

Lecture 4: Sequence Models I

Alan Ritter

(many slides from Greg Durrett, Dan Klein, Vivek Srikumar, Chris Manning, Yoav Artzi)

This Lecture

- ▶ Sequence modeling
- ▶ HMMs for POS tagging
- ▶ HMM parameter estimation
- ▶ Viterbi, forward-backward

Linguistic Structures

- ▶ Language is tree-structured

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I ate the spaghetti with chopsticks

Linguistic Structures

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I ate the spaghetti with chopsticks

I ate the spaghetti with meatballs

Linguistic Structures

- ▶ Language is tree-structured



I ate the spaghetti with chopsticks

The diagram illustrates the tree structure of the sentence "I ate the spaghetti with chopsticks". It features four curved arrows above the text: a small black arrow from "I" to "ate", a larger black arrow from "ate" to "spaghetti", a long orange arrow from "ate" to "chopsticks", and a small black arrow from "spaghetti" to "with".

I ate the spaghetti with meatballs

Linguistic Structures

- ▶ Language is tree-structured

I ate the spaghetti with chopsticks



I ate the spaghetti with meatballs



Linguistic Structures

- ▶ Language is tree-structured

I ate the spaghetti with chopsticks

A diagram illustrating shallow syntactic analysis for the sentence "I ate the spaghetti with chopsticks". It features four curved arrows above the words: a black arrow from "I" to "ate", a black arrow from "ate" to "the", a black arrow from "the" to "spaghetti", and a black arrow from "spaghetti" to "with". A single, larger orange arrow spans from "I" to "with", representing a shallow analysis that groups the entire object phrase "the spaghetti with chopsticks" as a single unit.

I ate the spaghetti with meatballs

A diagram illustrating shallow syntactic analysis for the sentence "I ate the spaghetti with meatballs". It features four curved arrows above the words: a black arrow from "I" to "ate", a black arrow from "ate" to "the", a black arrow from "the" to "spaghetti", and a black arrow from "spaghetti" to "with". A single, larger orange arrow spans from "I" to "with", representing a shallow analysis that groups the entire object phrase "the spaghetti with meatballs" as a single unit.

- ▶ Understanding syntax fundamentally requires trees — the sentences have the same shallow analysis

Linguistic Structures

- ▶ Language is tree-structured



I ate the spaghetti with chopsticks



I ate the spaghetti with meatballs

- ▶ Understanding syntax fundamentally requires trees — the sentences have the same shallow analysis

PRP VBZ DT NN IN NNS
I ate the spaghetti with chopsticks

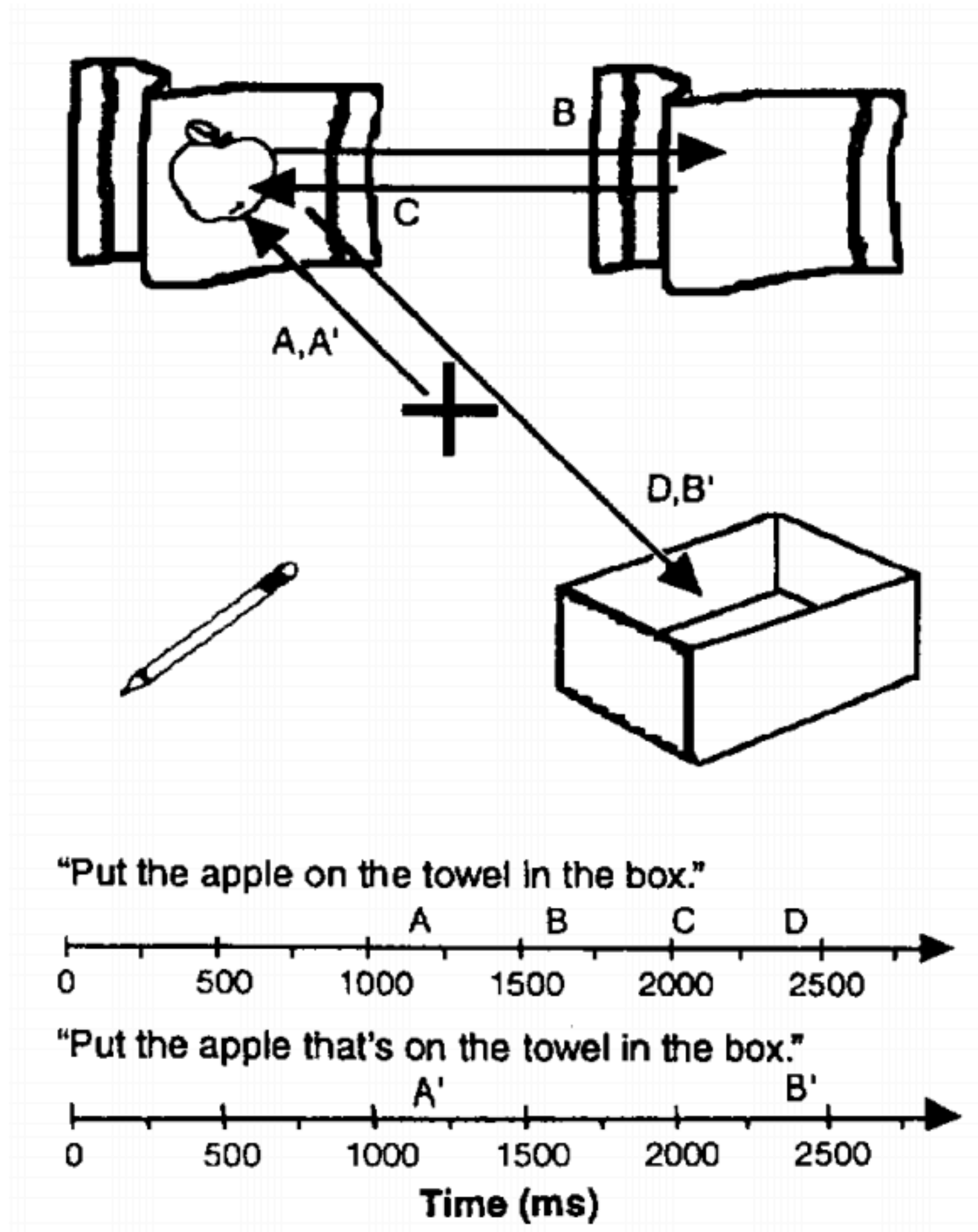
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Linguistic Structures

- ▶ Language is sequentially structured: interpreted in an online way

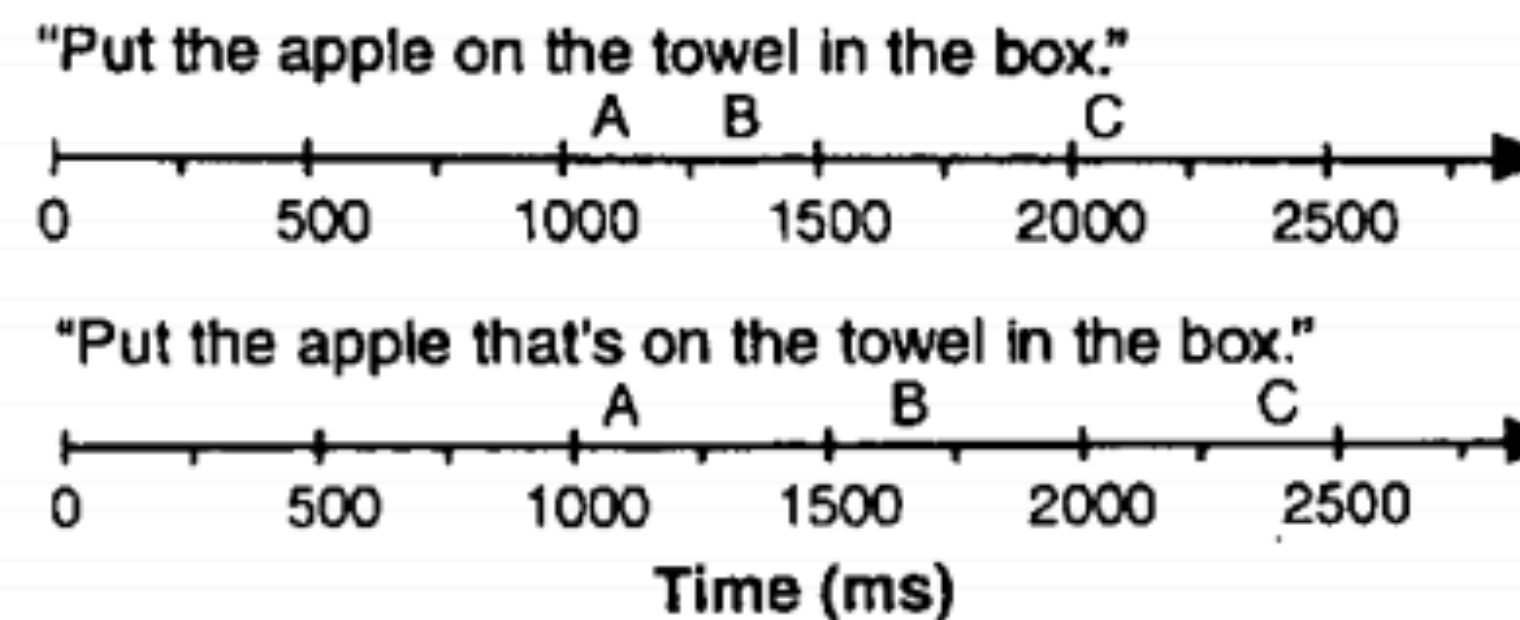
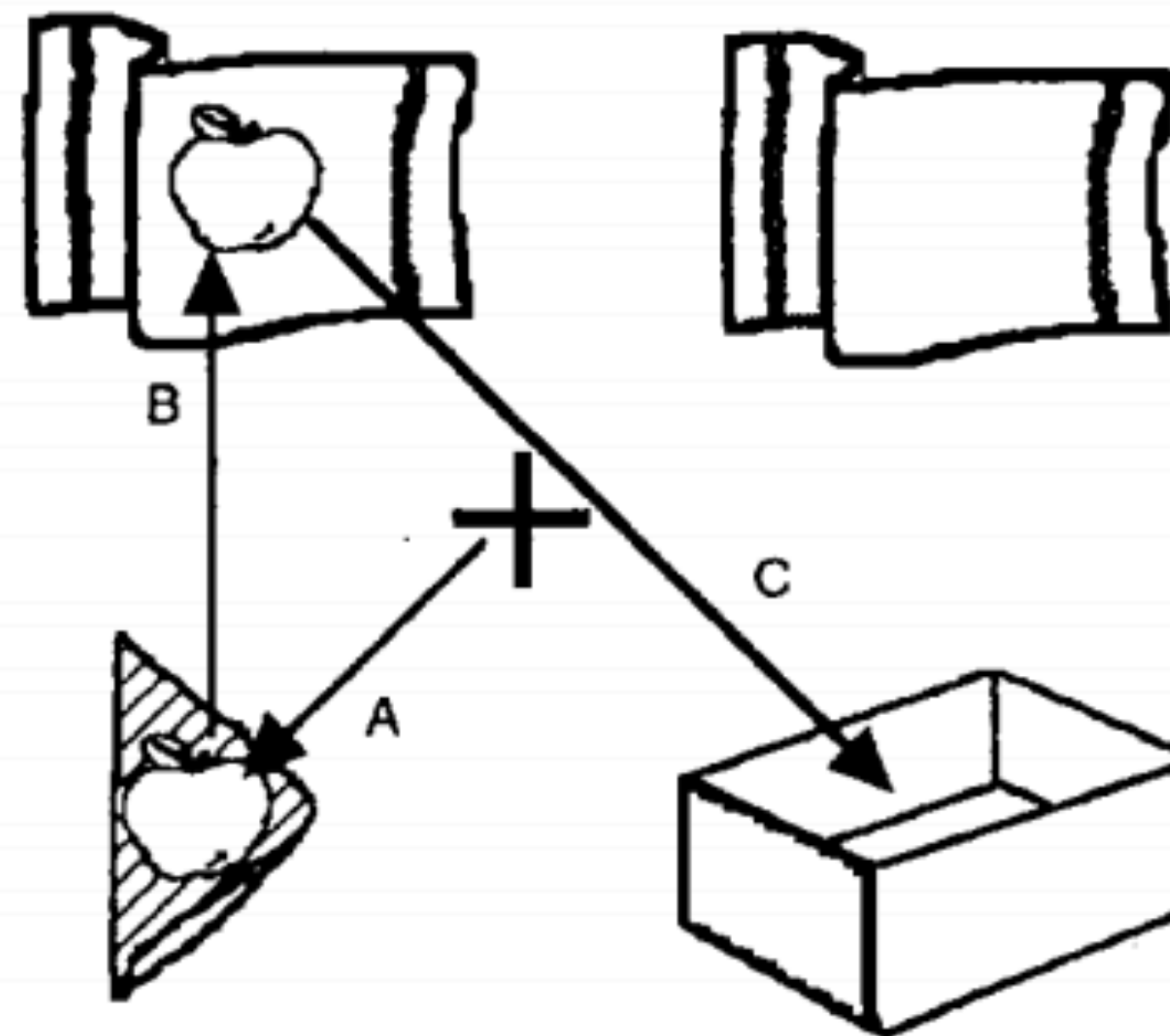
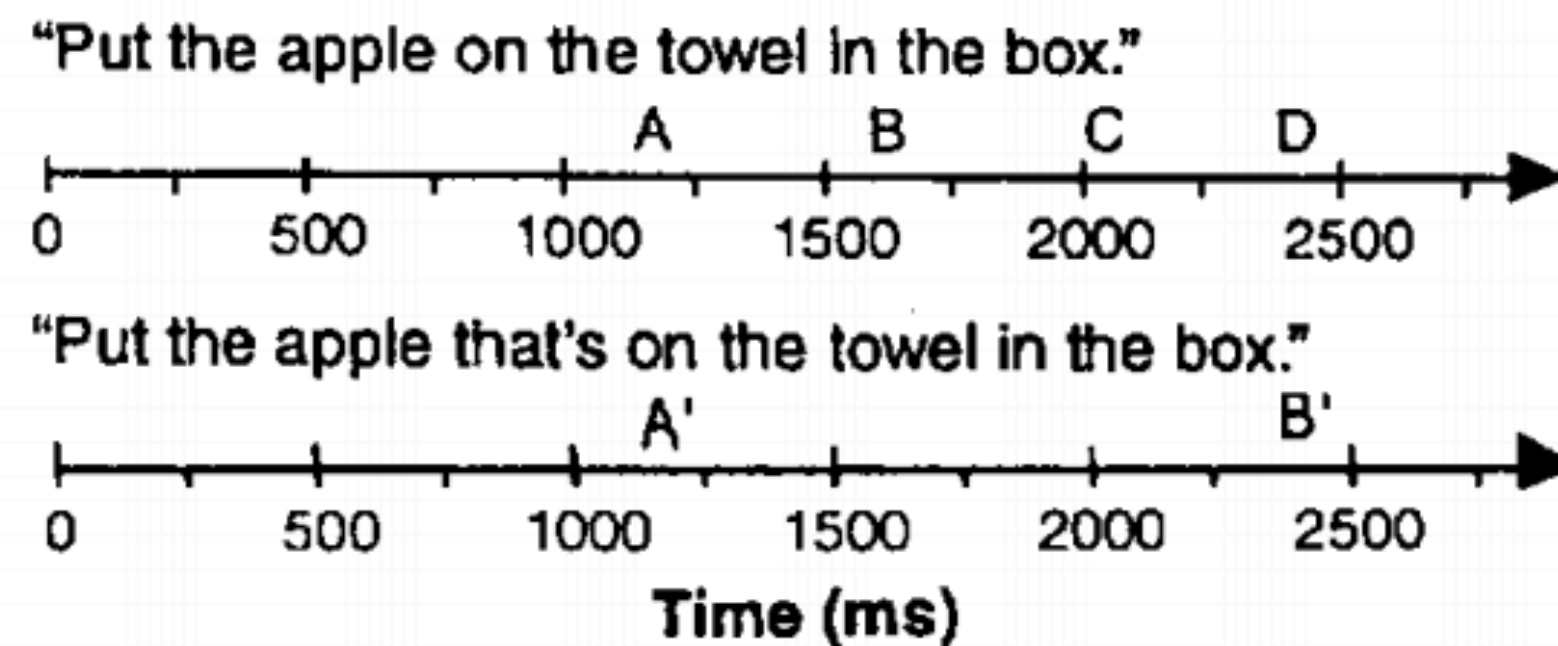
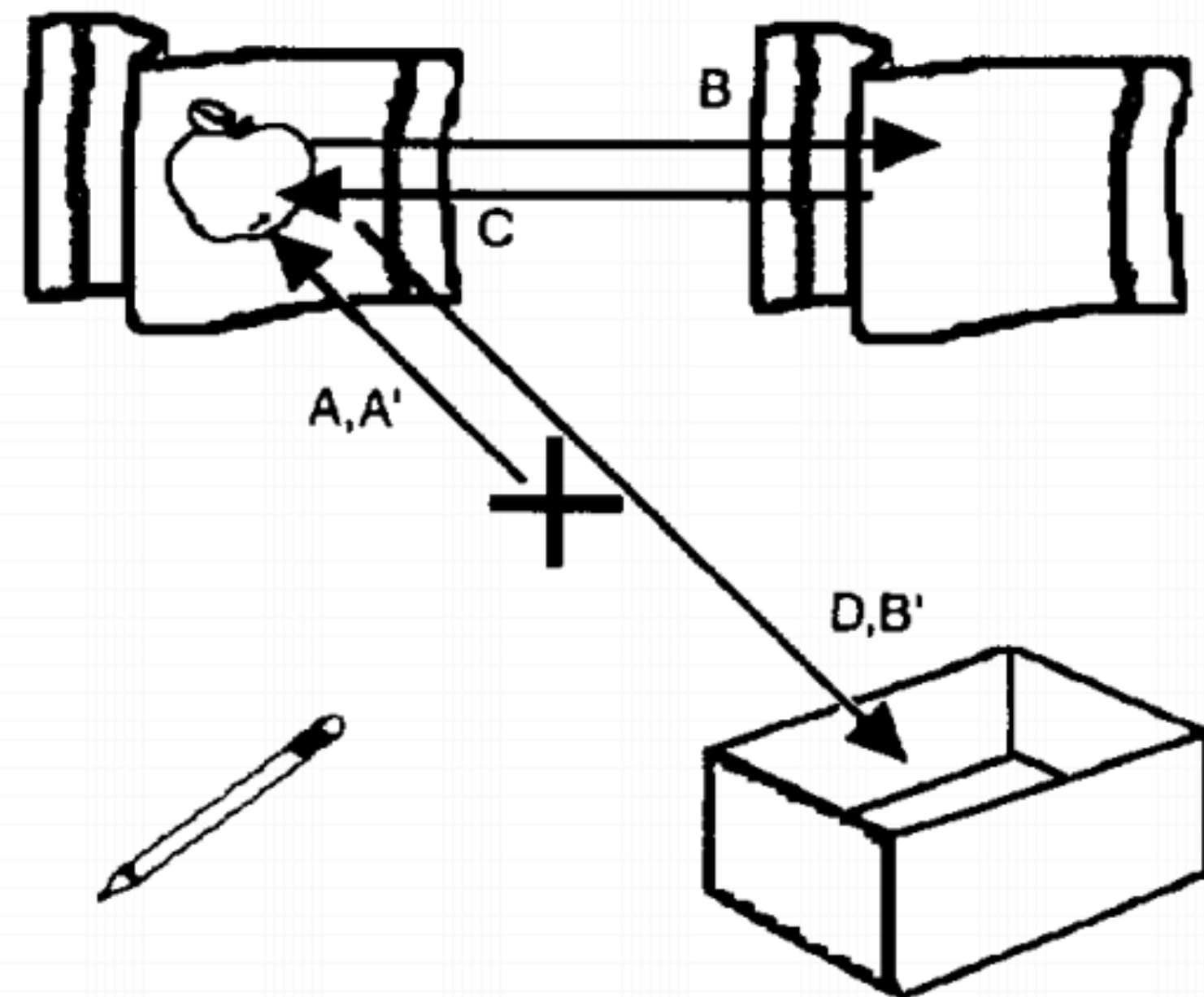
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Linguistic Structures

