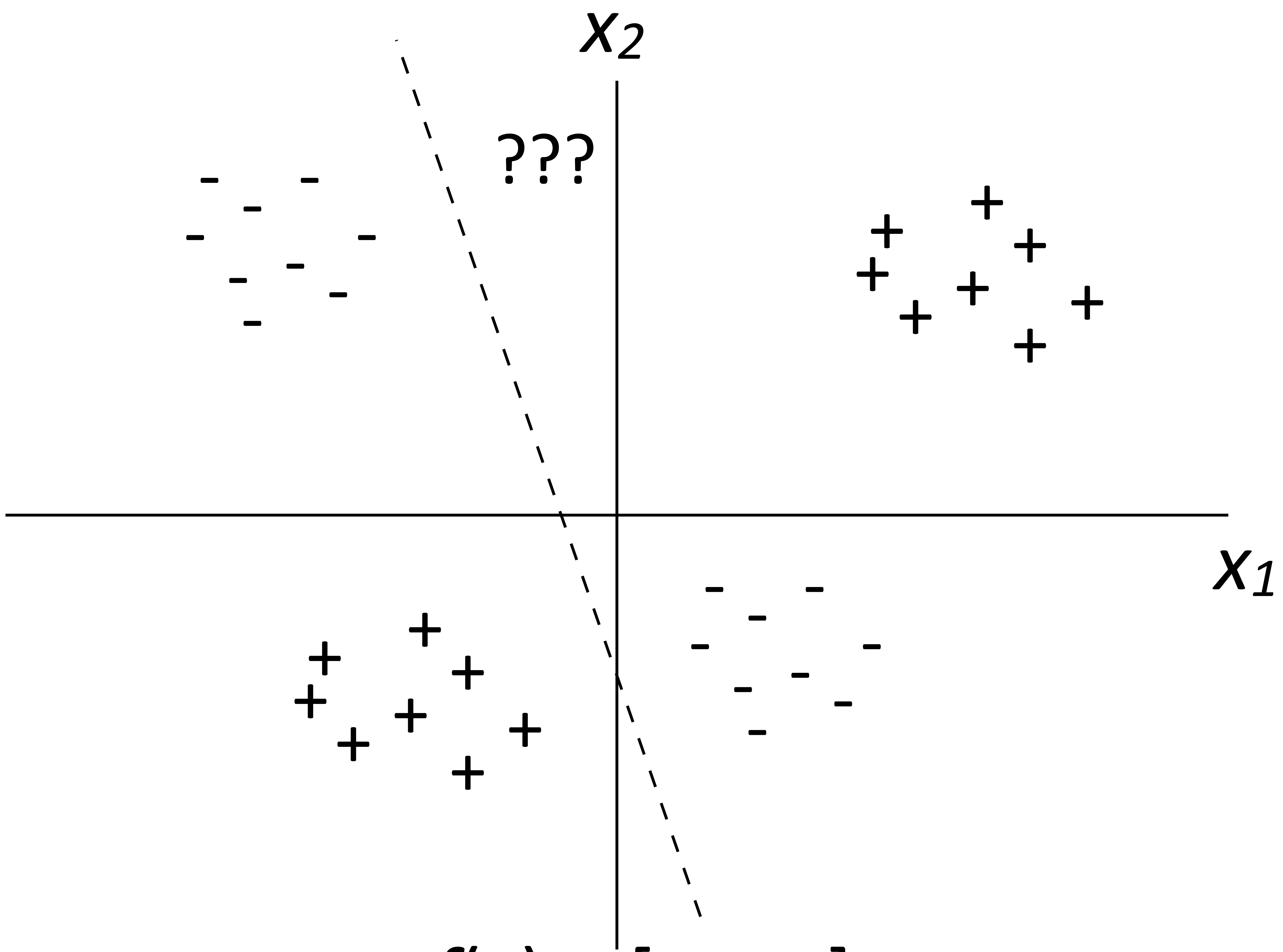

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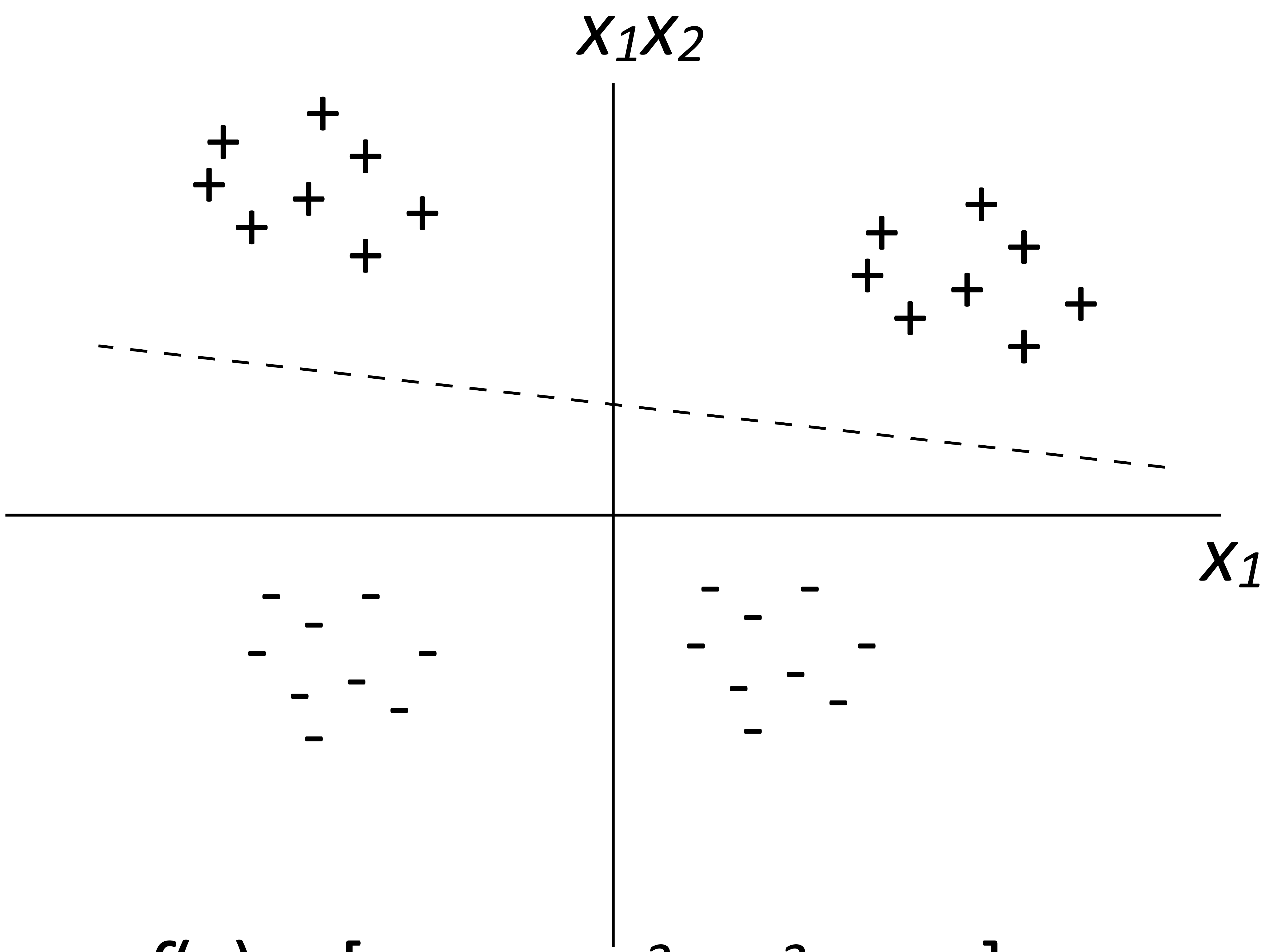

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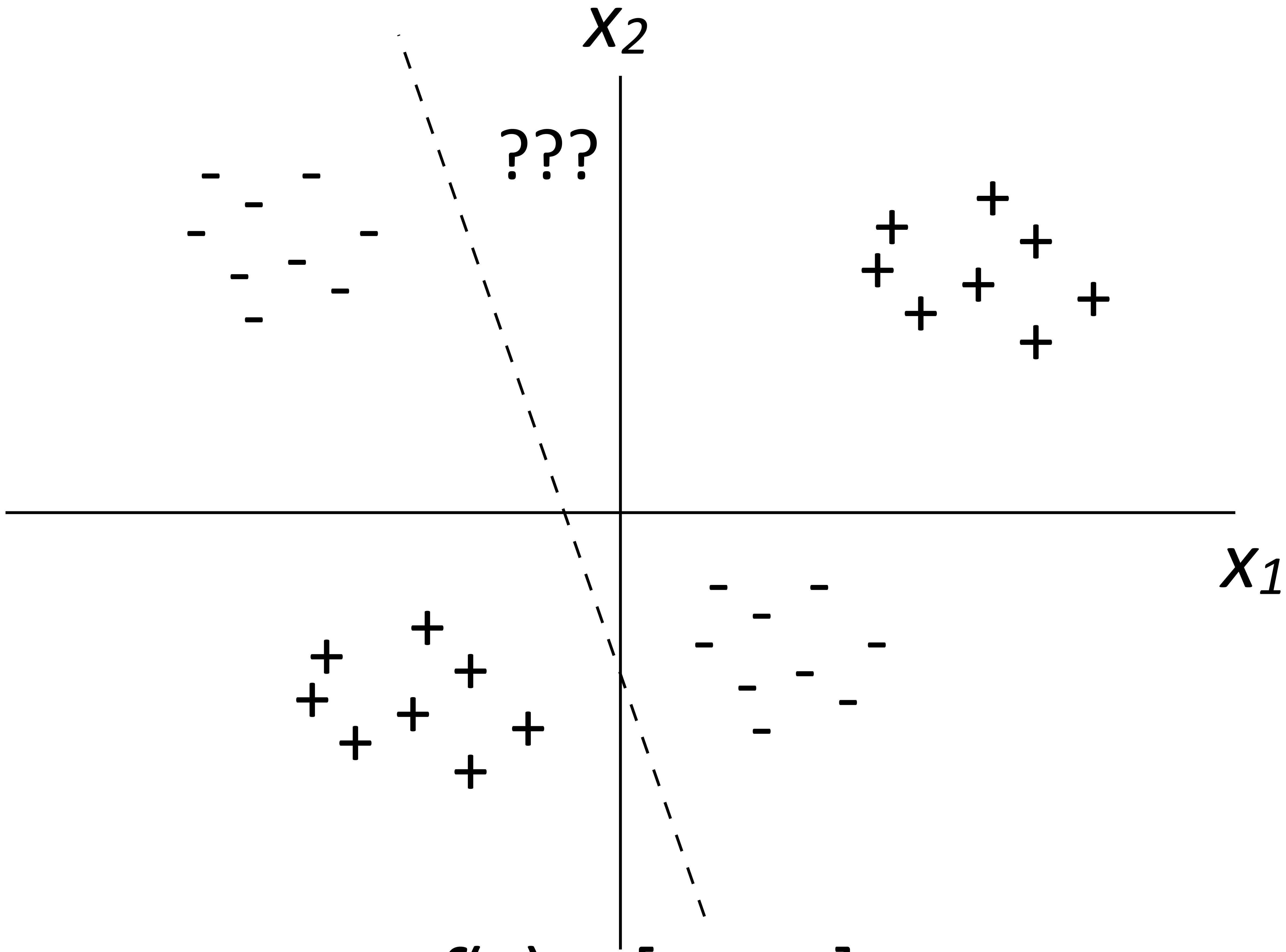

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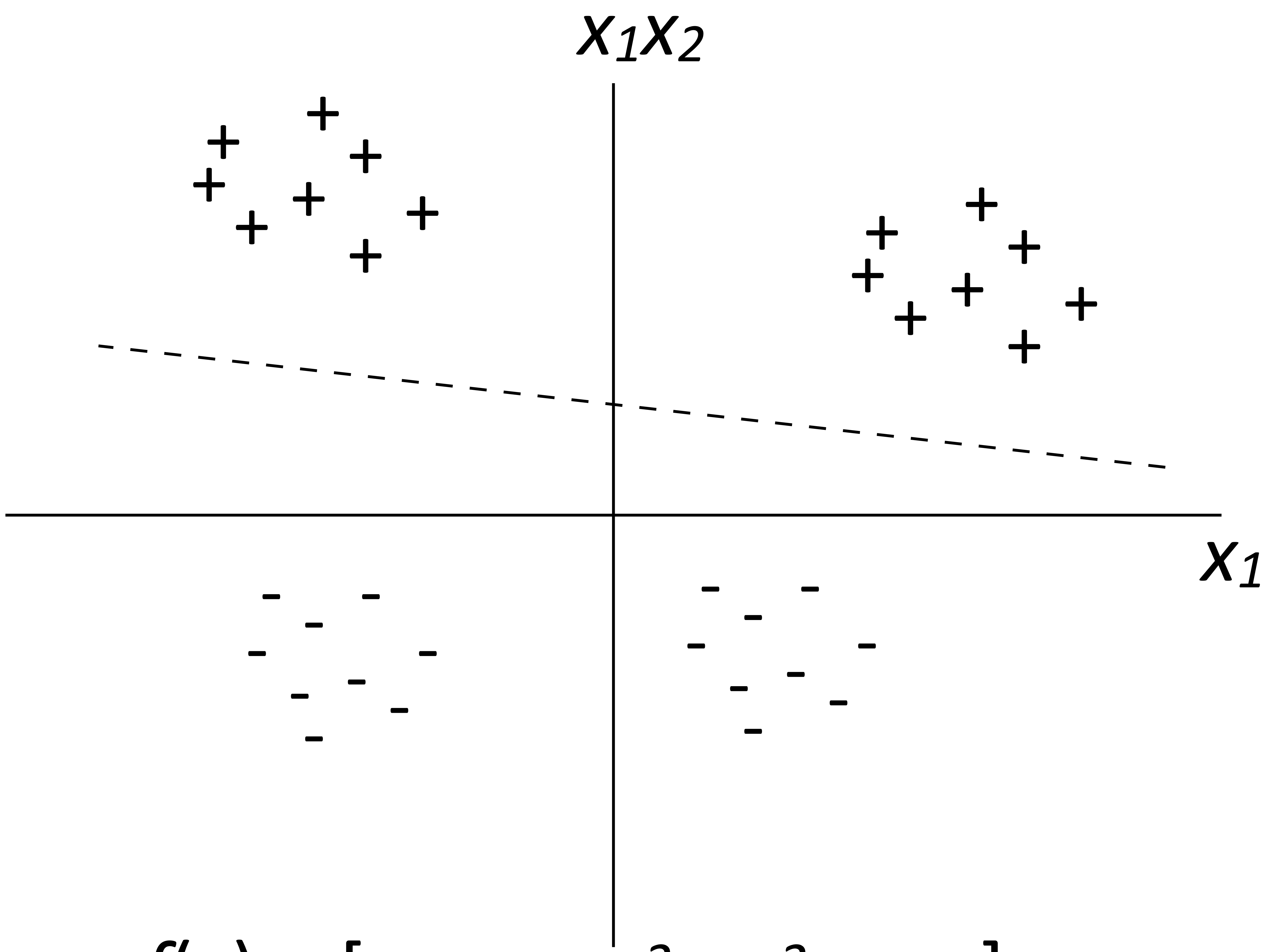

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- "Kernel trick" does this for "free," but is too expensive to use in NLP applications, training is $O(n^2)$ instead of $O(n \cdot (\text{num feats}))$

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  - Train weights on data to get our classifier

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- Convert this example to a vector using *bag-of-words features*

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- More sophisticated feature mappings possible (tf-idf), as well as lots of other features: character n-grams, parts of speech, lemmas, ...

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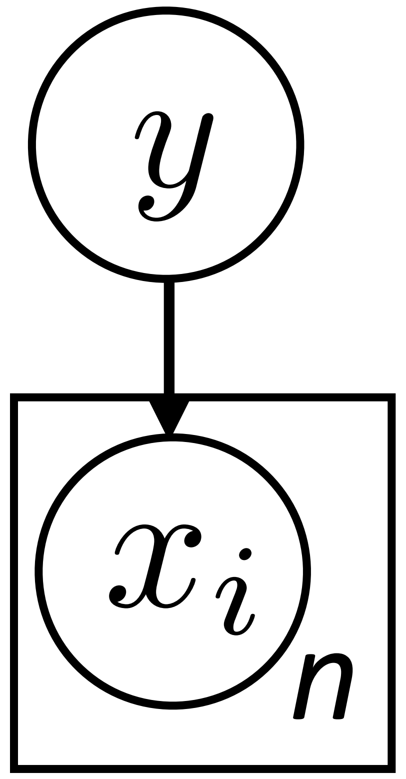

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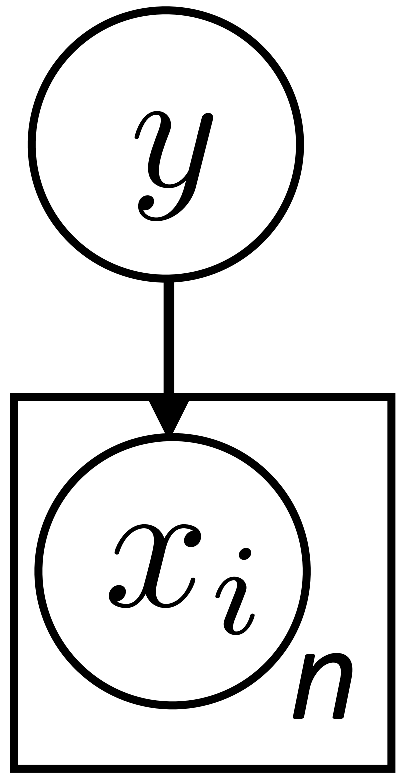

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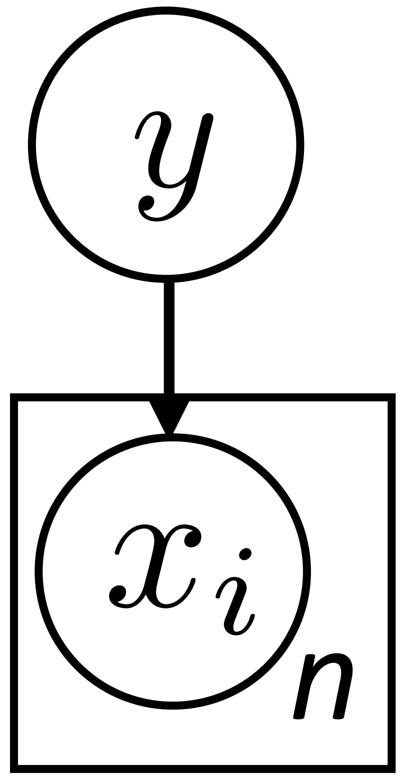

linear model!

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Naive Bayes Example

it was great $\longrightarrow$ $P(y|x) \propto \Big[\qquad \Big]$

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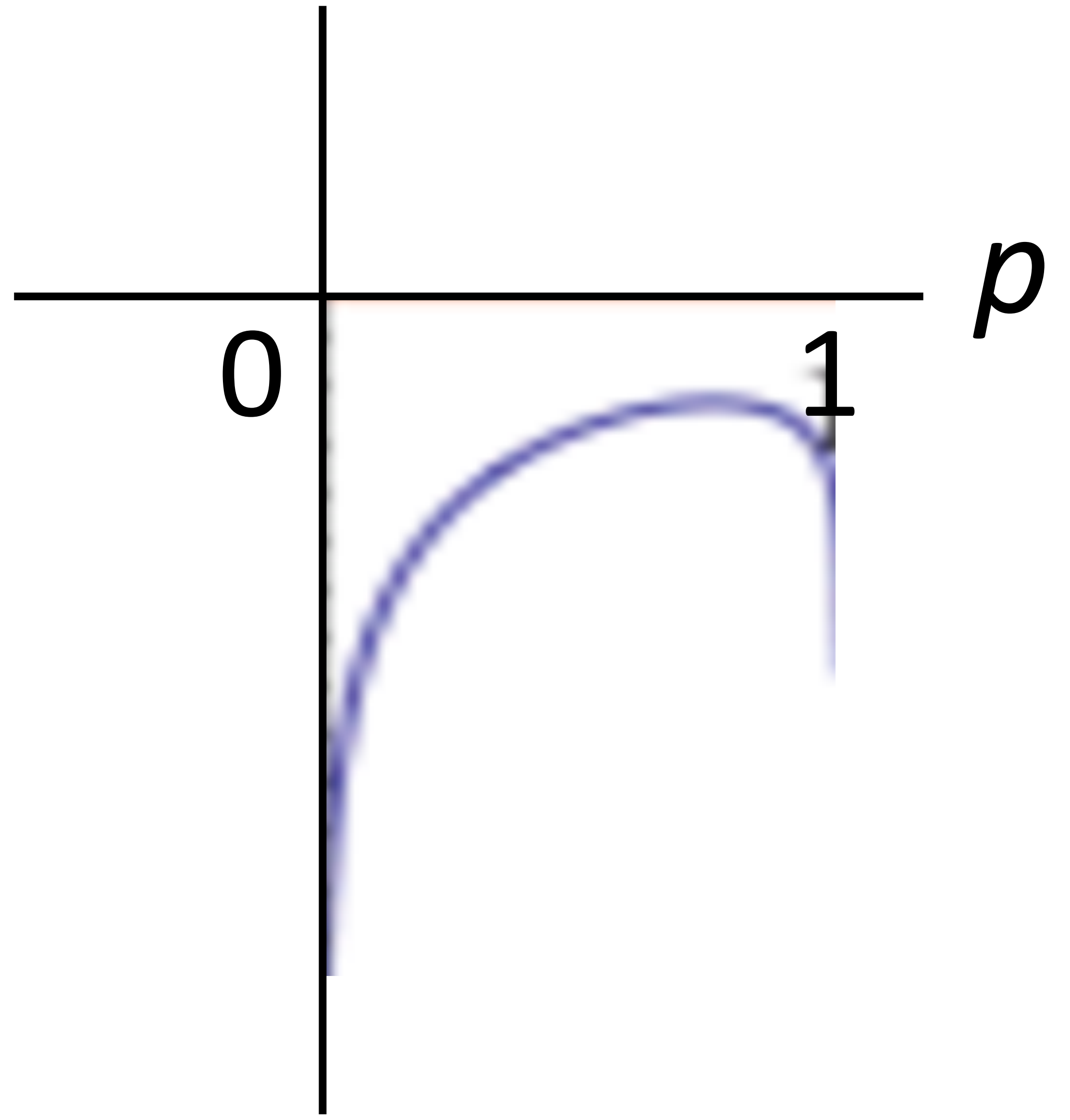