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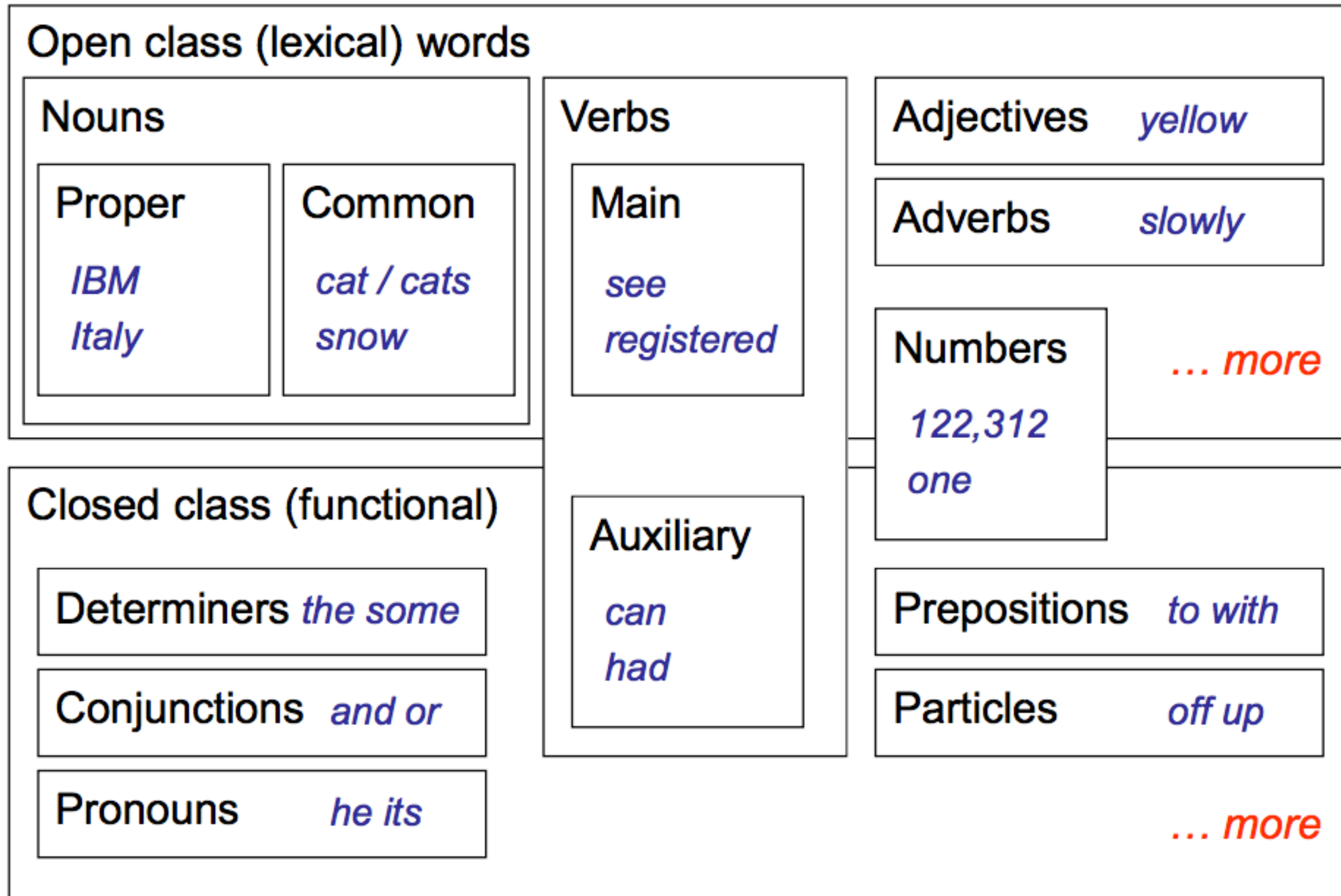


POS Tagging

- ▶ What tags are out there?

Ghana 's ambassador should have set up the big meeting in DC yesterday .

POS Tagging



POS Tagging

POS Tagging

Fed raises interest rates 0.5 percent

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I hereby
increase interest
rates 0.5%



POS Tagging

VBD

VBN

NNP

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POS Tagging

VBD

VBN VBZ

NNP NNS

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POS Tagging

VBD VB
VBN **VBZ** VBP
NNP NNS **NN**

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POS Tagging

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POS Tagging

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- ▶ What governs the correct choice? Word + context

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- ▶ Other paths are also plausible but even more semantically weird...
- ▶ What governs the correct choice? Word + context
 - ▶ Word identity: most words have ≤ 2 tags, many have one (*percent*, *the*)
 - ▶ Context: nouns start sentences, nouns follow verbs, etc.

POS Tagging

| | | |
|--------------|--|---|
| CC | conjunction, coordinating | and both but either or |
| CD | numeral, cardinal | mid-1890 nine-thirty 0.5 one |
| DT | determiner | a all an every no that the |
| EX | existential there | there |
| FW | foreign word | gemeinschaft hund ich jeux |
| IN | preposition or conjunction, subordinating | among whether out on by if |
| JJ | adjective or numeral, ordinal | third ill-mannered regrettable |
| JJR | adjective, comparative | braver cheaper taller |
| JJS | adjective, superlative | bravest cheapest tallest |
| MD | modal auxiliary | can may might will would |
| NN | noun, common, singular or mass | cabbage thermostat investment subhumanity |
| NNP | noun, proper, singular | Motown Cougar Yvette Liverpool |
| NNPS | noun, proper, plural | Americans Materials States |
| NNS | noun, common, plural | undergraduates bric-a-brac averages |
| POS | genitive marker | 's |
| PRP | pronoun, personal | hers himself it we them |
| PRP\$ | pronoun, possessive | her his mine my our ours their thy your |
| RB | adverb | occasionally maddeningly adventurously |
| RBR | adverb, comparative | further gloomier heavier less-perfectly |
| RBS | adverb, superlative | best biggest nearest worst |
| RP | particle | aboard away back by on open through |
| TO | "to" as preposition or infinitive marker | to |
| UH | interjection | huh howdy uh whammo shucks heck |
| VB | verb, base form | ask bring fire see take |
| VBD | verb, past tense | pleaded swiped registered saw |
| VBG | verb, present participle or gerund | stirring focusing approaching erasing |
| VBN | verb, past participle | dilapidated imitated reunified unsettled |
| VBP | verb, present tense, not 3rd person singular | twist appear comprise mold postpone |
| VBZ | verb, present tense, 3rd person singular | bases reconstructs marks uses |
| WDT | WH-determiner | that what whatever which whichever |
| WP | WH-pronoun | that what whatever which who whom |
| WP\$ | WH-pronoun, possessive | whose |
| WRB | Wh-adverb | however whenever where why |

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- ▶ Text-to-speech: *record, lead*
- ▶ Preprocessing step for syntactic parsers
- ▶ Domain-independent disambiguation for other tasks
- ▶ (Very) shallow information extraction

Sequence Models

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- ▶ Input $\mathbf{x} = (x_1, \dots, x_n)$ Output $\mathbf{y} = (y_1, \dots, y_n)$

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Sequence Models

- ▶ Input $\mathbf{x} = (x_1, \dots, x_n)$ Output $\mathbf{y} = (y_1, \dots, y_n)$
- ▶ POS tagging: \mathbf{x} is a sequence of words, \mathbf{y} is a sequence of tags
- ▶ Today: generative models $P(\mathbf{x}, \mathbf{y})$; discriminative models next time

Hidden Markov Models

- ▶ Input $\mathbf{x} = (x_1, \dots, x_n)$ Output $\mathbf{y} = (y_1, \dots, y_n)$

Hidden Markov Models

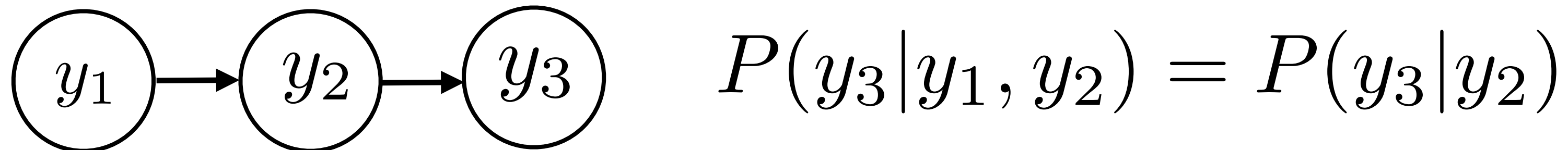
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- ▶ Model the sequence of y as a Markov process (dynamics model)