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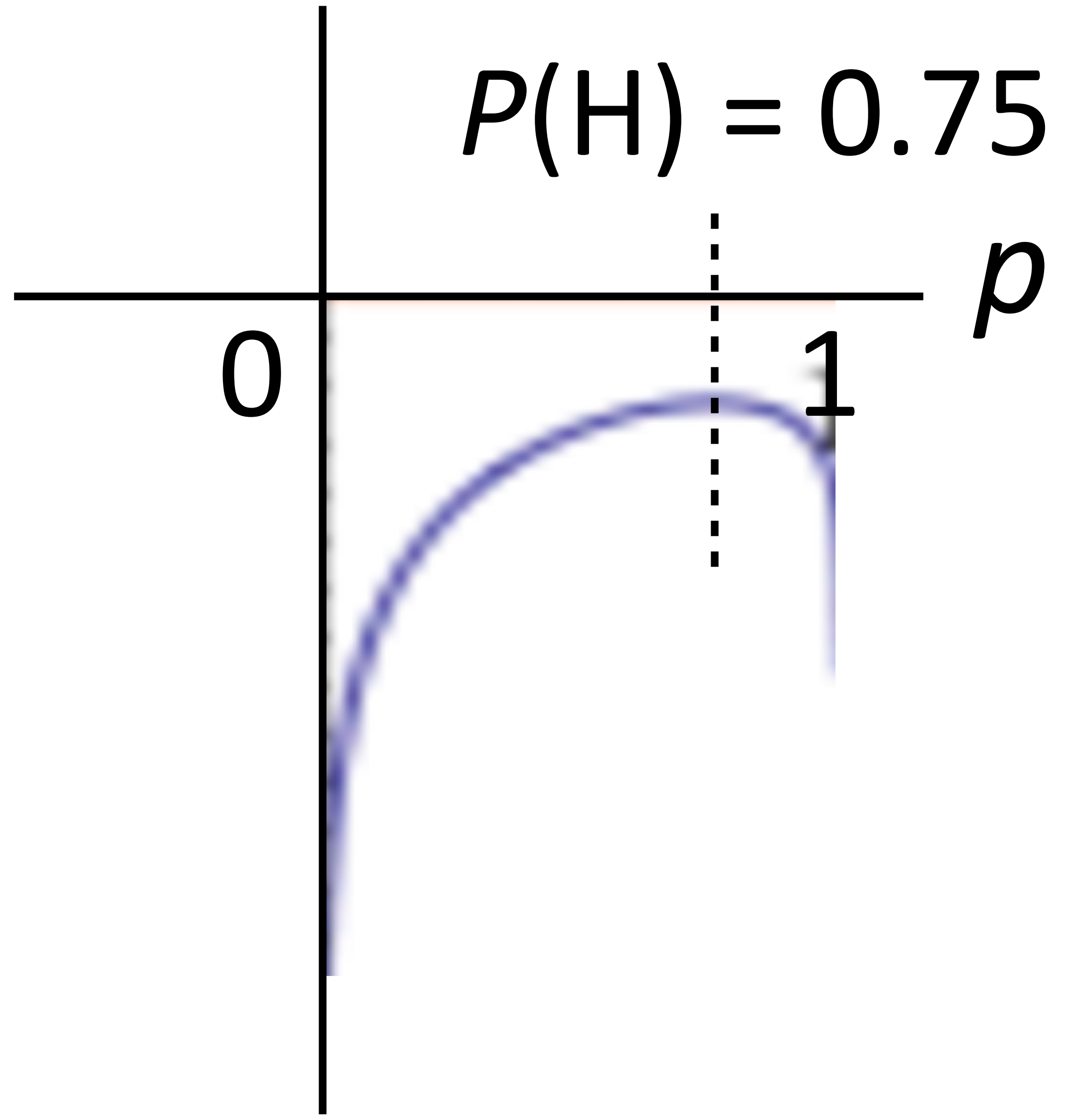

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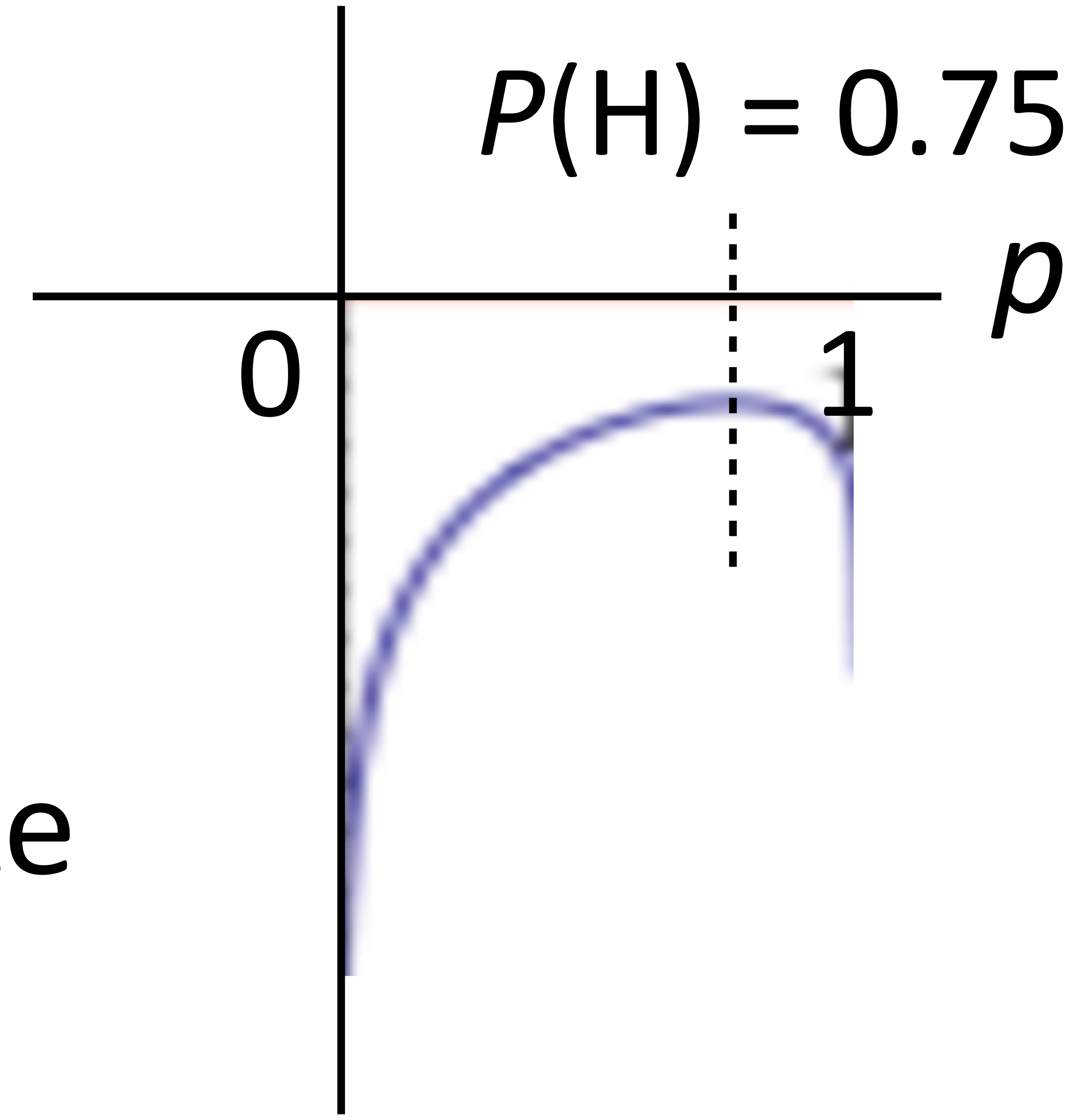

- Maximum likelihood parameters for binomial/ multinomial = read counts off of the data + normalize

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data points (*j*) features (*i*) *i*th feature of *j*th example

- Equivalent to maximizing logarithm of data likelihood:

$$\sum_{j=1}^{m} \log P(y_j, x_j) = \sum_{j=1}^{m} \left[\log P(y_j) + \sum_{i=1}^{n} \log P(x_{ji}|y_j) \right]$$

Maximum Likelihood for Naive Bayes

this movie was great! would watch again	+
I liked it well enough for an action flick	+
I expected a great film and left happy	+
brilliant directing and stunning visuals	+
that film was awful, I'll never watch again	—
I didn't really like that movie	—
dry and a bit distasteful, it misses the mark	—
great potential but ended up being a flop	—

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it was great

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it was great $\longrightarrow$ $P(y|x) \propto \begin{bmatrix} P(+)P(\text{great}|+) \\ P(-)P(\text{great}|-) \end{bmatrix}$

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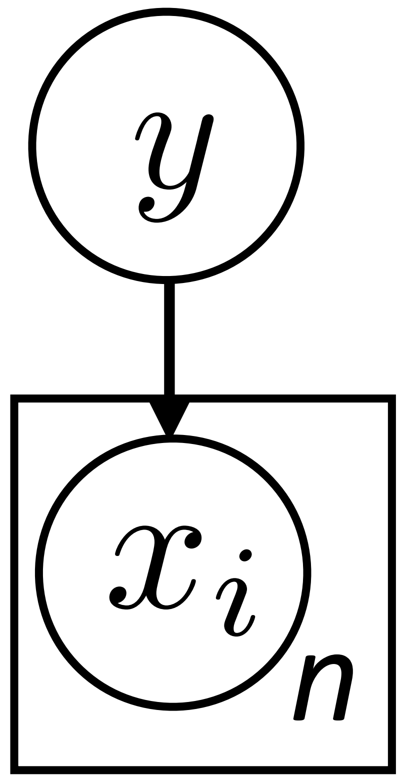
Naive Bayes: Summary

- Model

$$P(x, y) = P(y) \prod_{i=1}^{n} P(x_i | y)$$



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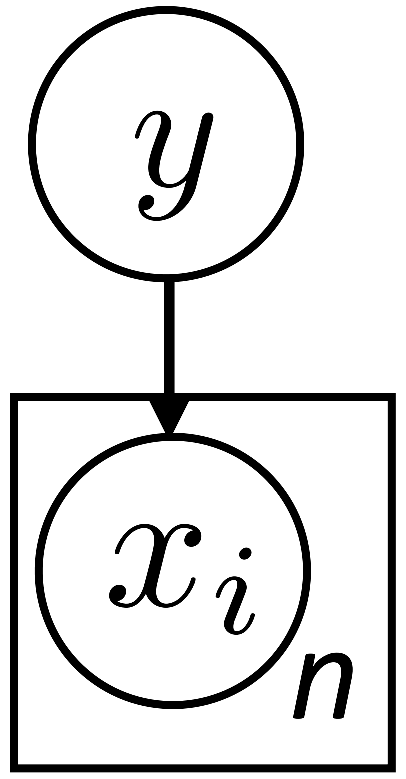

- Inference

$$\text{argmax}_y \log P(y|x) = \text{argmax}_y \left[\log P(y) + \sum_{i=1}^{n} \log P(x_i|y) \right]$$

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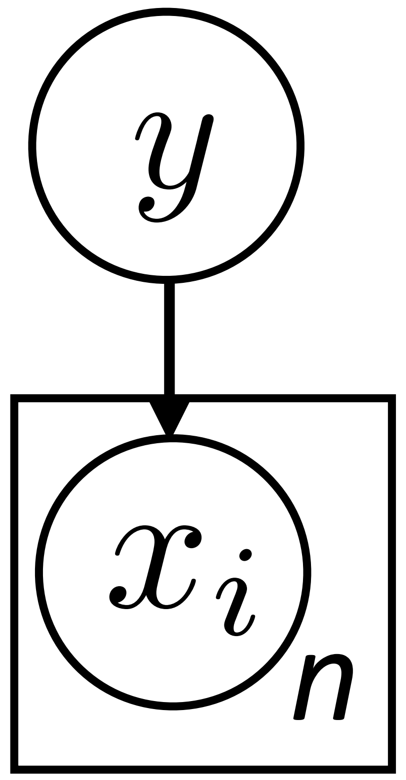

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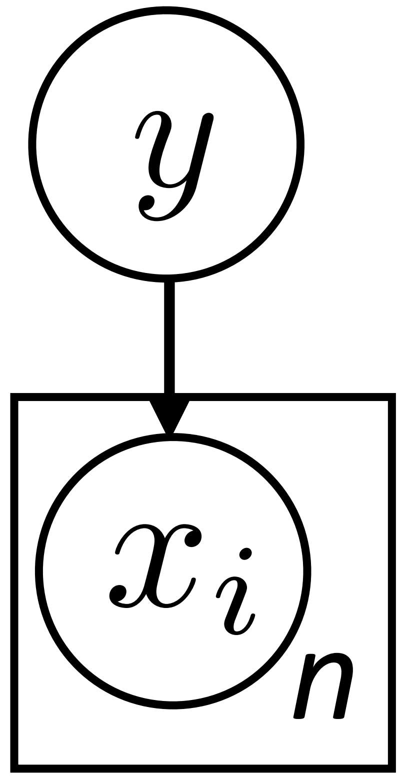

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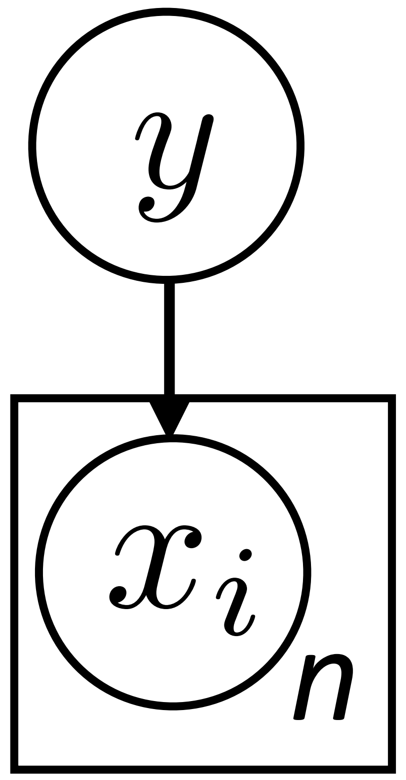

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- Learning: maximize $P(x, y)$ by reading counts off the data

Problems with Naive Bayes

the film was beautiful, stunning cinematography and gorgeous sets, but boring —

$$P(x_{\text{beautiful}}|+) = 0.1 \qquad P(x_{\text{beautiful}}|-) = 0.01$$

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$$P(x_{\text{boring}}|+) = 0.01 \qquad P(x_{\text{boring}}|-) = 0.1$$

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- Naive Bayes is naive, but another problem is that it's *generative*: spends capacity modeling P(x,y), when what we care about is P(y|x)
- Discriminative models model P(y|x) directly (SVMs, most neural networks, …)

Logistic Regression

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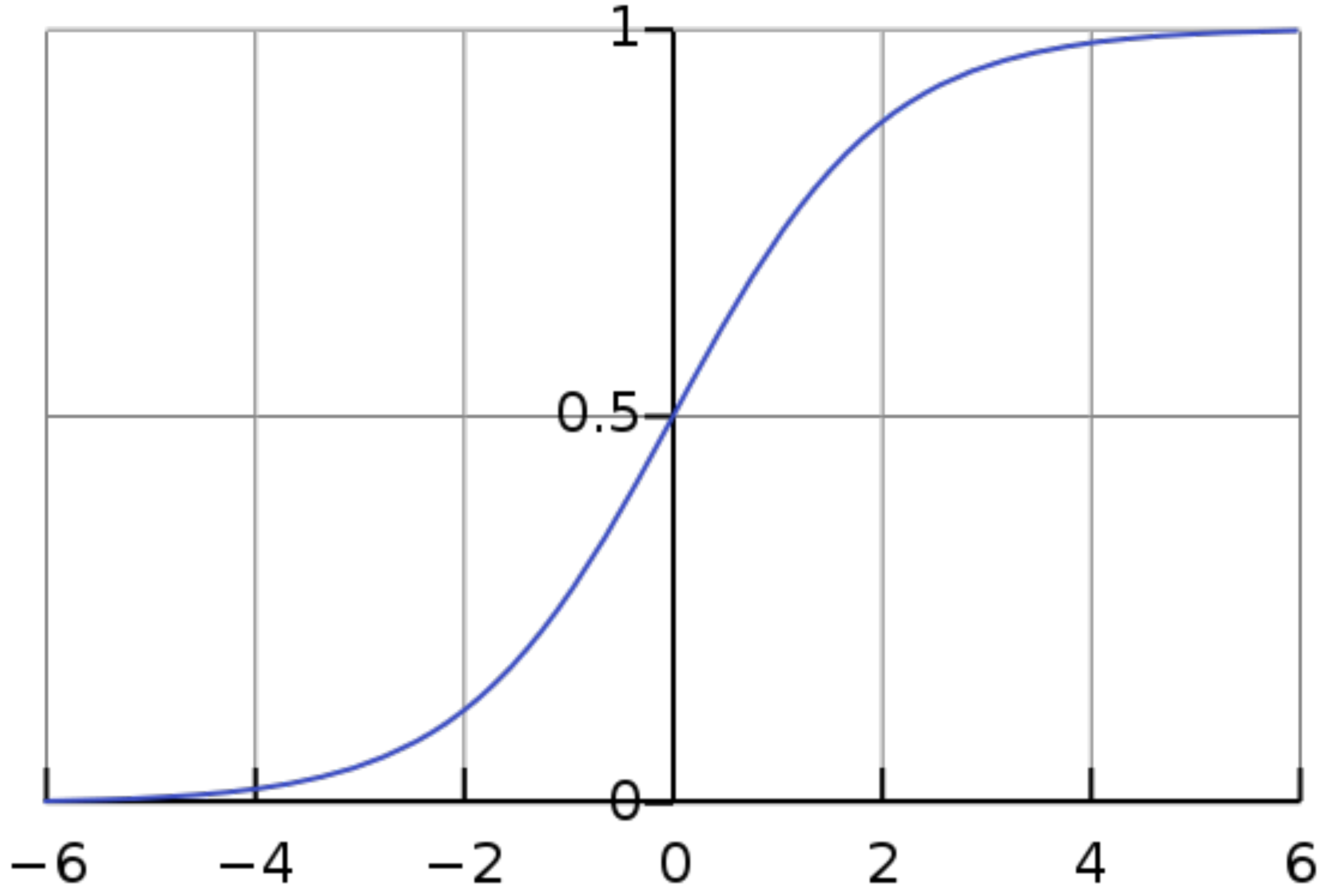

Logistic Regression

$$P(y=+|x) = \text{logistic}(w^\top x)$$

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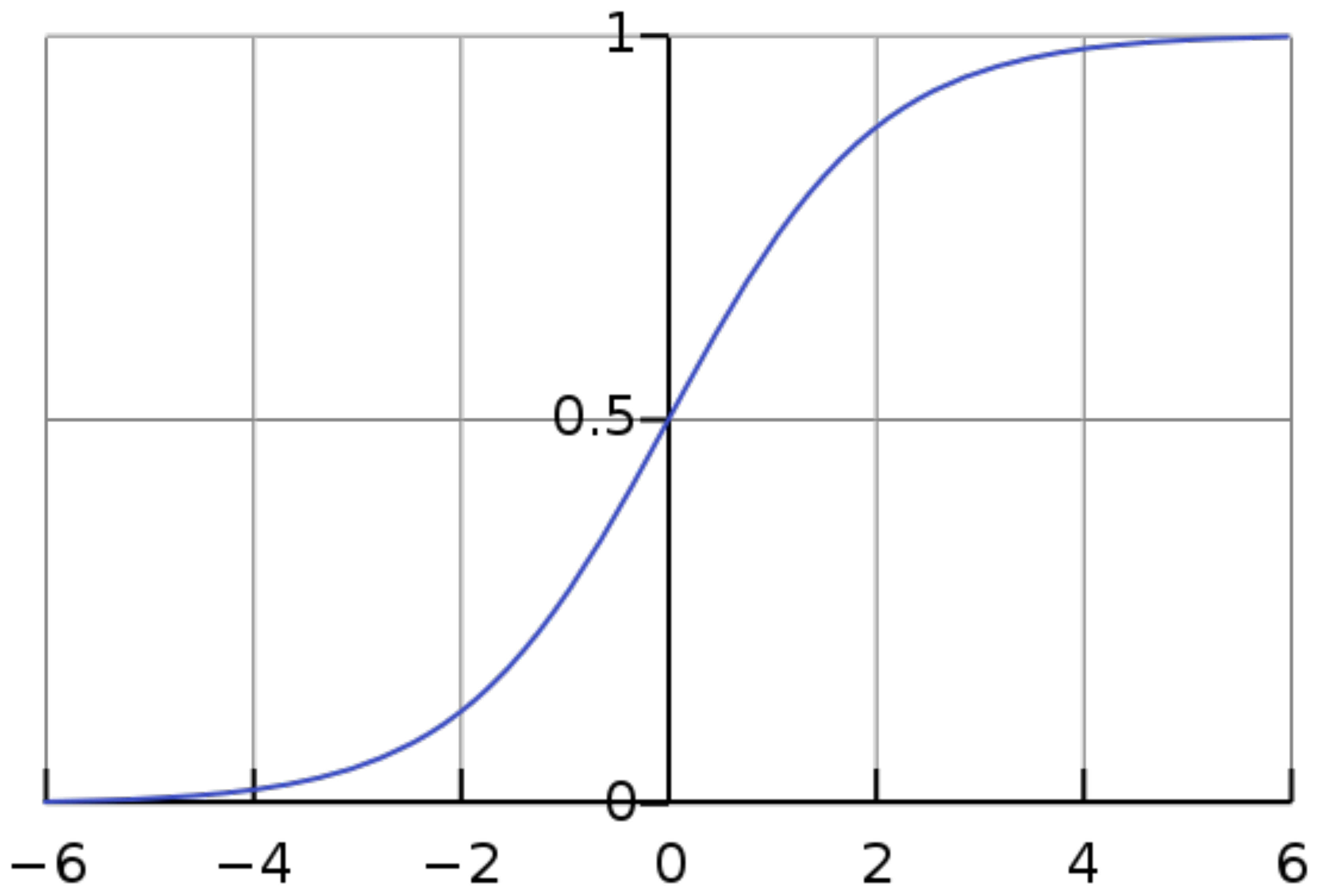

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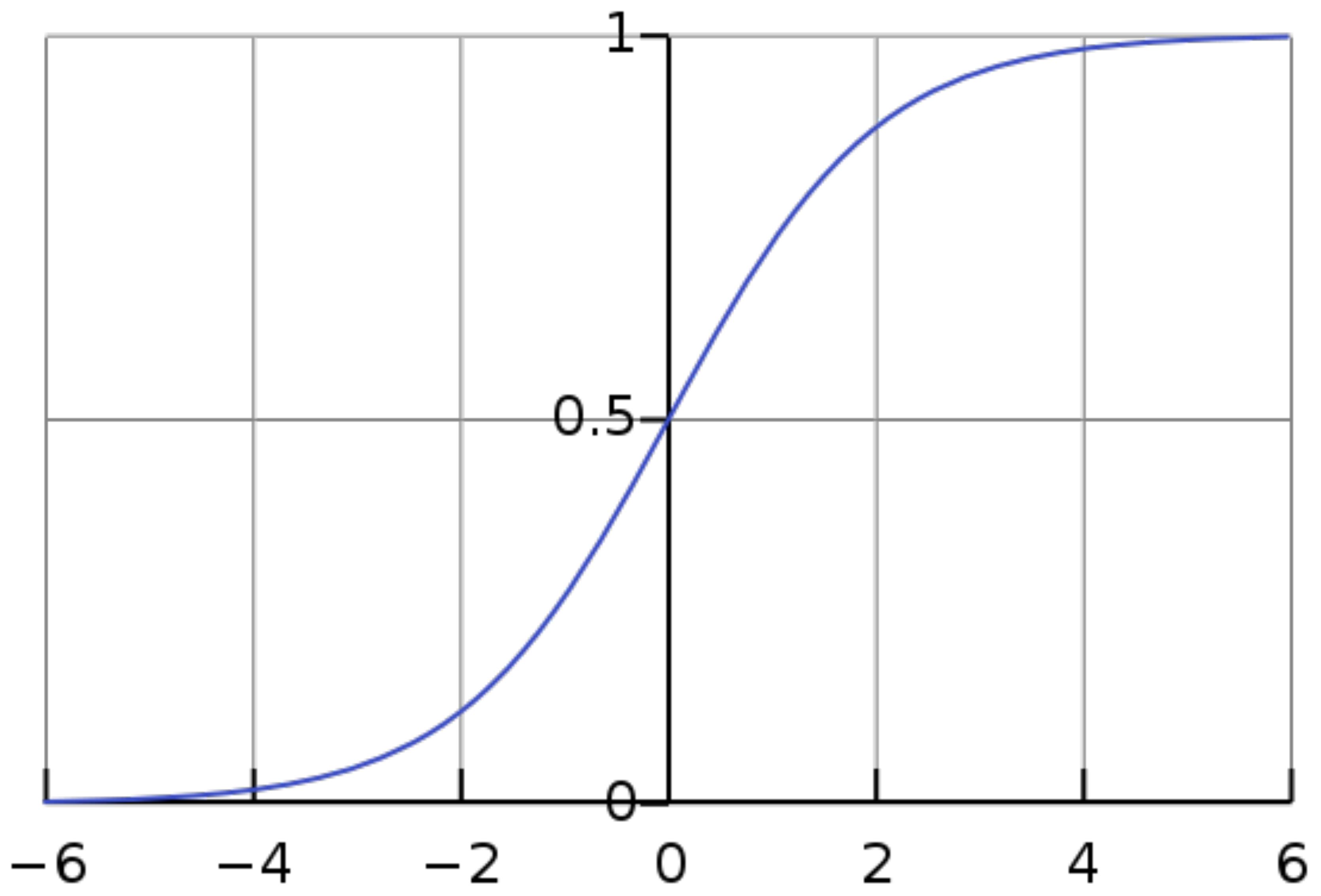