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- ▶ Markov property: future is conditionally independent of the past given the present

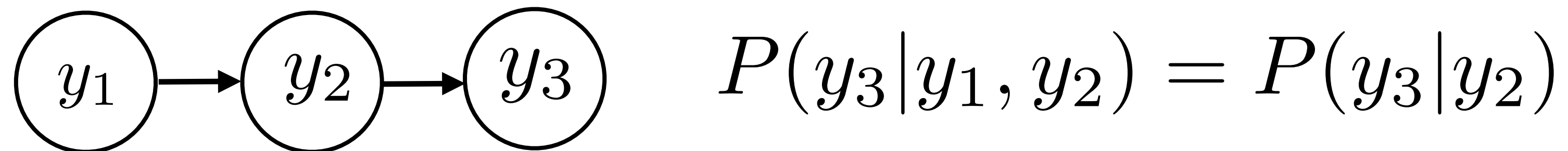
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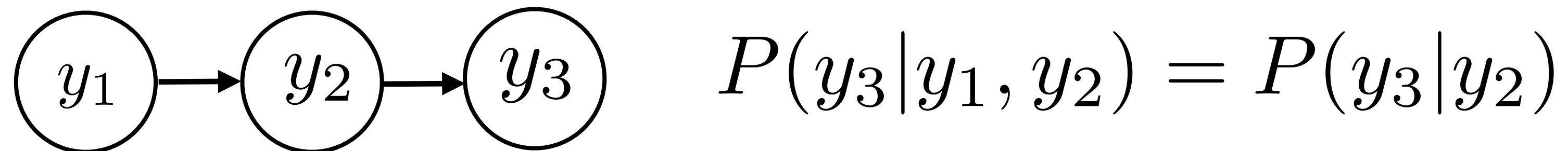
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- ▶ Lots of mathematical theory about how Markov chains behave

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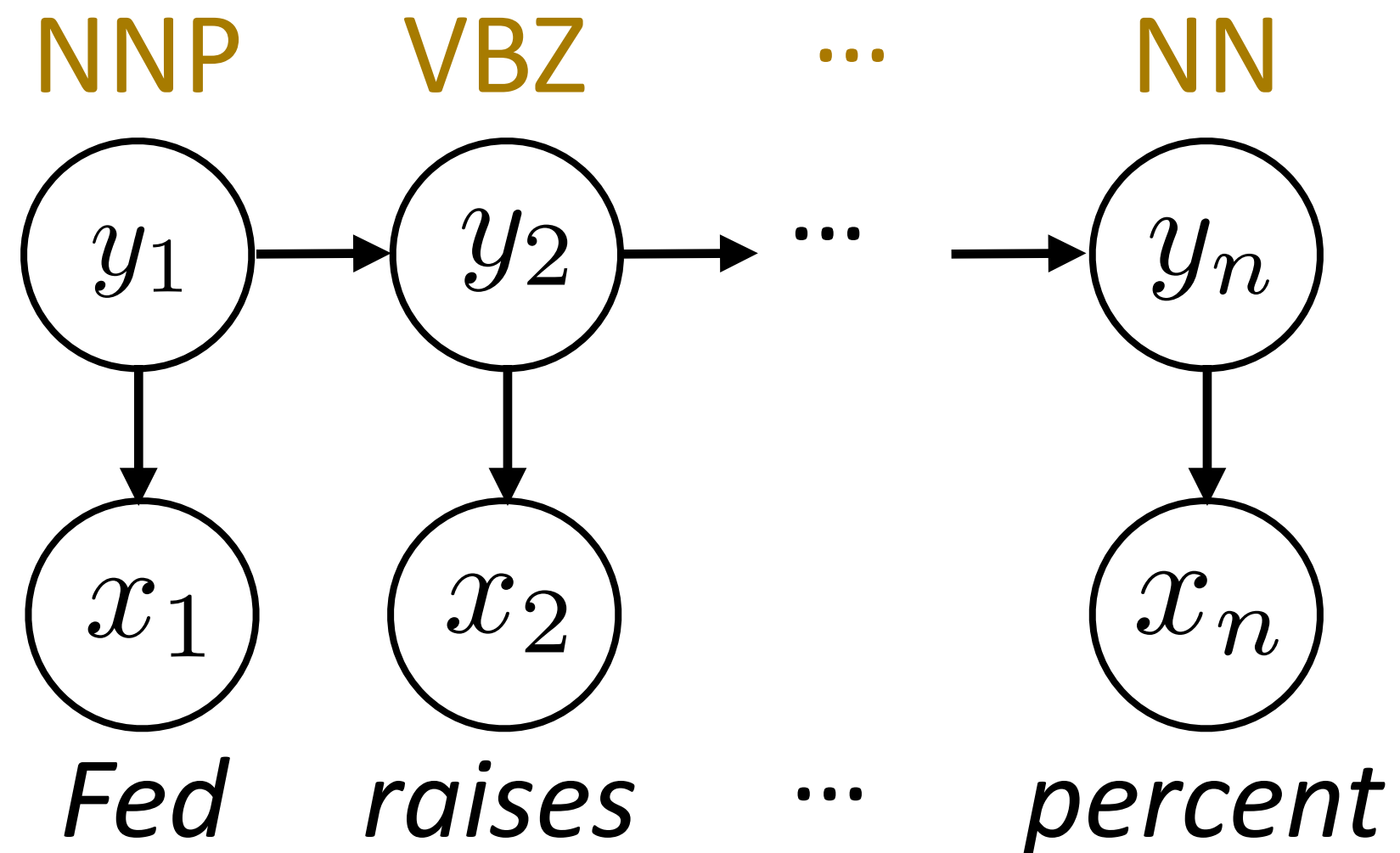
- ▶ Lots of mathematical theory about how Markov chains behave
- ▶ If y are tags, this roughly corresponds to assuming that the next tag only depends on the current tag, not anything before

Hidden Markov Models

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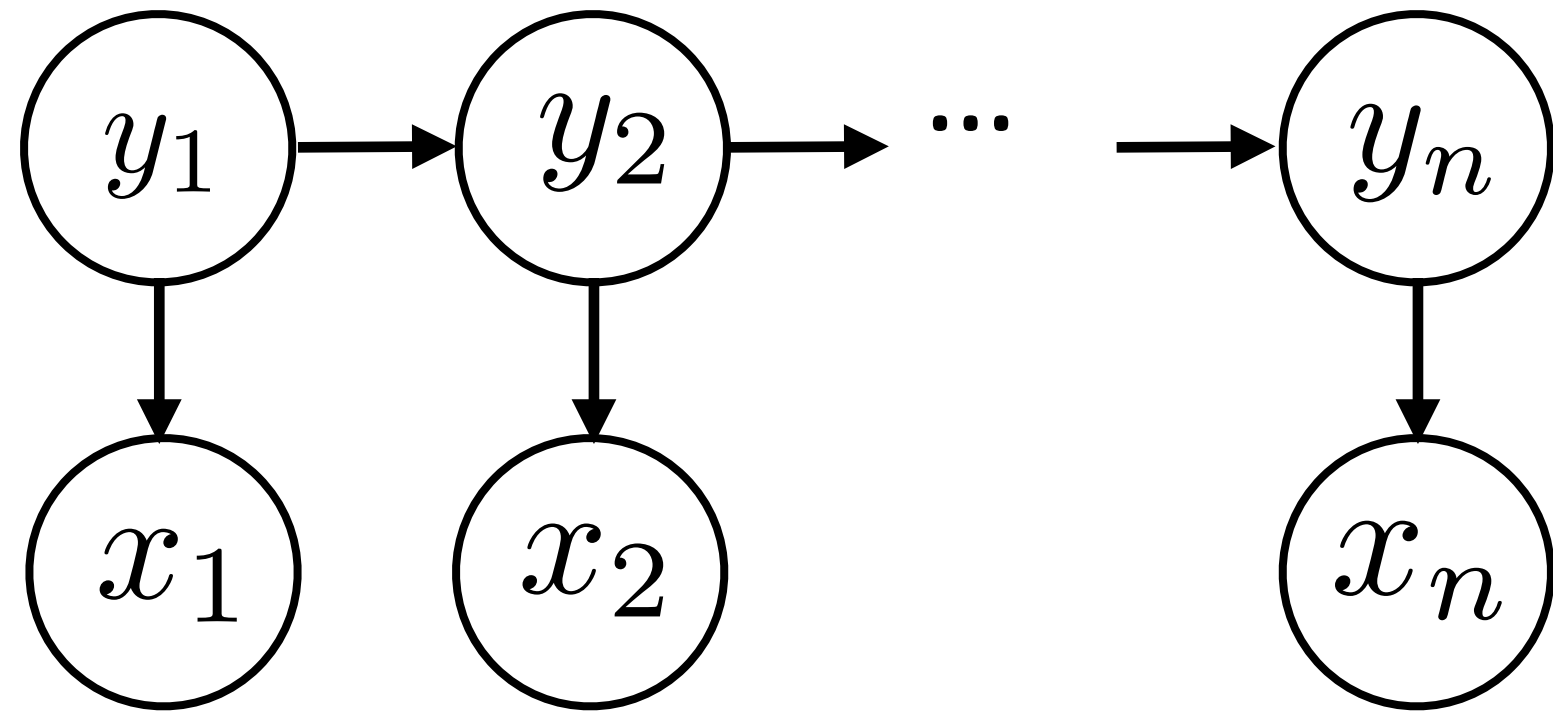
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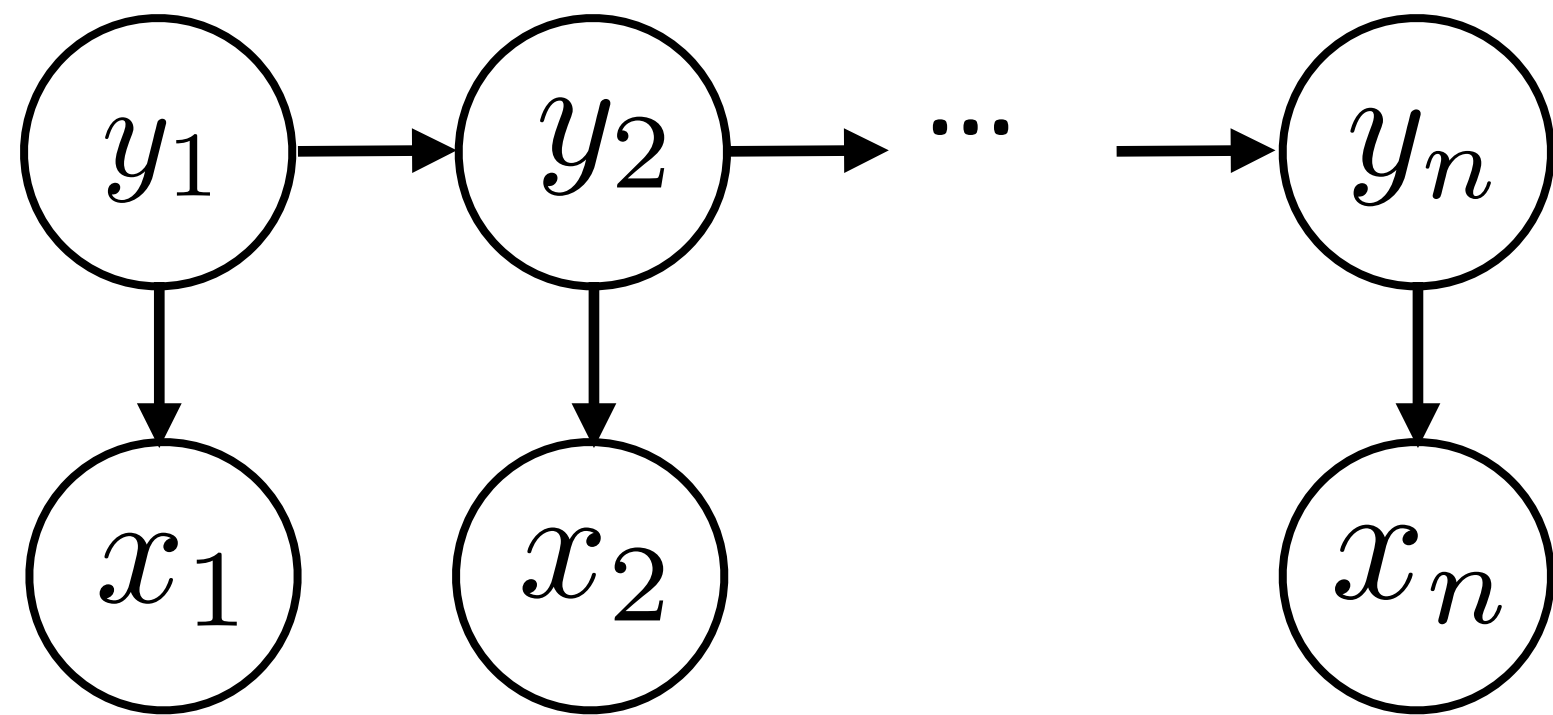
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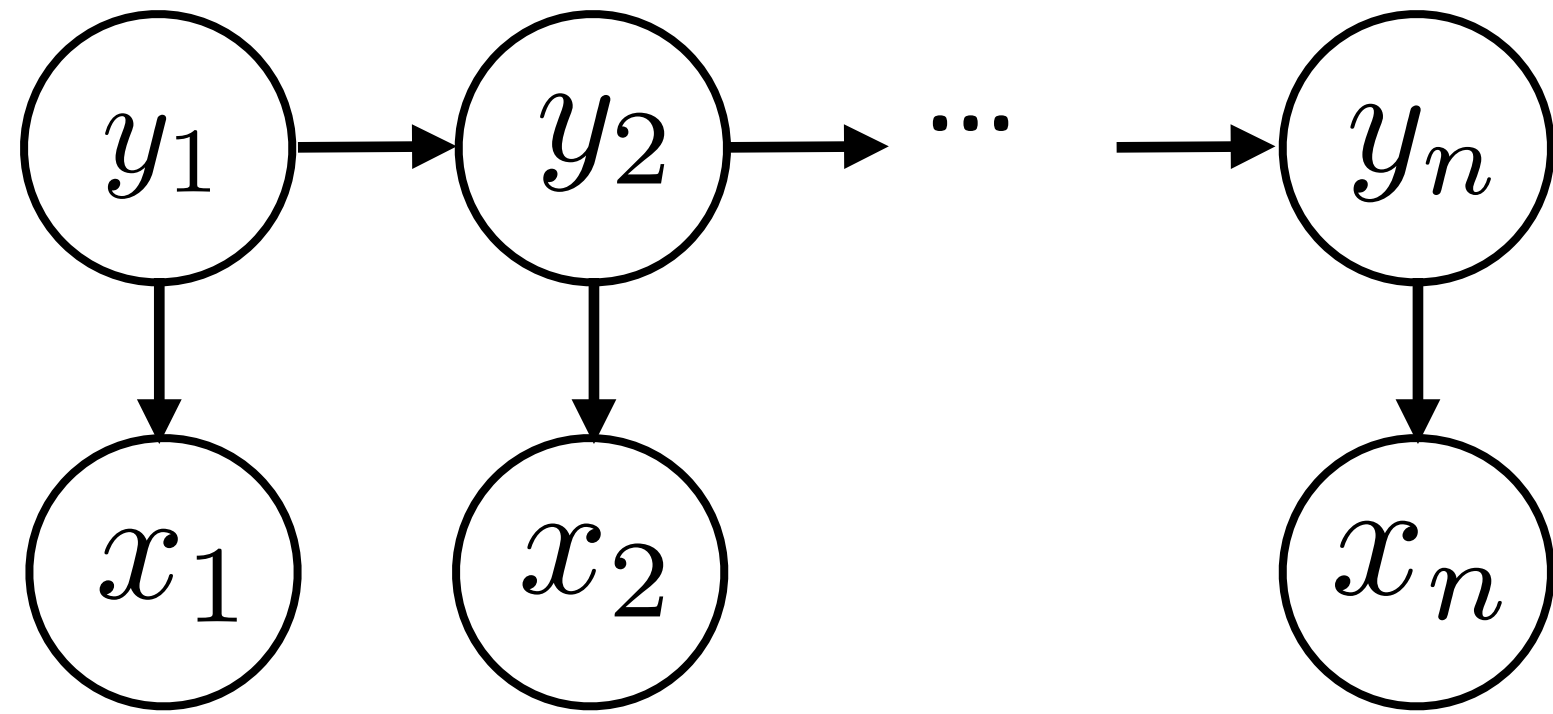
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$$P(\mathbf{y}, \mathbf{x}) = P(y_1) \prod_{i=2}^n P(y_i | y_{i-1}) \prod_{i=1}^n P(x_i | y_i)$$