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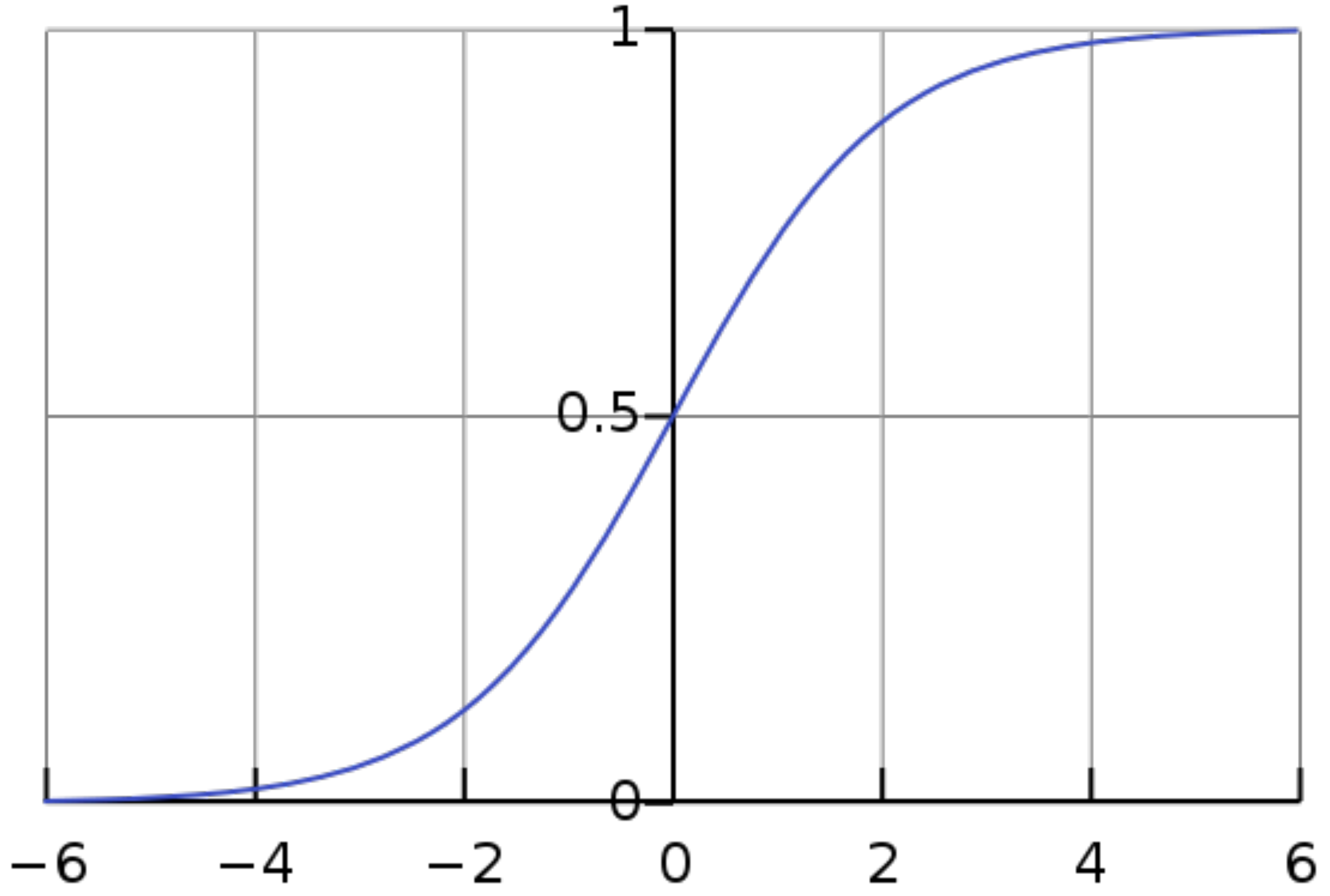

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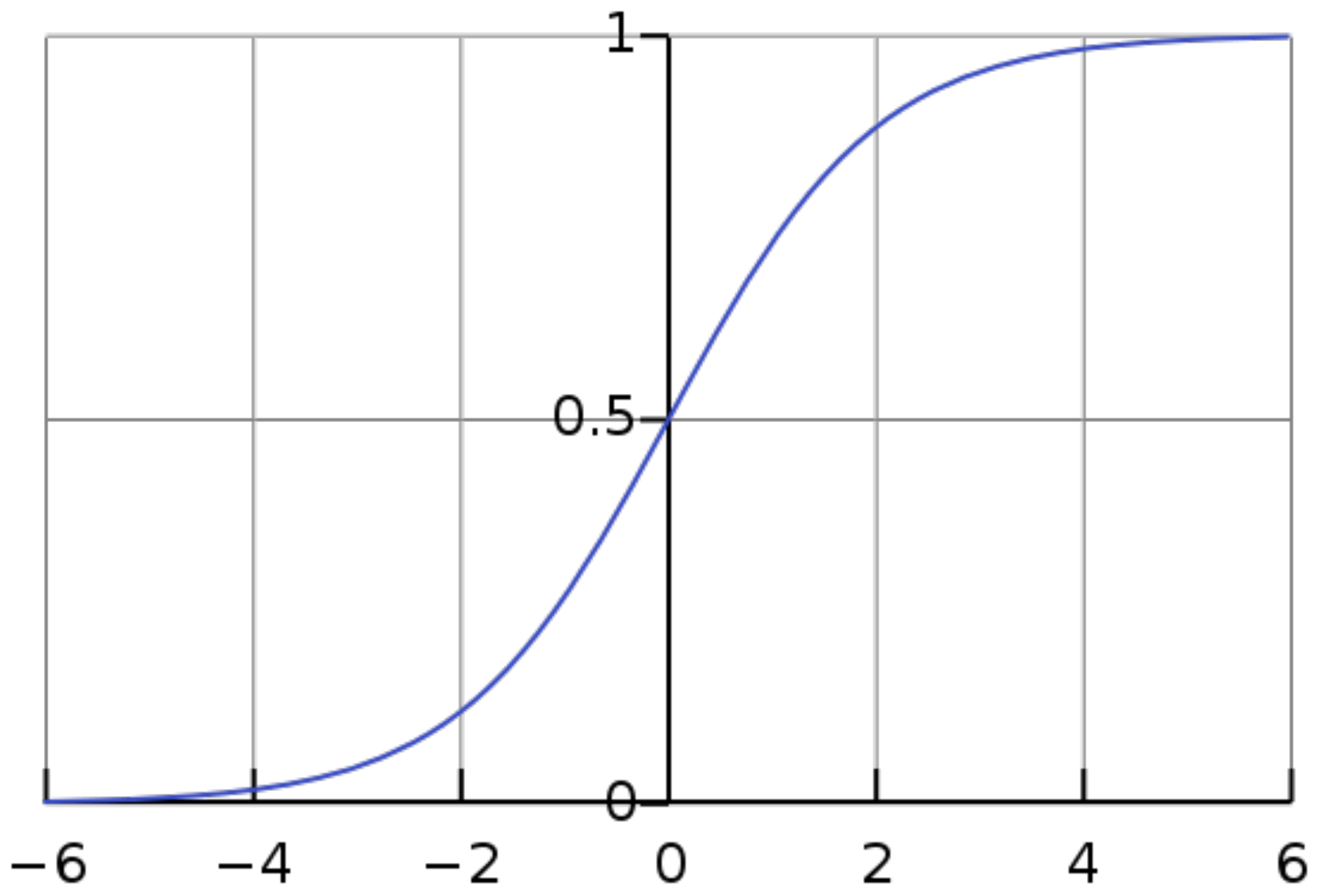

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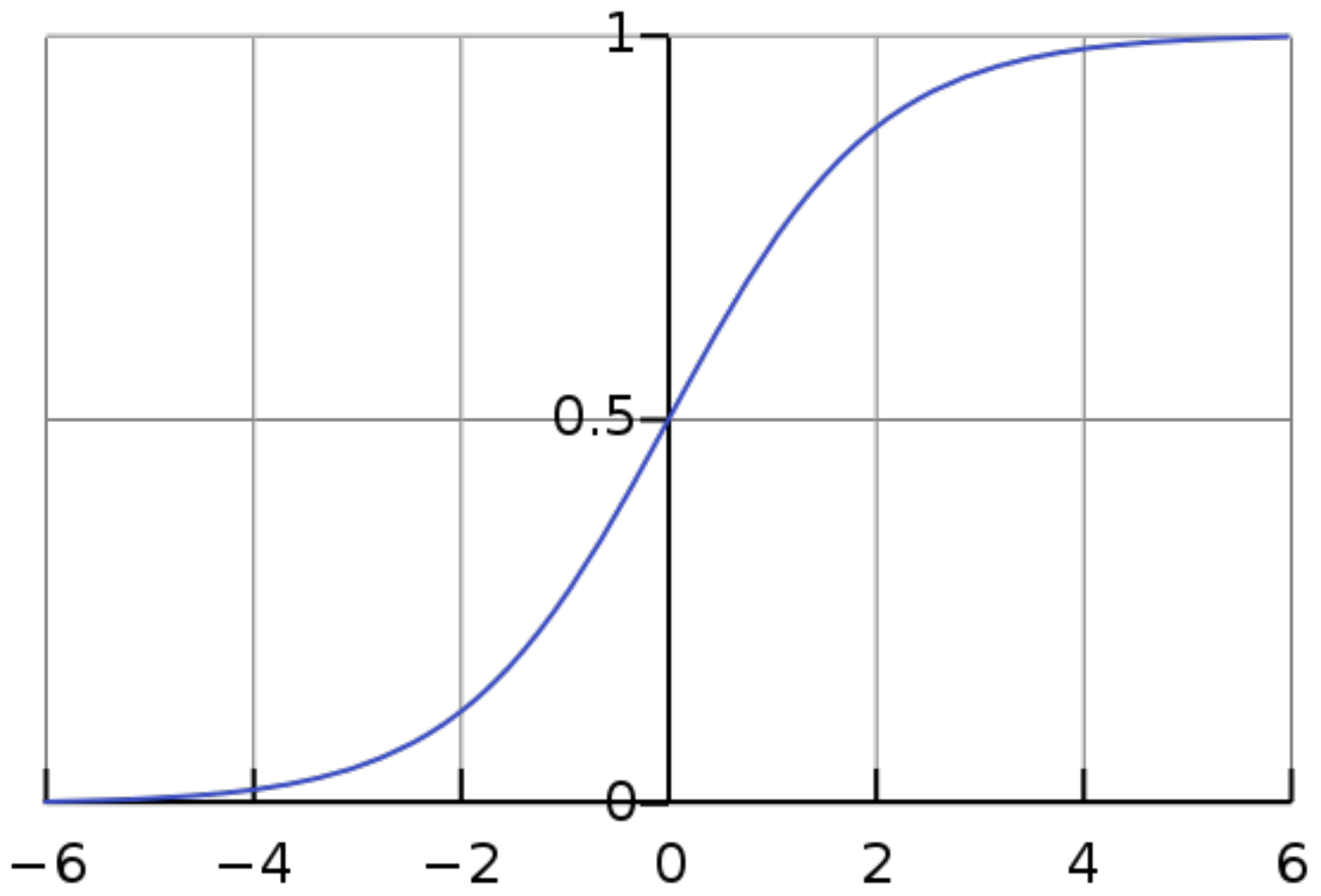