Hidden Markov Models

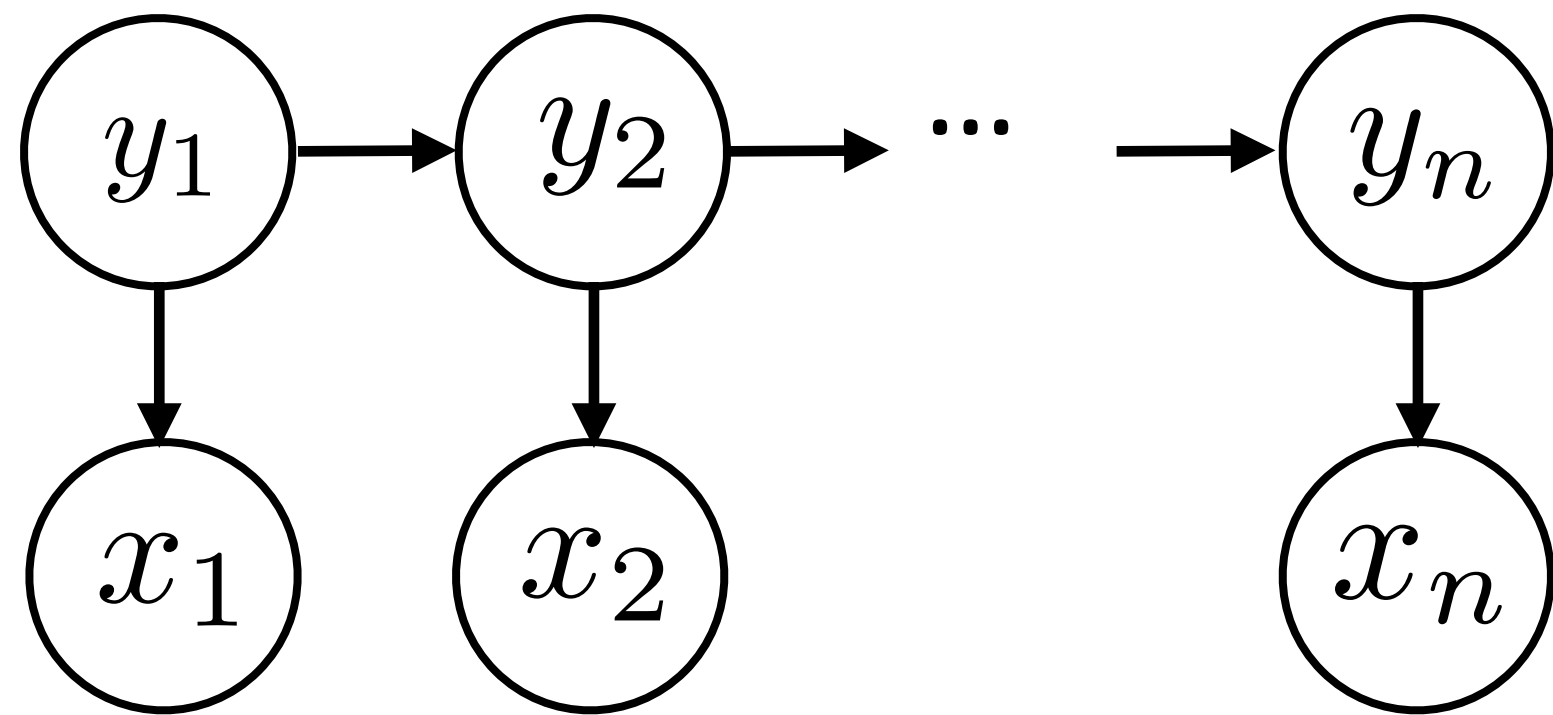
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$$P(\mathbf{y}, \mathbf{x}) = \underbrace{P(y_1)}_{\text{Initial distribution}} \prod_{i=2}^n P(y_i | y_{i-1}) \prod_{i=1}^n P(x_i | y_i)$$

Hidden Markov Models

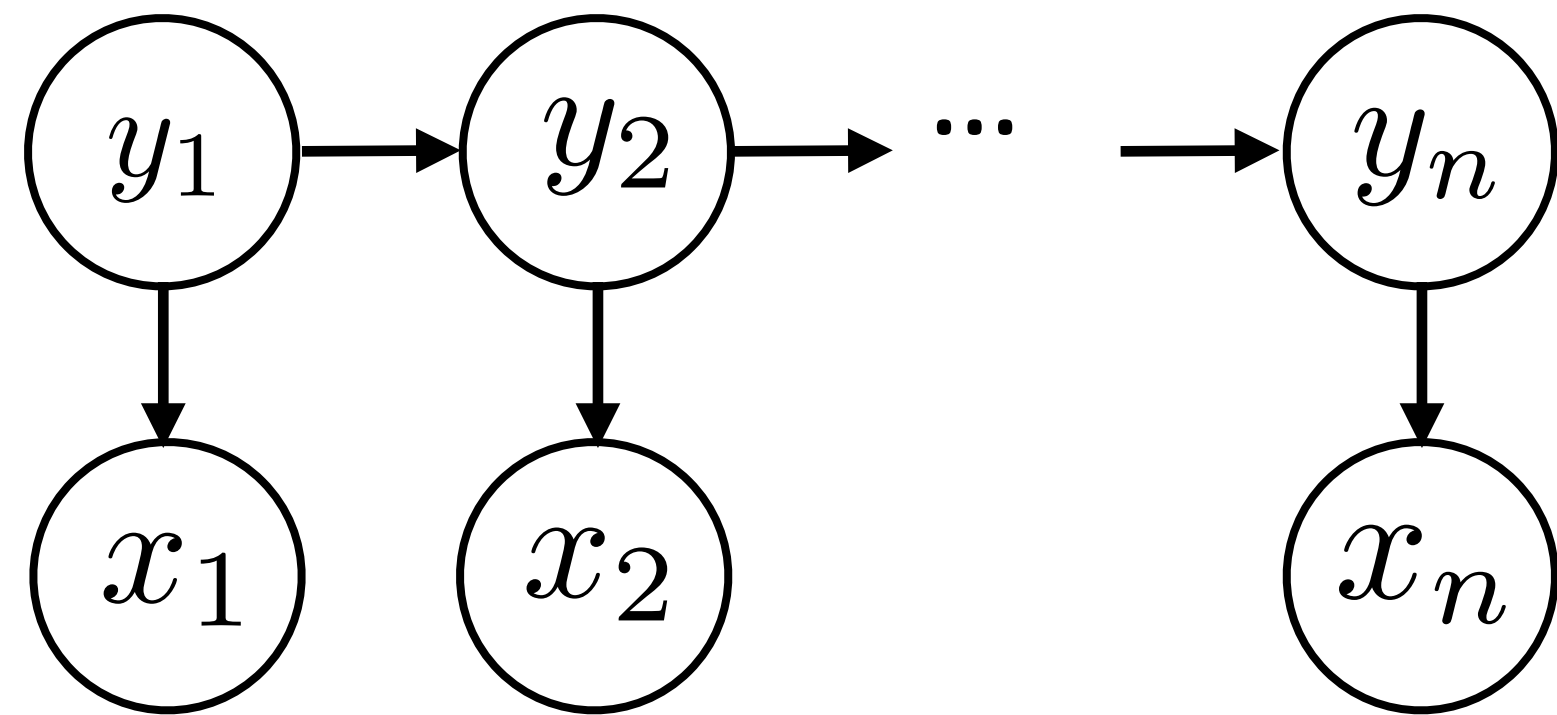
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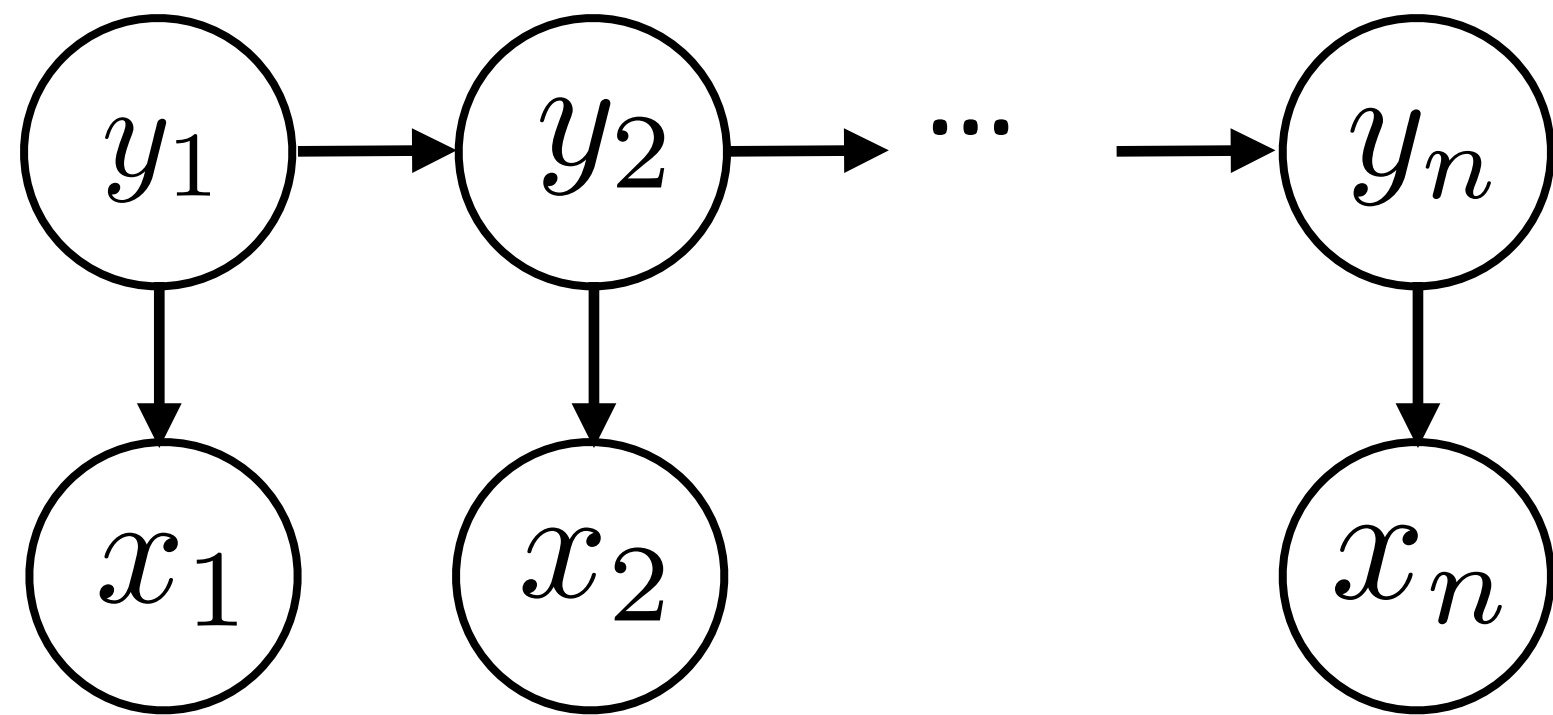
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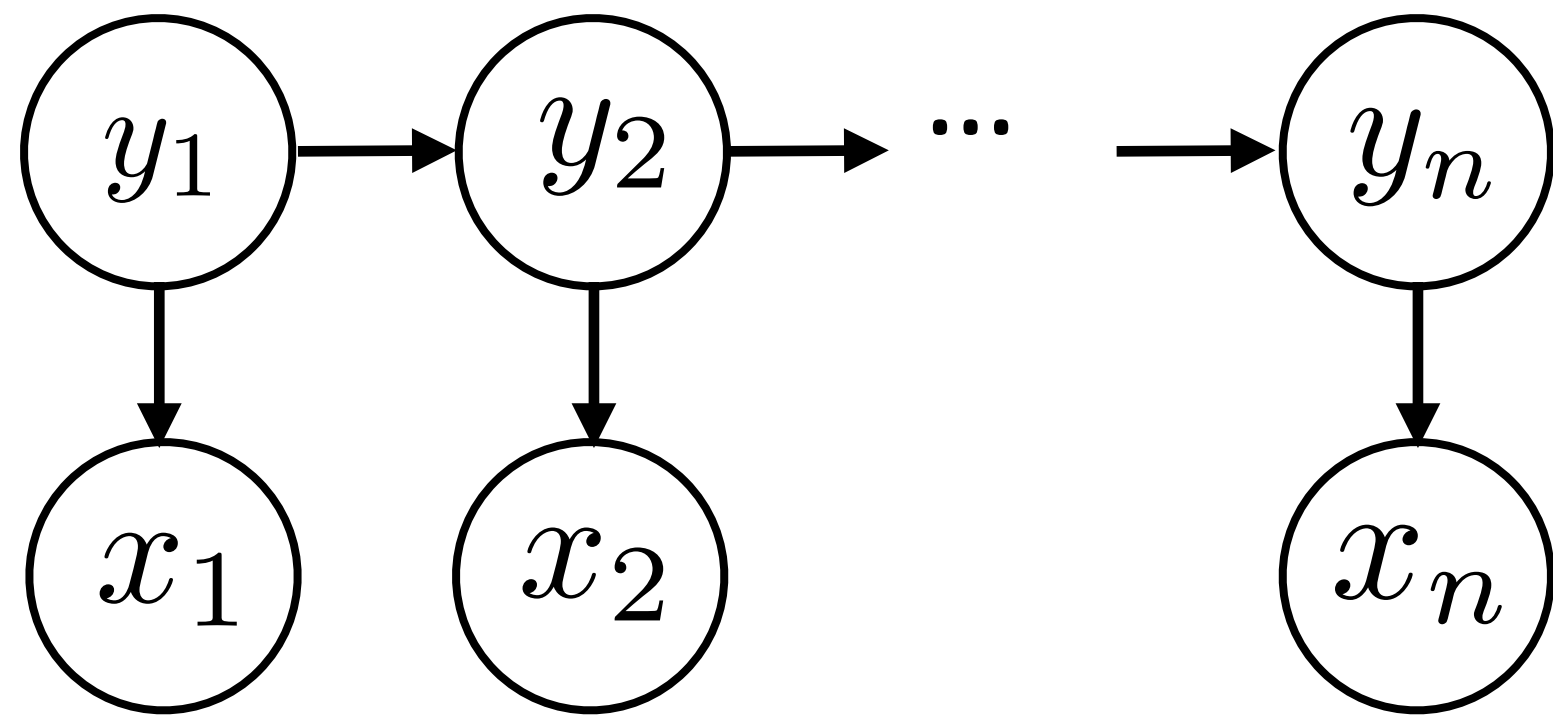