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sum over features

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deriv
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Logistic Regression

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deriv of log

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deriv of exp

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$$= x_{ji} - x_{ji} \frac{\exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)}{1 + \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)} = x_{ji}(1 - P(y_j = +|x_j))$$

Logistic Regression

- Recall that $y_j = 1$ for positive instances, $y_j = 0$ for negative instances.
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- Gradient of w_i on negative example $= x_{ji}(-P(y_j = +|x_j))$

  If P(+) is close to 0, make very little update

  Otherwise make w_i look less like x_{ji}, which will decrease P(+)

- Can combine these gradients as $x_j(y_j - P(y_j = 1|x_j))$

Regularization

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- Regularizing an objective can mean many things, including an L2-norm penalty to the weights:

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  - Early stopping
  - Large numbers of sparse features are hard to overfit in a really bad way

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- Keeping weights small can prevent overfitting
- For most of the NLP models we build, explicit regularization isn't necessary
  - Early stopping
  - Large numbers of sparse features are hard to overfit in a really bad way
  - For neural networks: dropout and gradient clipping

Logistic Regression: Summary

- Model

$$P(y=+|x) = \frac{\exp(\sum_{i=1}^{n} w_i x_i)}{1+\exp(\sum_{i=1}^{n} w_i x_i)}$$

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- Learning: gradient ascent on the (regularized) discriminative log-likelihood

Perceptron/SVM

Perceptron

- Invented in 1958
  - By Frank Rosenblatt
  - At the Cornell Aeronautical Laboratory

- Implemented in custom-built hardware

- Connected to a camera with 20×20 cadmium sulfide photocells to make a 400-pixel image.

- Weights were encoded in potentiometers, and weight updates during learning were performed by electric motors.

Source: Wikipedia

Perceptron

- Simple error-driven learning approach similar to logistic regression

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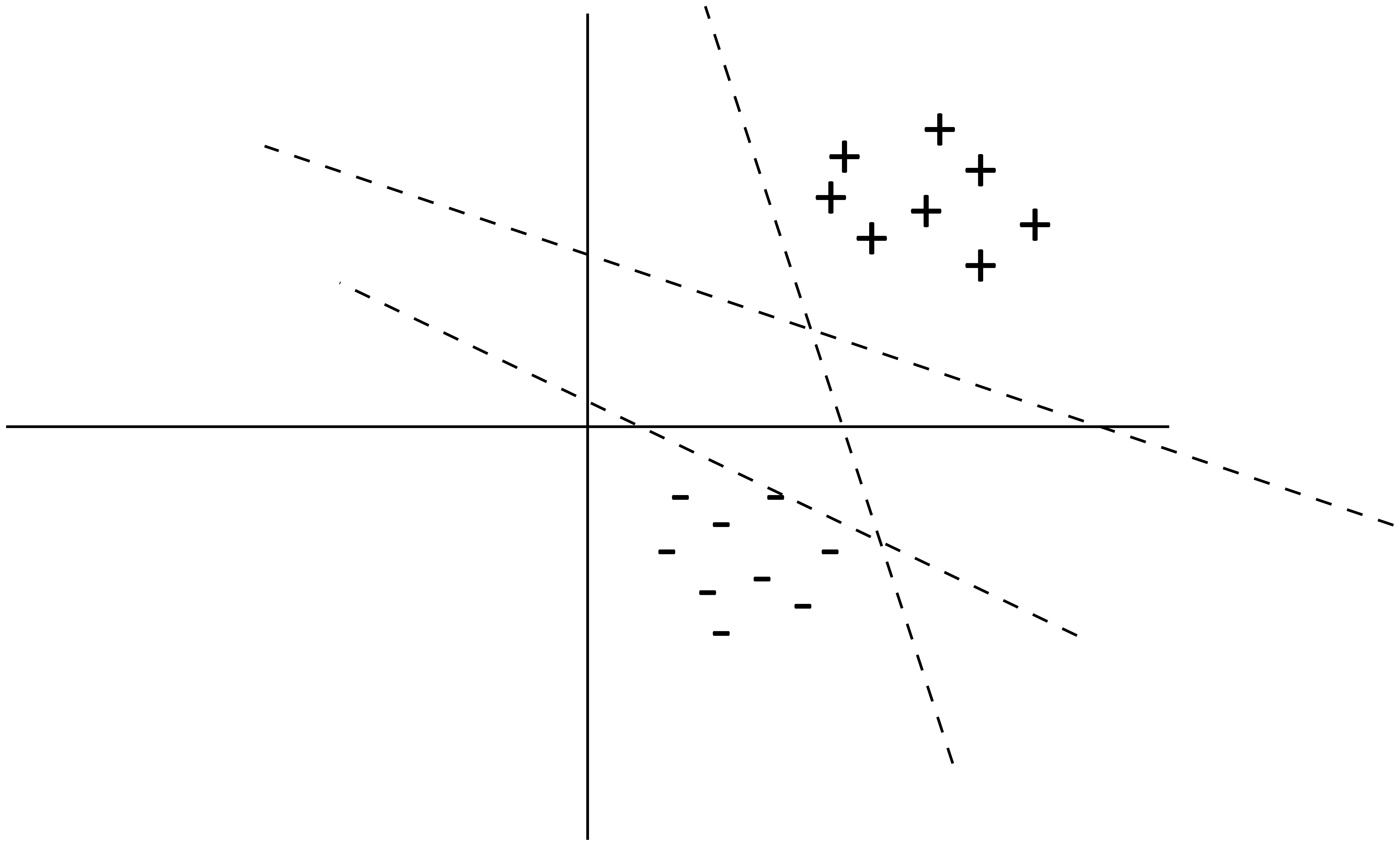
- Guaranteed to eventually separate the data if the data are separable

Support Vector Machines

- Many separating hyperplanes — is there a best one?



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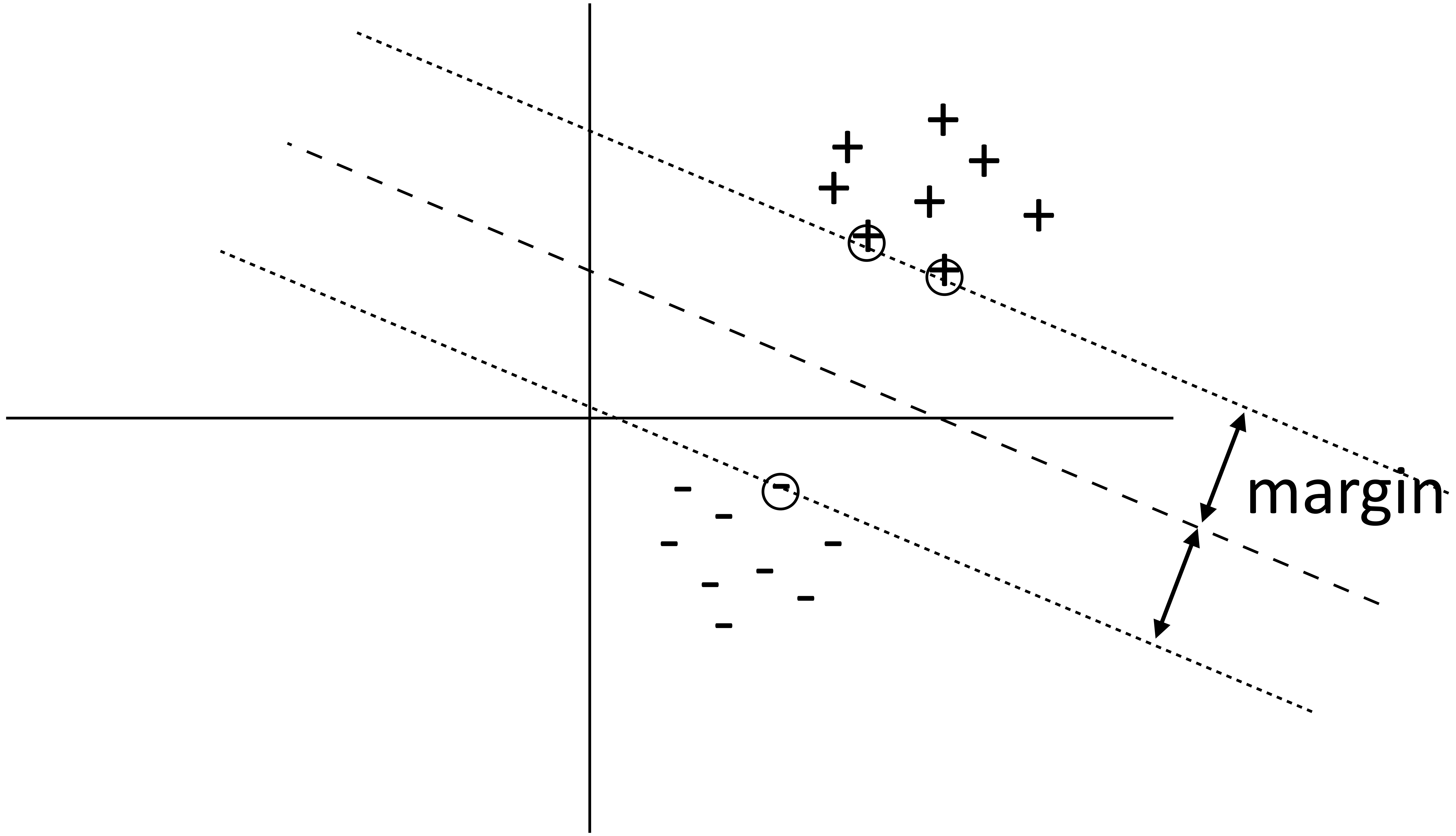

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- Generally no solution (data is generally non-separable) — need slack!