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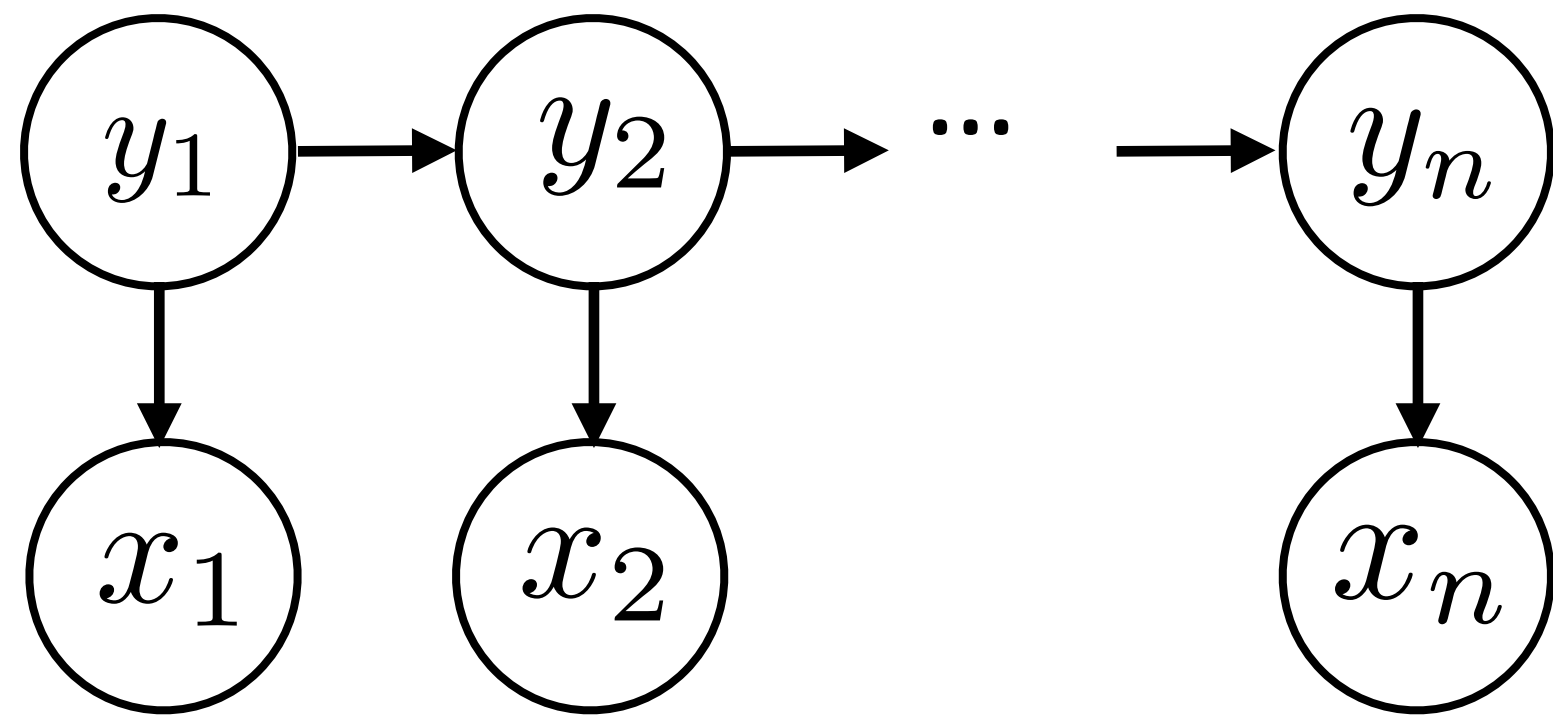


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- ▶ Multinomials: tag x tag transitions, tag x word emissions

Hidden Markov Models

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- ▶ Observation (x) depends only on current state (y)
- ▶ Multinomials: tag x tag transitions, tag x word emissions
- ▶ $P(x|y)$ is a distribution over all words in the vocabulary — not a distribution over features (but could be!)

Transitions in POS Tagging

► Dynamics model $P(y_1) \prod_{i=2}^n P(y_i | y_{i-1})$

VBD

VB

VBN VBZ

VBP

VBZ

NNP NNS

NN

NNS

CD

NN

.

Fed raises interest rates 0.5 percent .

NNP - proper noun, singular

VBZ - verb, 3rd ps. sing. present

NN - noun, singular or mass

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- ▶ $P(y_2 = \text{VBZ} | y_1 = \text{NNP})$ likely because verb often follows noun

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- ▶ $P(y_1 = \text{NNP})$ likely because start of sentence
- ▶ $P(y_2 = \text{VBZ} | y_1 = \text{NNP})$ likely because verb often follows noun
- ▶ $P(y_3 = \text{NN} | y_2 = \text{VBZ})$ direct object follows verb, other verb rarely follows past tense verb (main verbs can follow modals though!)

Estimating Transitions

NNP VBZ NN NNS CD NN .
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- ▶ $P(\text{tag} \mid \text{NN})$

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$$P(\text{tag} \mid \text{tag}_{-1}) = (1 - \lambda) \hat{P}(\text{tag} \mid \text{tag}_{-1}) + \lambda \hat{P}(\text{tag})$$

Estimating Transitions

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$$P(\text{tag} \mid \text{tag}_{-1}) = (1 - \lambda) \hat{P}(\text{tag} \mid \text{tag}_{-1}) + \lambda \hat{P}(\text{tag})$$

\hat{P} = empirical distribution (read off from data)

Emissions in POS Tagging

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- ▶ $P(\text{word} | \text{NN}) = (0.05 \textit{ person}, 0.04 \textit{ official}, 0.03 \textit{ interest}, 0.03 \textit{ percent} \dots)$

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- ▶ When you compute the posterior for a given word's tags, the distribution favors tags that are more likely to generate that word

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- ▶ How should we smooth this?

Estimating Emissions

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Estimating Emissions

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Fed raises interest rates 0.5 percent

- ▶ $P(\text{word} \mid \text{NN}) = (0.5 \textit{ interest}, 0.5 \textit{ percent})$ — hard to smooth!

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- ▶ Fancy techniques from language modeling, e.g. look at type fertility — $P(\text{tag} \mid \text{word})$ is flatter for some kinds of words than for others)

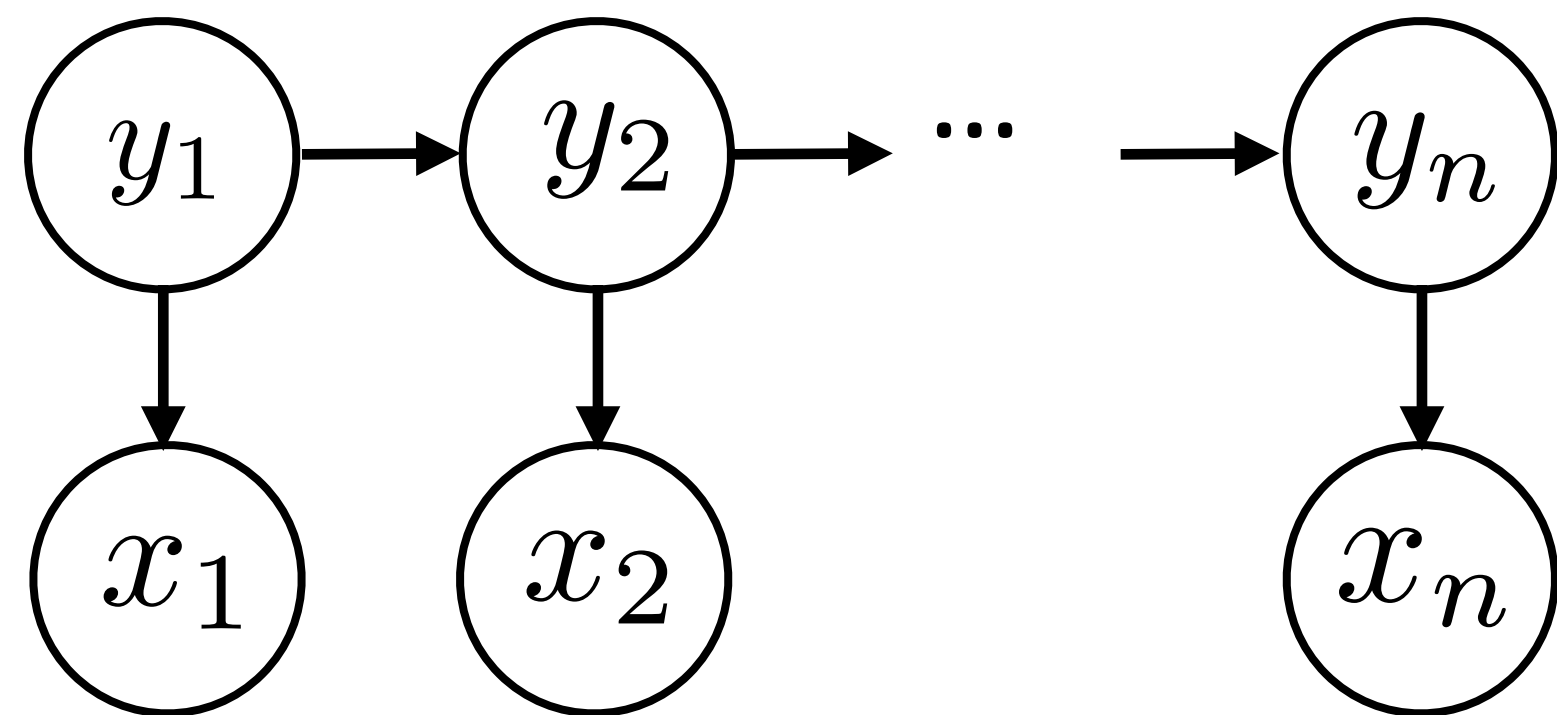
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- ▶ $P(\text{word} \mid \text{tag})$ can be a log-linear model — we'll see this in a few lectures

Inference in HMMs

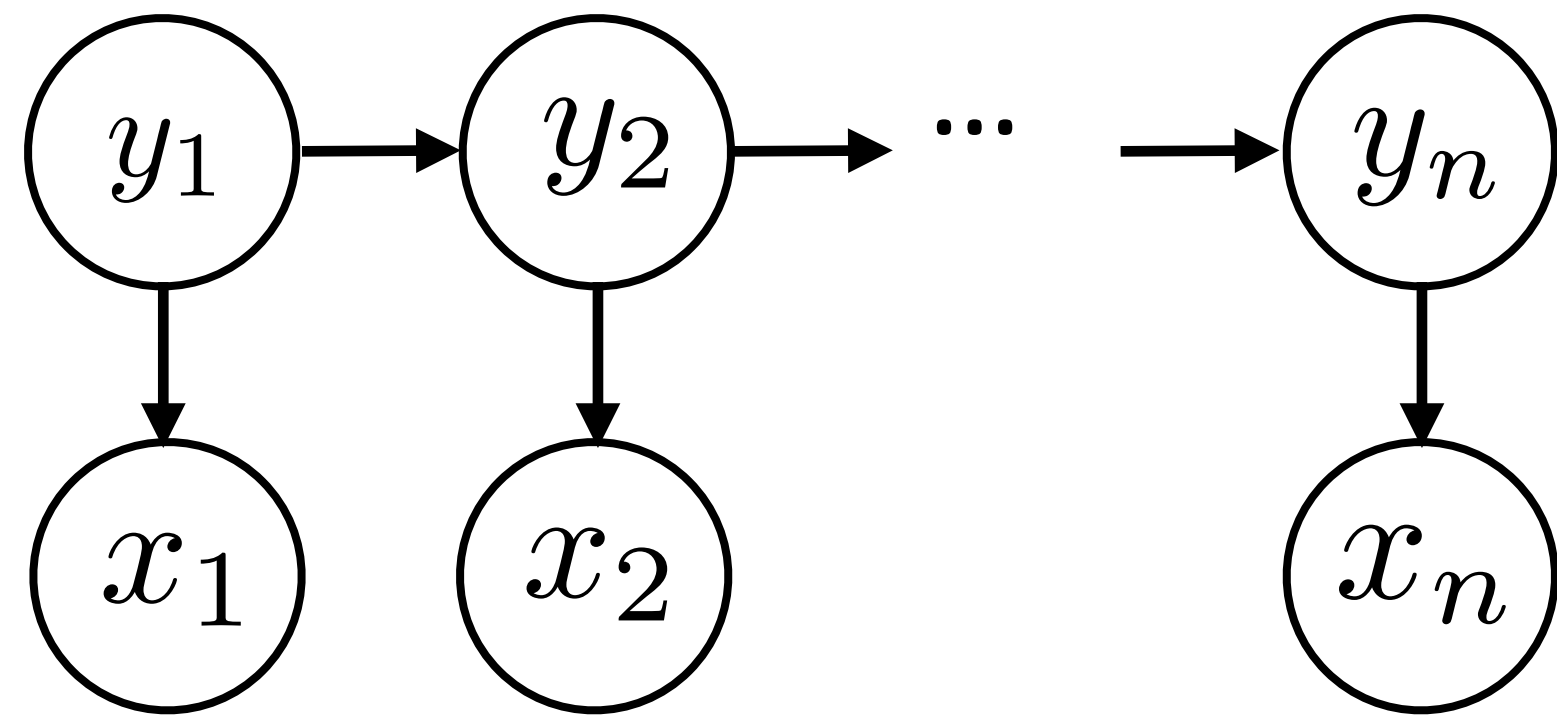
- ▶ Input $\mathbf{x} = (x_1, \dots, x_n)$ Output $\mathbf{y} = (y_1, \dots, y_n)$



$$P(\mathbf{y}, \mathbf{x}) = P(y_1) \prod_{i=2}^n P(y_i | y_{i-1}) \prod_{i=1}^n P(x_i | y_i)$$

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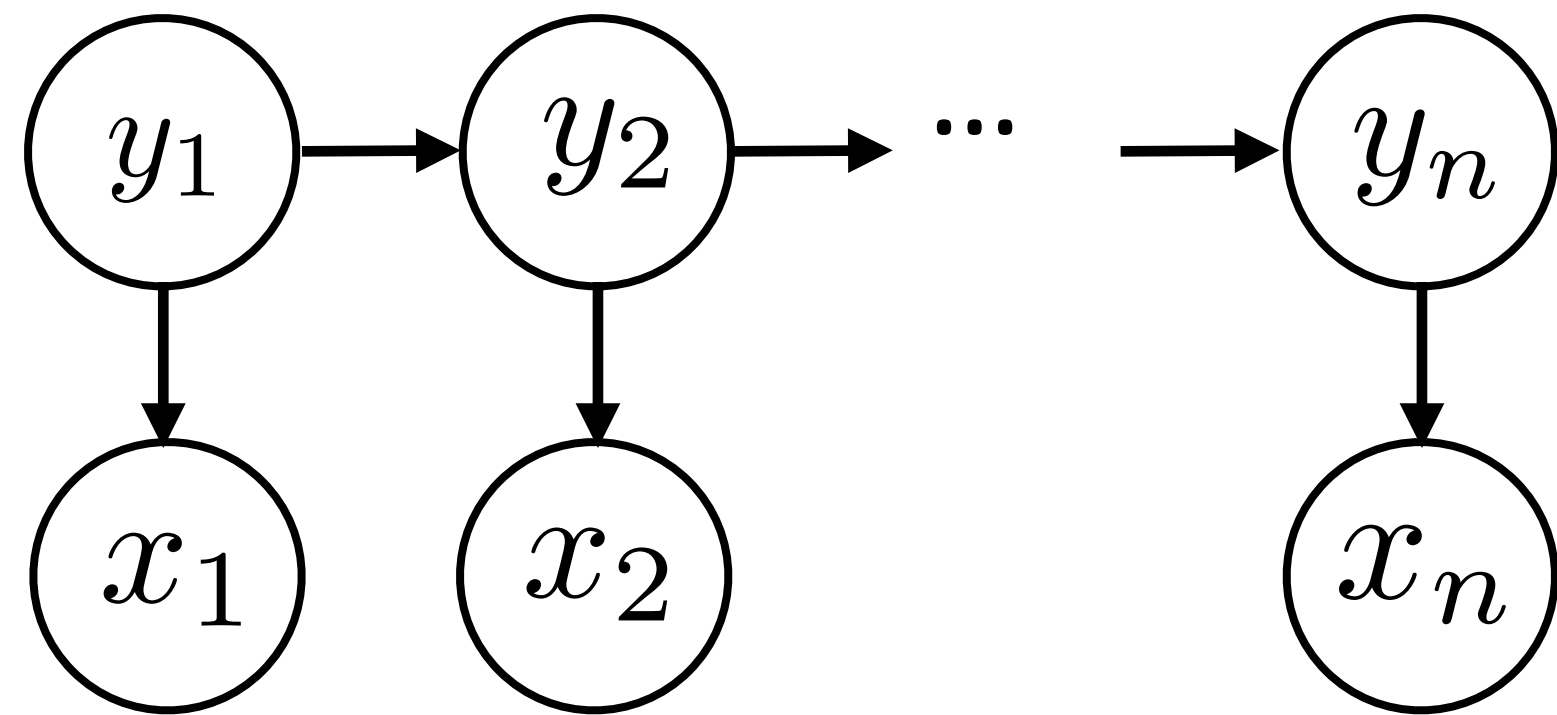


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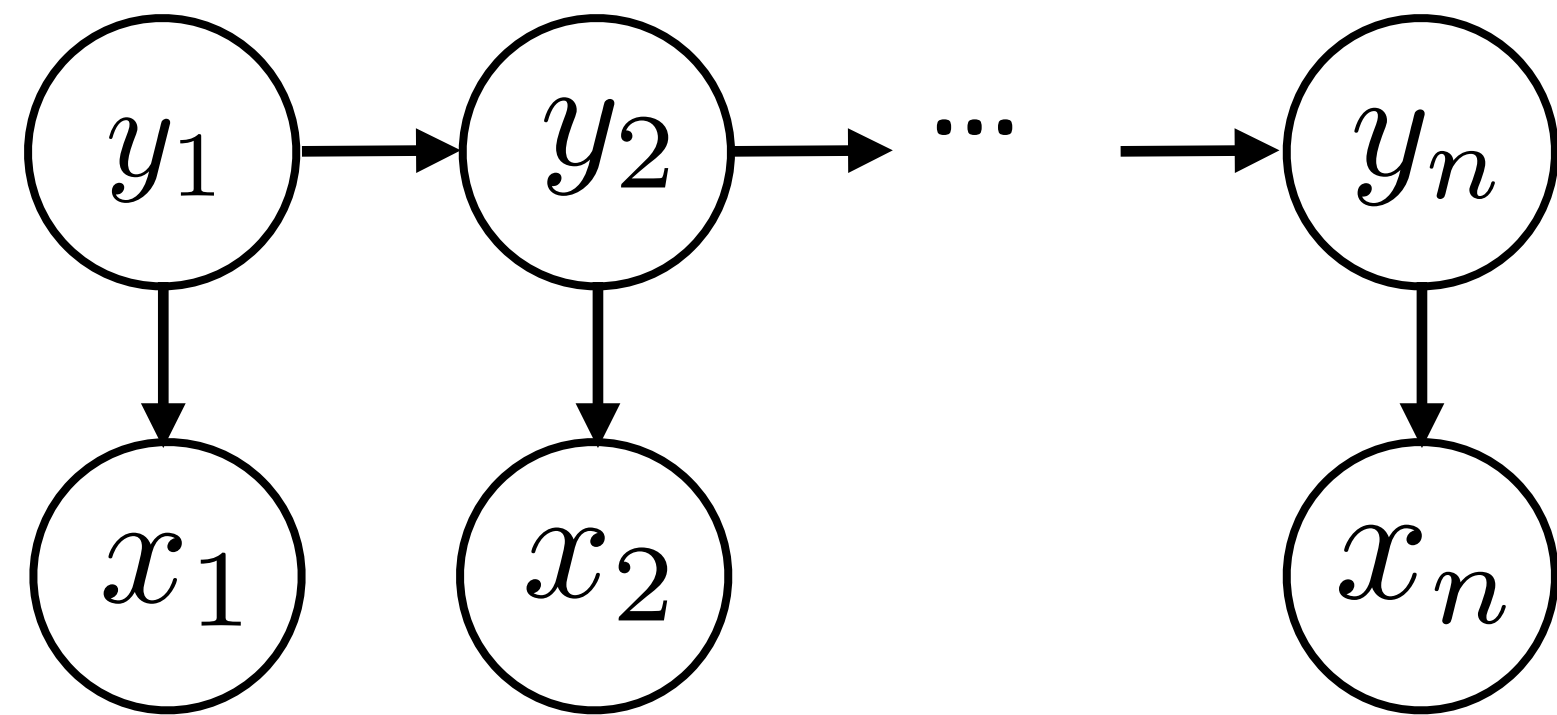


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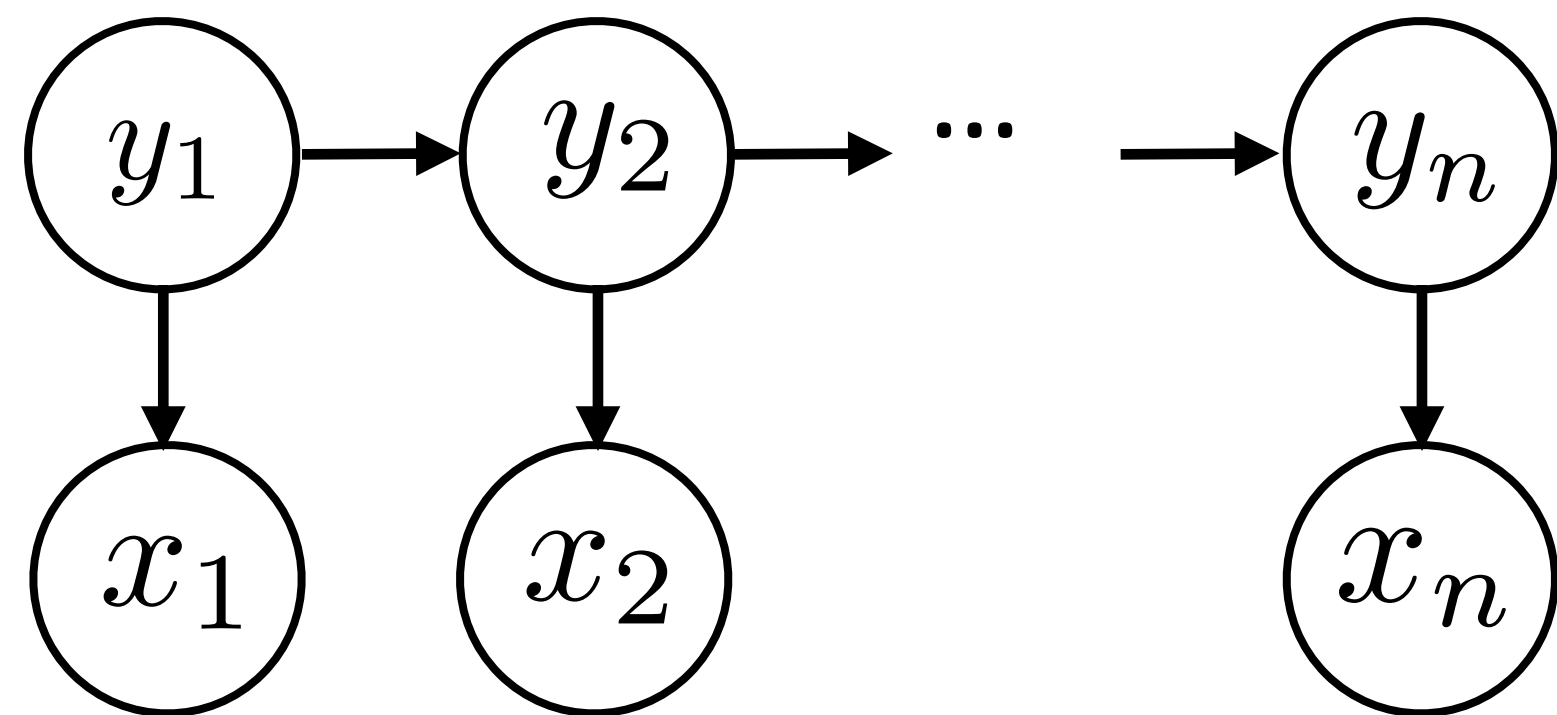