N-Slack SVMs

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- Looks like the perceptron! But updates more frequently

Gradients on Positive Examples

Logistic regression

$$x(1 - \text{logistic}(w^\top x))$$

Perceptron

$$x \text{ if } w^\top x < 0, \text{ else } 0$$

SVM (ignoring regularizer)

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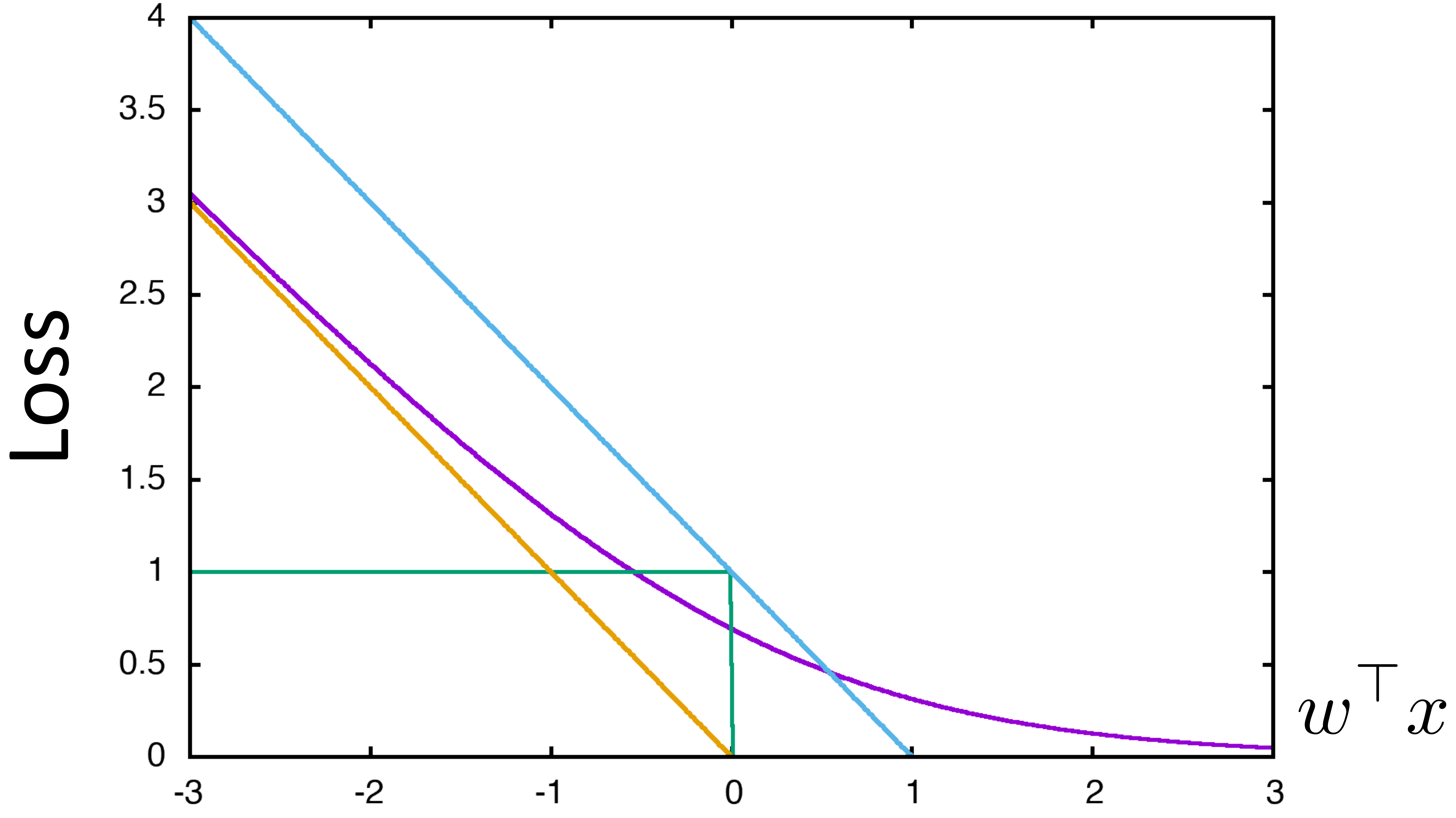

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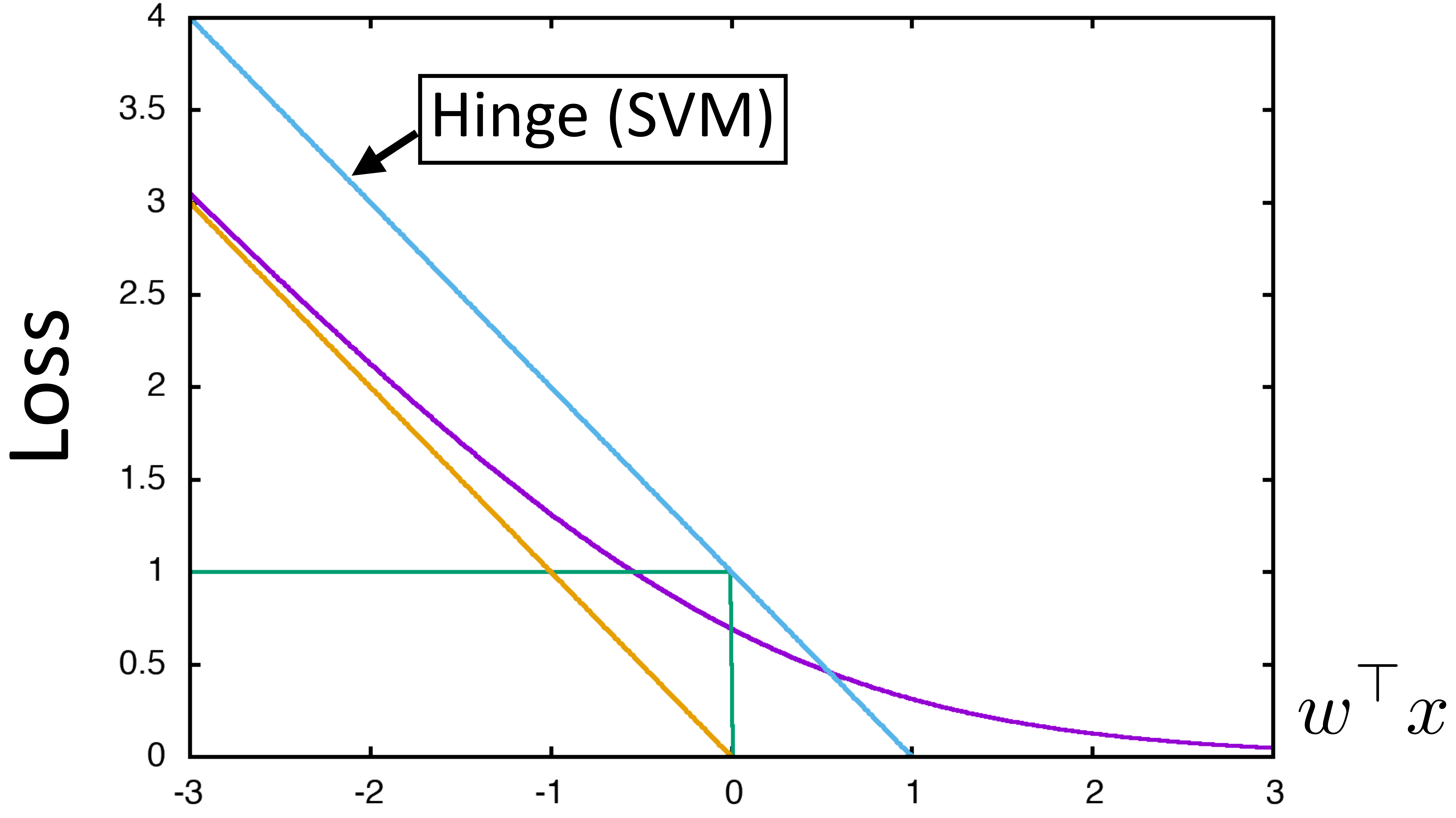

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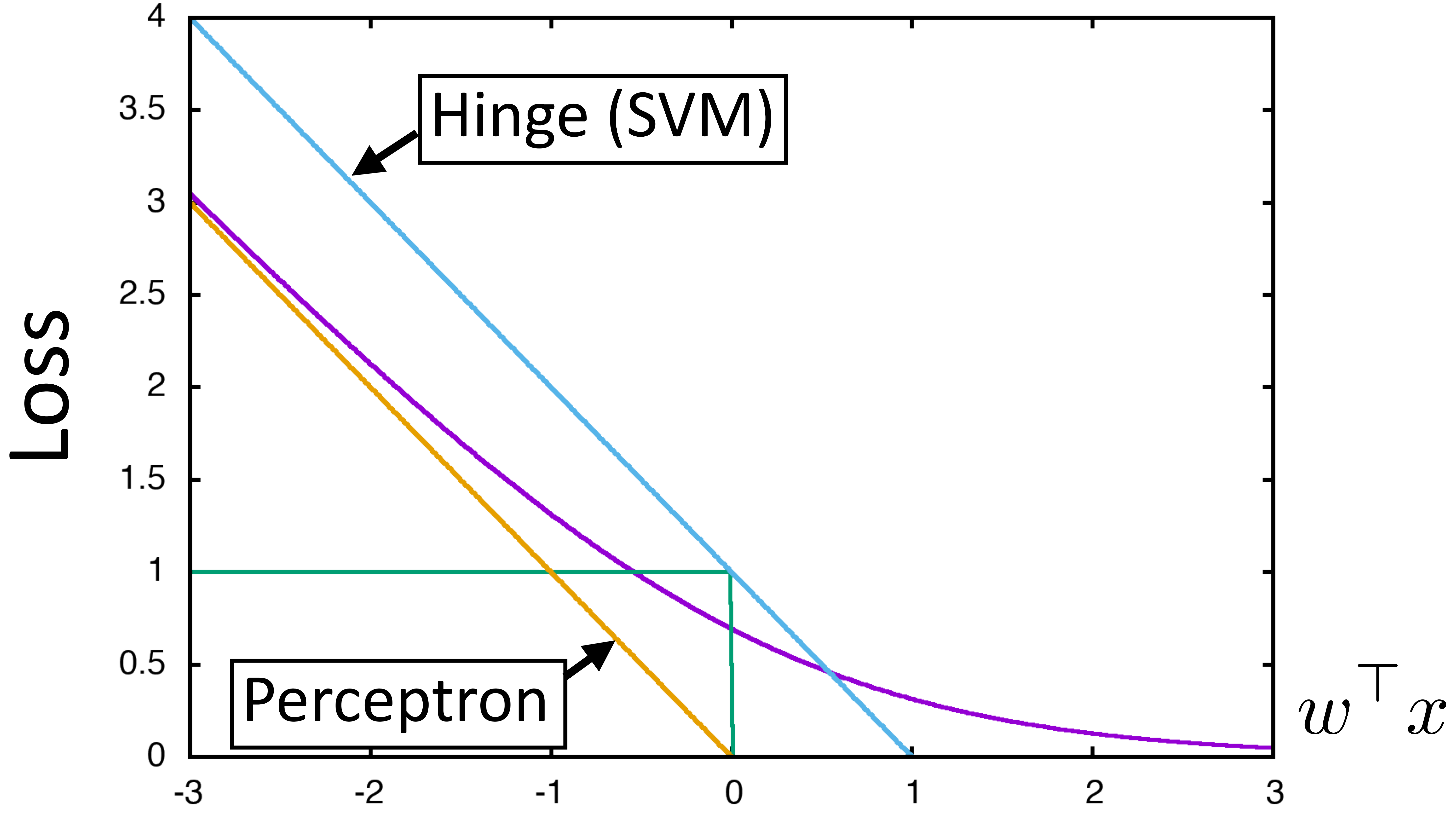