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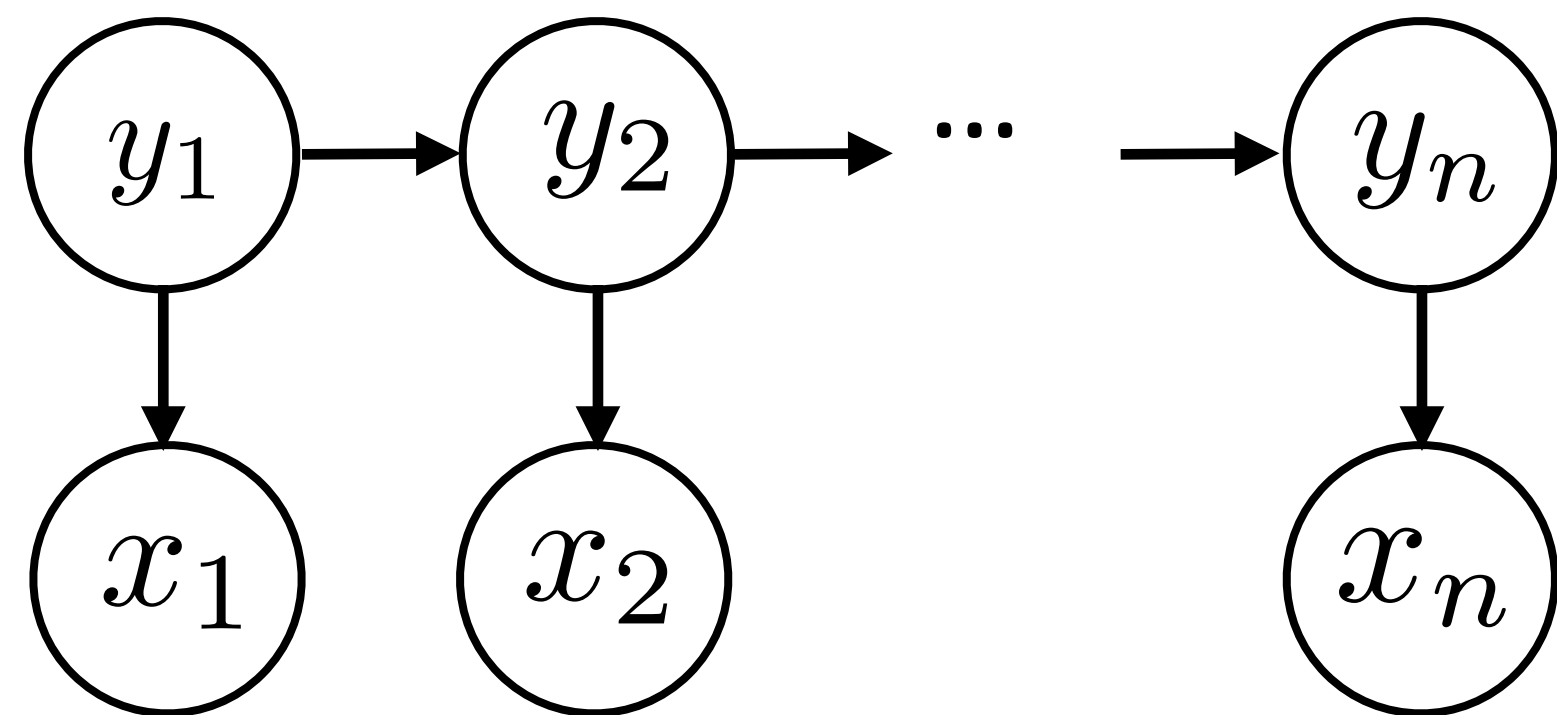


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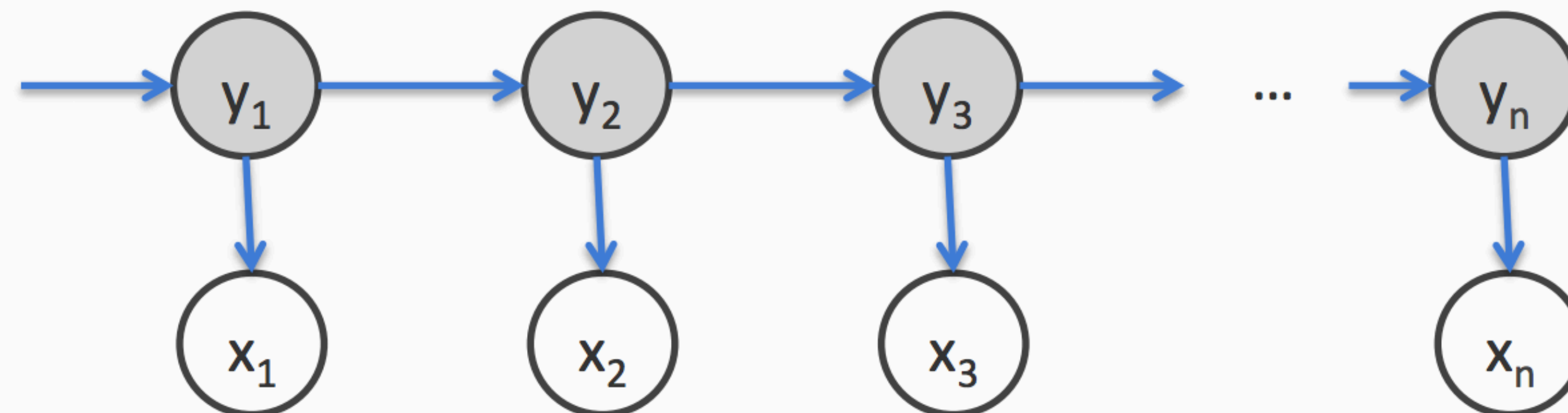
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- ▶ Exponentially many possible \mathbf{y} here!
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 - ▶ Many neural sequence models depend on entire previous tag sequence, need to use approximations like beam search

Viterbi Algorithm

$$P(x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n) = P(y_1) \prod_{i=1}^{n-1} P(y_{i+1}|y_i) \prod_{i=1}^n P(x_i|y_i)$$

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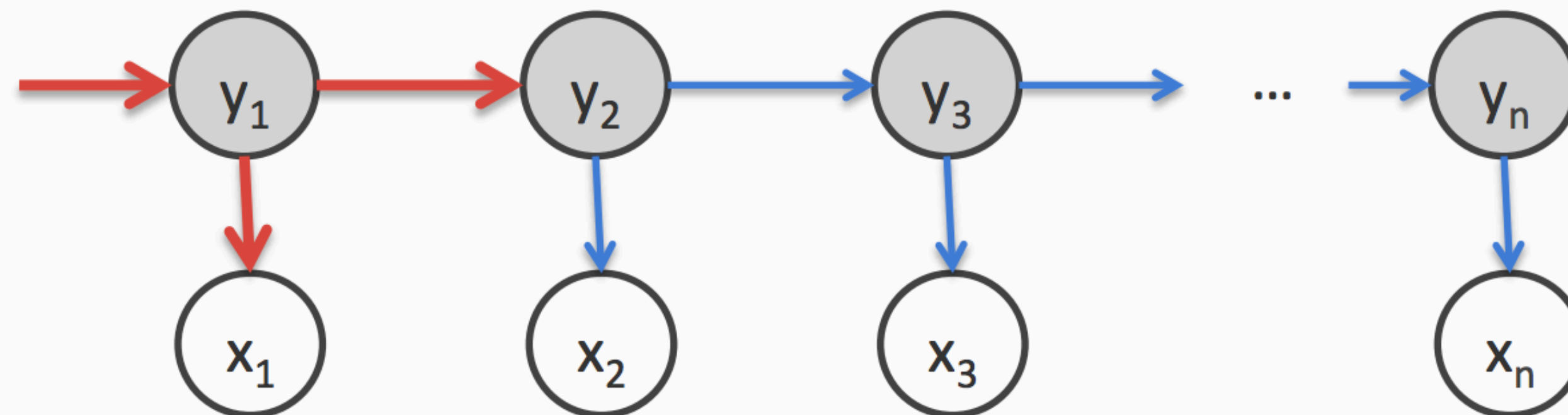


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The only terms that depend on y_1



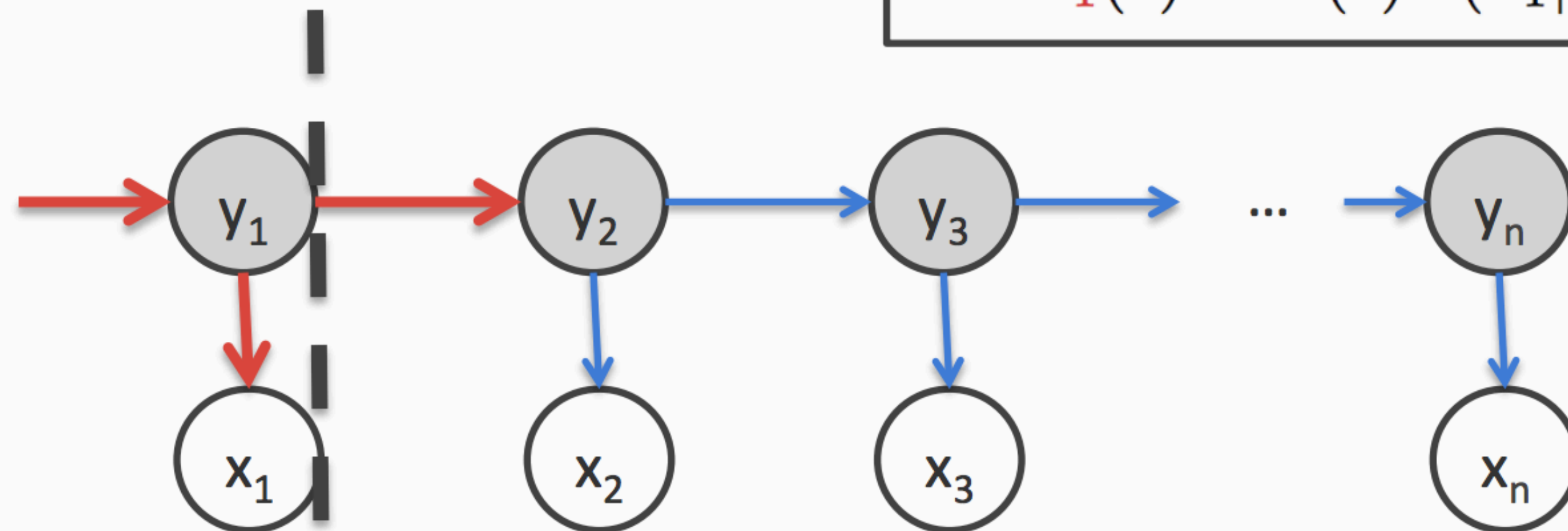
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Abstract away the score for all decisions till here into **score**

$$\mathbf{score}_1(s) = P(s)P(x_1|s)$$



Viterbi Algorithm

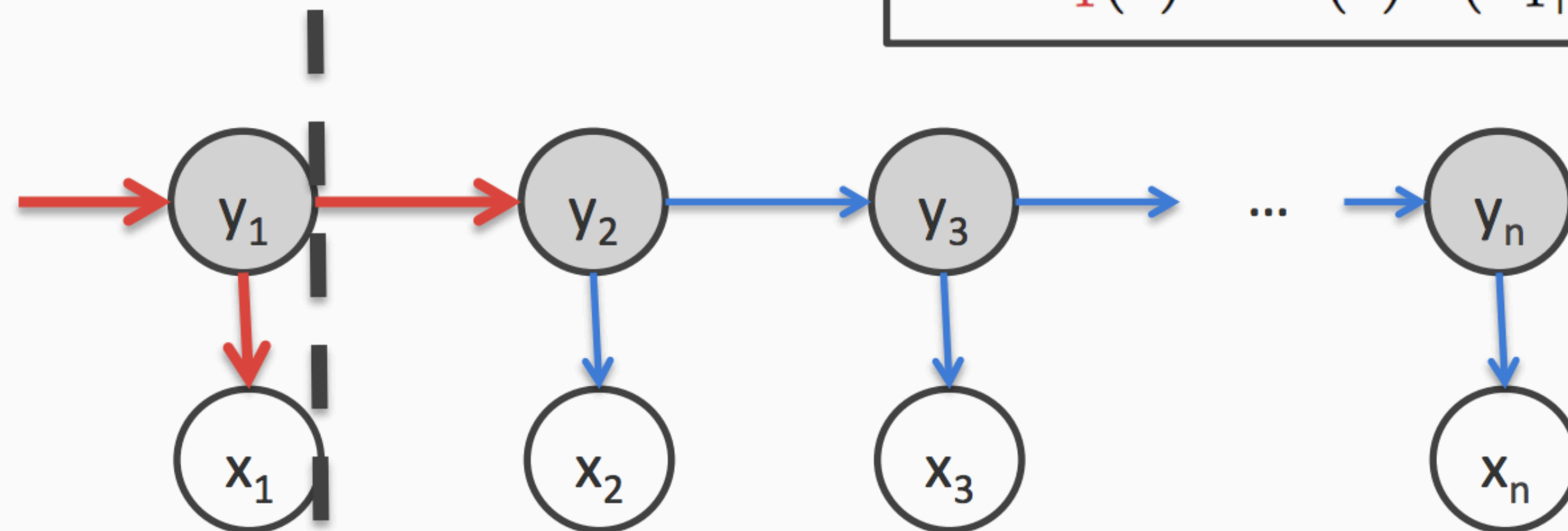
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best (partial) score for a sequence ending in state s