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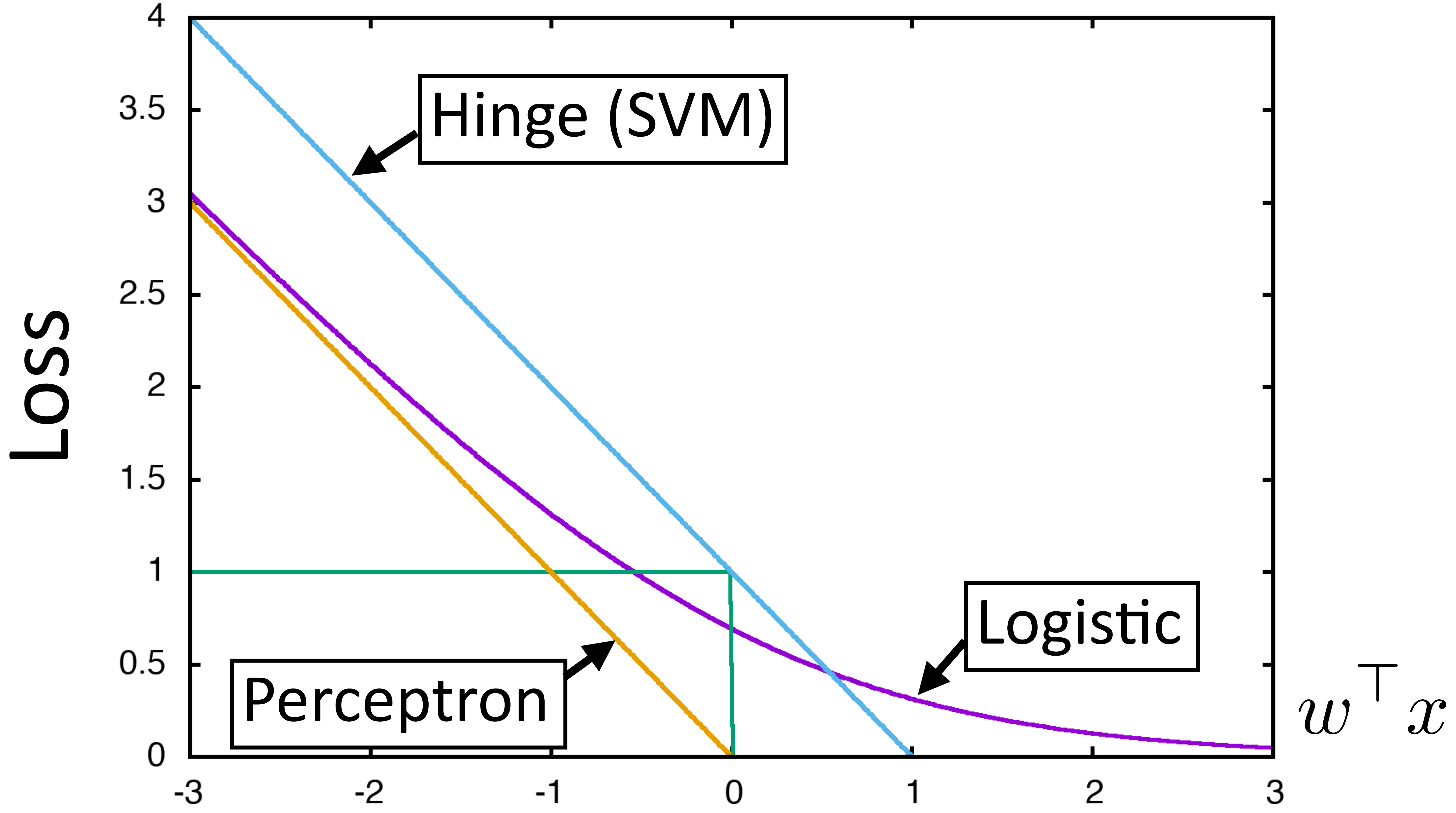

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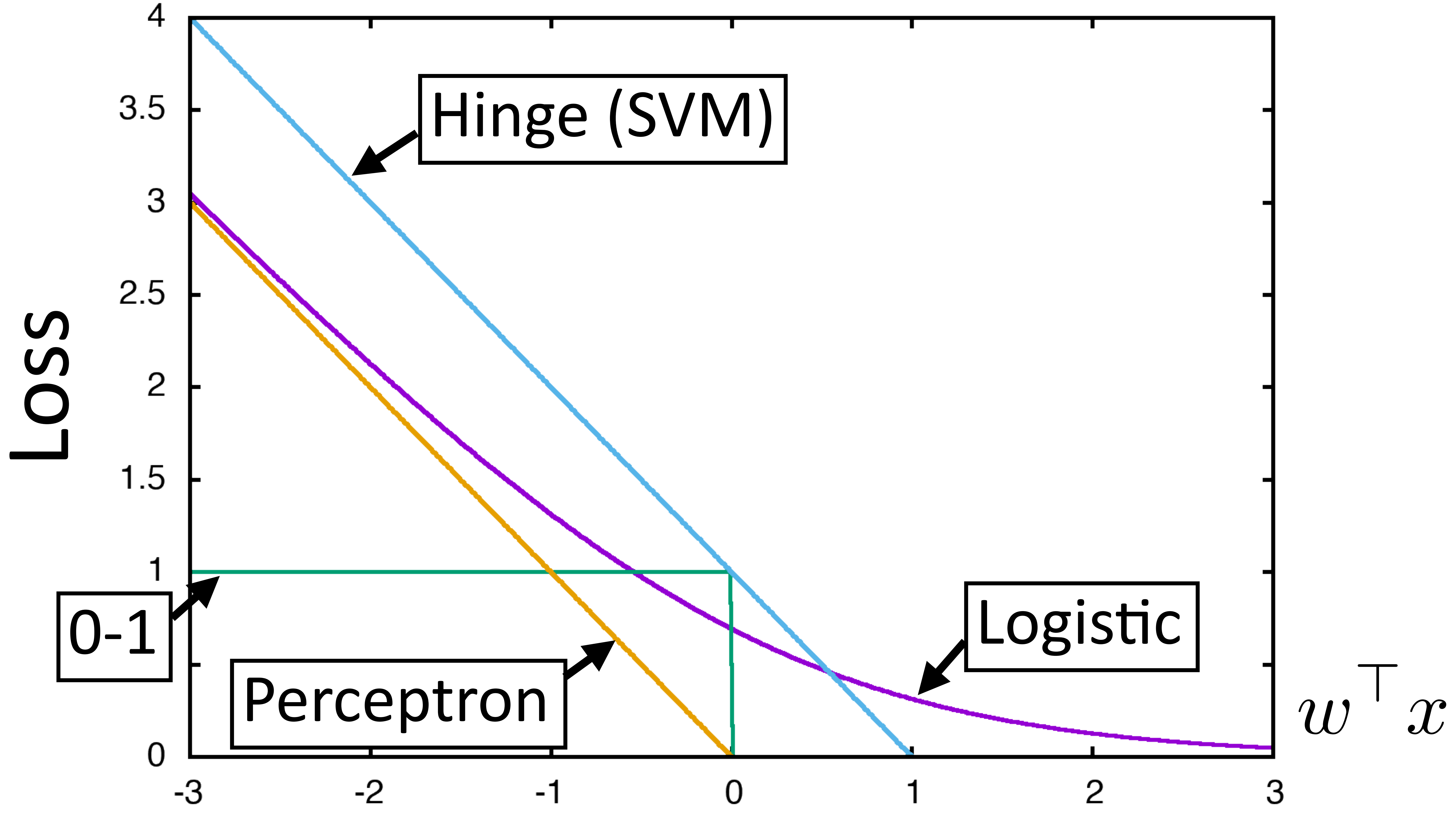

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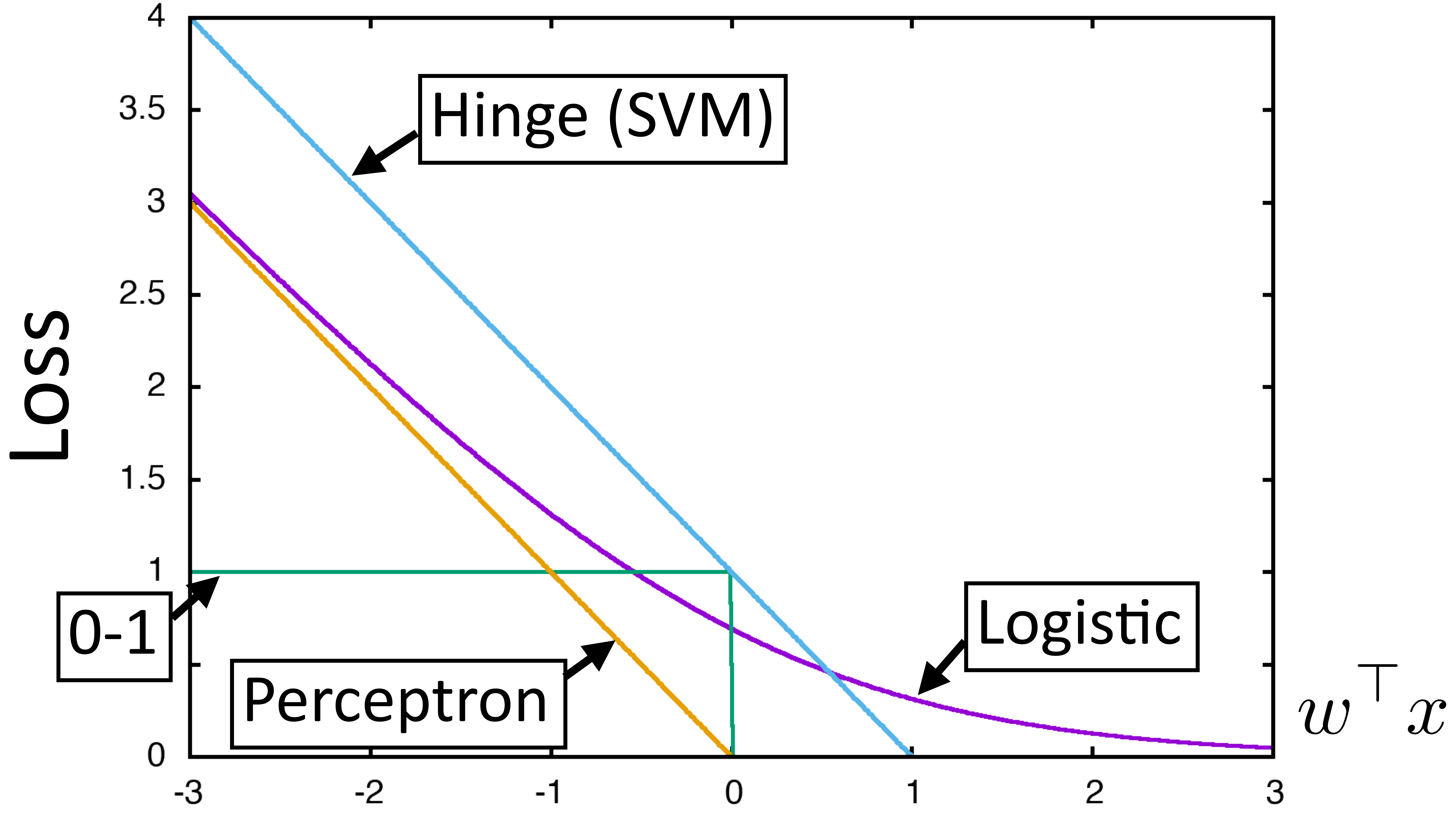

*gradients are for maximizing things, which is why they are flipped

Comparing Gradient Updates (Reference)

Logistic regression (unregularized)

$x(y - P(y = 1|x))$ = $x(y - \mathrm{logistic}(w^\top x))$

y = 1 for pos,
0 for neg

Perceptron

$(2y - 1)x$ if classified incorrectly

0 else

SVM

$(2y - 1)x$ if not classified correctly with margin of 1

0 else

Optimization — next time…

- Range of techniques from simple gradient descent (works pretty well) to more complex methods (can work better)
- Most methods boil down to: take a gradient and a step size, apply the gradient update times step size, incorporate estimated curvature information to make the update more effective

Sentiment Analysis

this movie was great! would watch again   +

Bo Pang, Lillian Lee, Shivakumar Vaithyanathan (2002)

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this movie was great! would watch again	+
the movie was gross and overwrought, but I liked it	+
this movie was not really very enjoyable	—

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this movie was great! would watch again | +

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- Bag-of-words doesn't seem sufficient (discourse structure, negation)
- There are some ways around this: extract bigram feature for "*not* X" for all X following the *not*

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Sentiment Analysis

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	Features	# of features	frequency or presence?	NB	ME	SVM
(1)	unigrams	16165	freq.	**78.7**	N/A	72.8
(2)	unigrams	"	pres.	81.0	80.4	**82.9**
(3)	unigrams+bigrams	32330	pres.	80.6	80.8	**82.7**
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- Simple feature sets can do pretty well!

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Sentiment Analysis

Wang and Manning (2012)

Sentiment Analysis

Method	RT-s	MPQA
MNB-uni	77.9	85.3
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SVM-uni	76.2	86.1
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Kim (2014) CNNs	**81.5**	**89.5**

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- SVM:

  Decision rule: $w^\top x \geq 0$

  (Sub)gradient (unregularized): 0 if correct with margin of 1, else $x(2y-1)$

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- SVM and perceptron inference require taking maxes, logistic regression has a similar update but is “softer” due to its probabilistic nature

- All gradient updates: “make it look more like the right thing and less like the wrong thing”