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Viterbi Algorithm

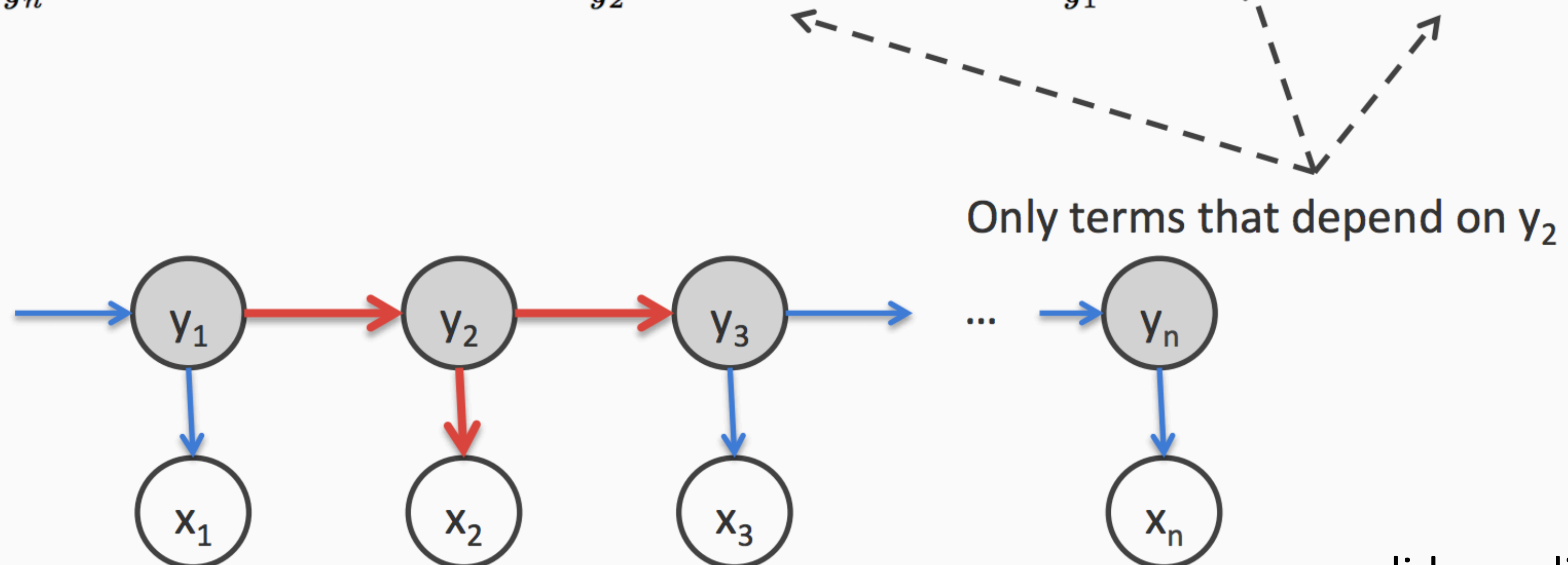
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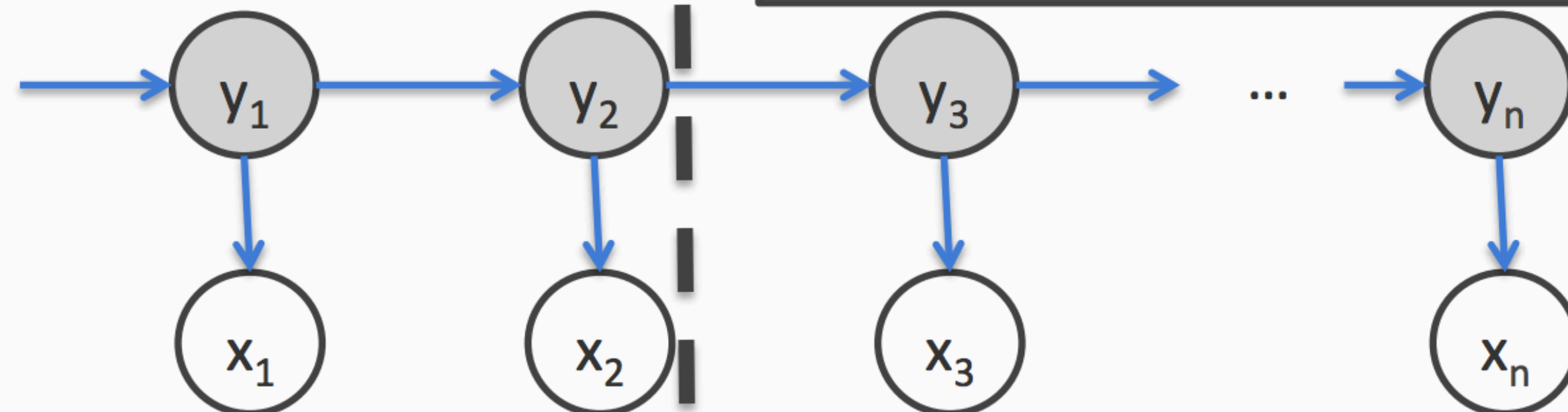
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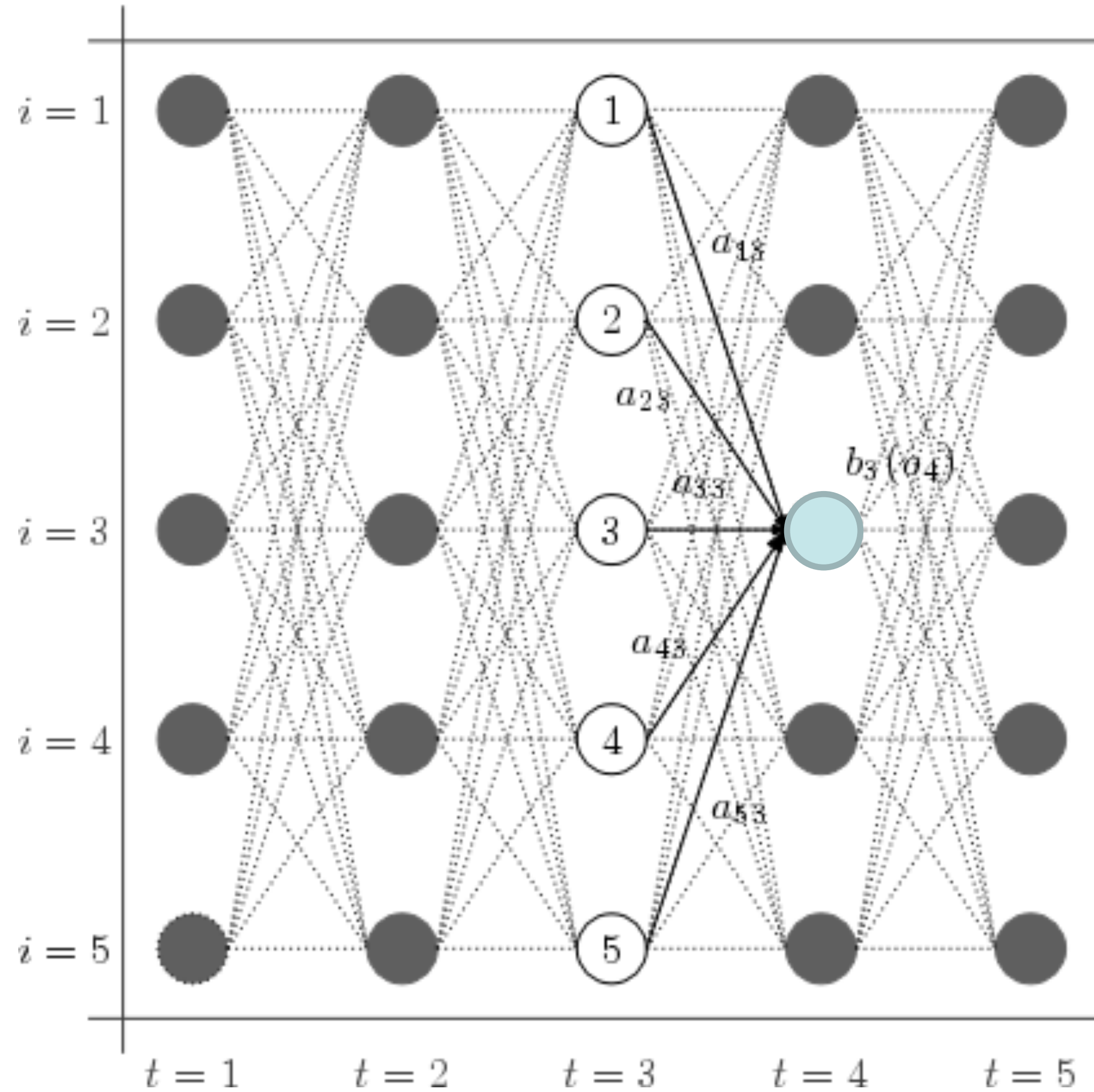
$$\text{score}_i(s) = \max_{y_{i-1}} P(s|y_{i-1})P(x_i|s) \text{score}_{i-1}(y_{i-1})$$



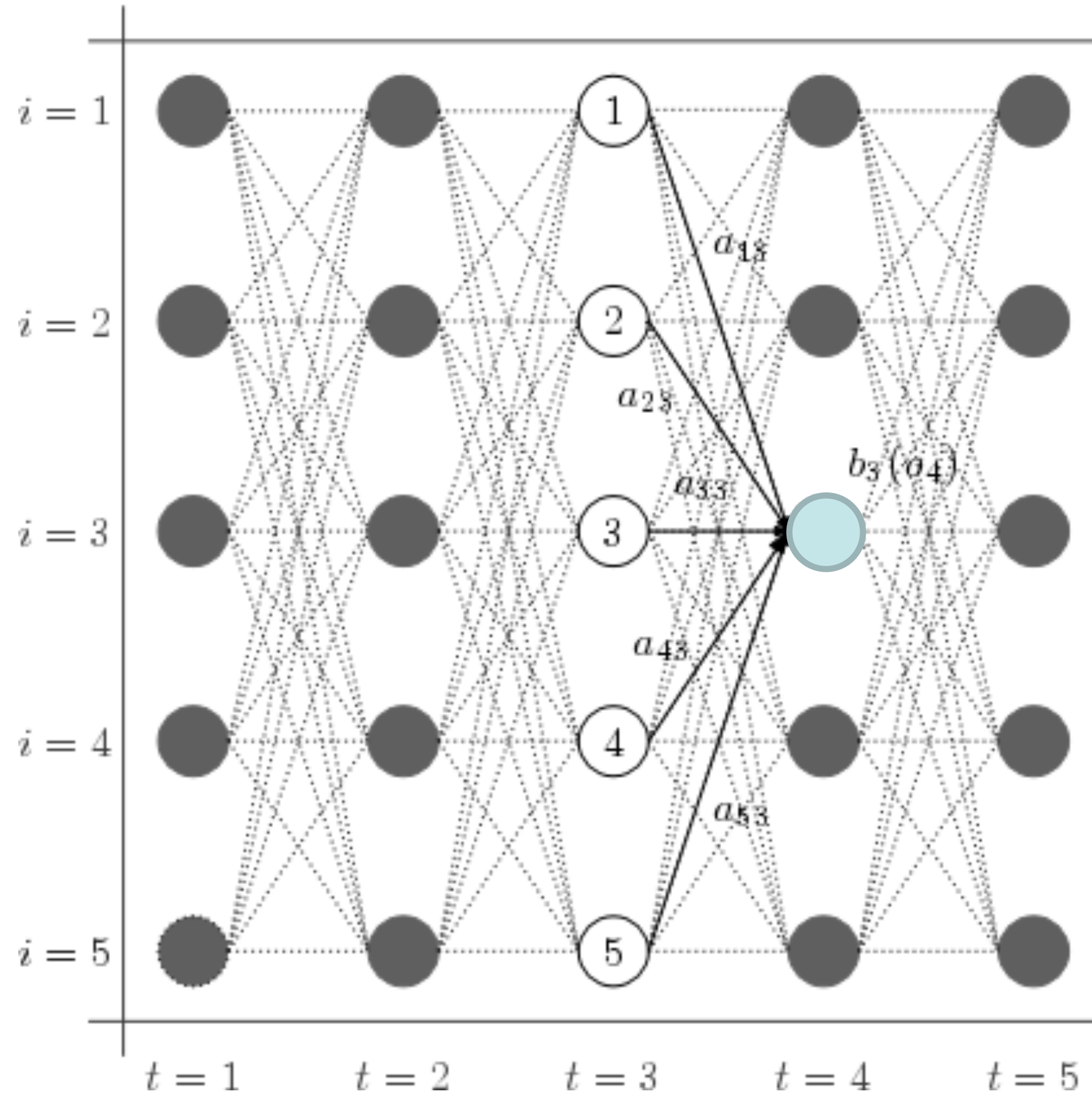
Abstract away the score for all decisions till here into **score**

slide credit: Vivek Srikumar

Viterbi Algorithm



Viterbi Algorithm



- ▶ “Think about” all possible immediate prior state values. Everything before that has already been accounted for by earlier stages.

Viterbi Algorithm

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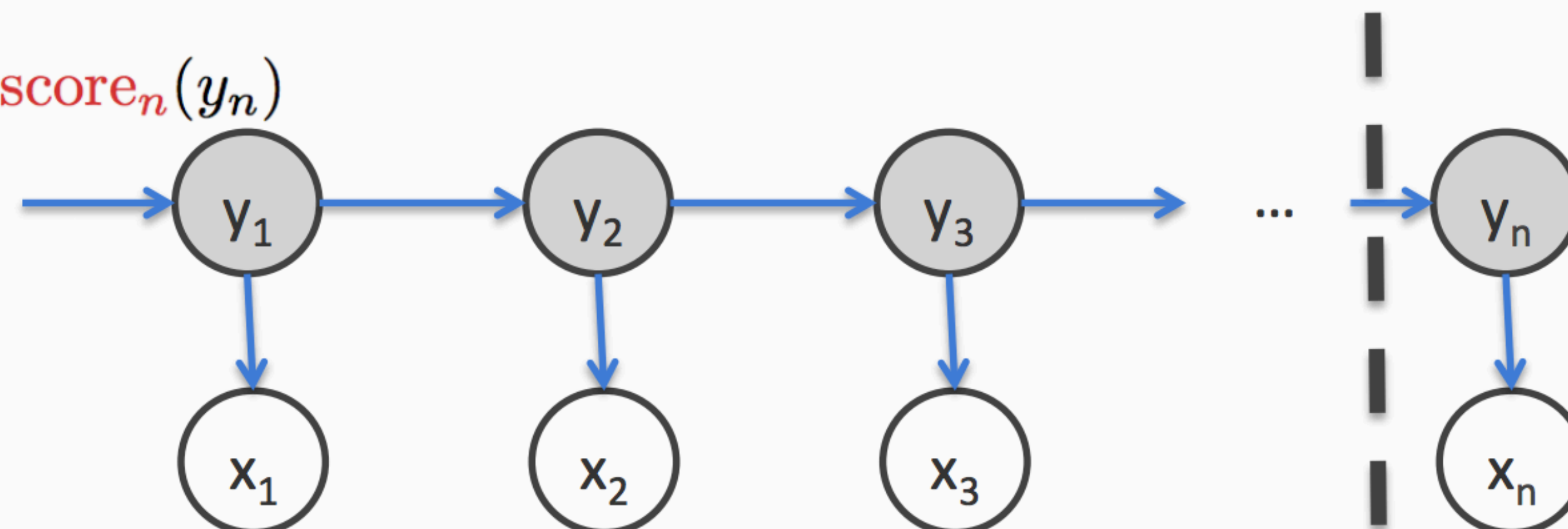
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⋮

$$= \max_{y_n} \text{score}_n(y_n)$$



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Viterbi Algorithm

1. **Initial:** For each state s , calculate

$$\text{score}_1(s) = P(s)P(x_1|s) = \pi_s B_{x_1,s}$$

2. **Recurrence:** For $i = 2$ to n , for every state s , calculate

$$\text{score}_i(s) = \max_{y_{i-1}} P(s|y_{i-1})P(x_i|s)\text{score}_{i-1}(y_{i-1})$$

$$= \max_{y_{i-1}} A_{y_{i-1},s} B_{s,x_i} \text{score}_{i-1}(y_{i-1})$$

3. **Final state:** calculate

$$\max_{\mathbf{y}} P(\mathbf{y}, \mathbf{x}|\pi, A, B) = \max_s \text{score}_n(s)$$

π : Initial probabilities

A: Transitions

B: Emissions

This only calculates the max. To get final answer (*argmax*),

- keep track of which state corresponds to the max at each step
- build the answer using these back pointers

Forward-Backward Algorithm

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Forward-Backward Algorithm

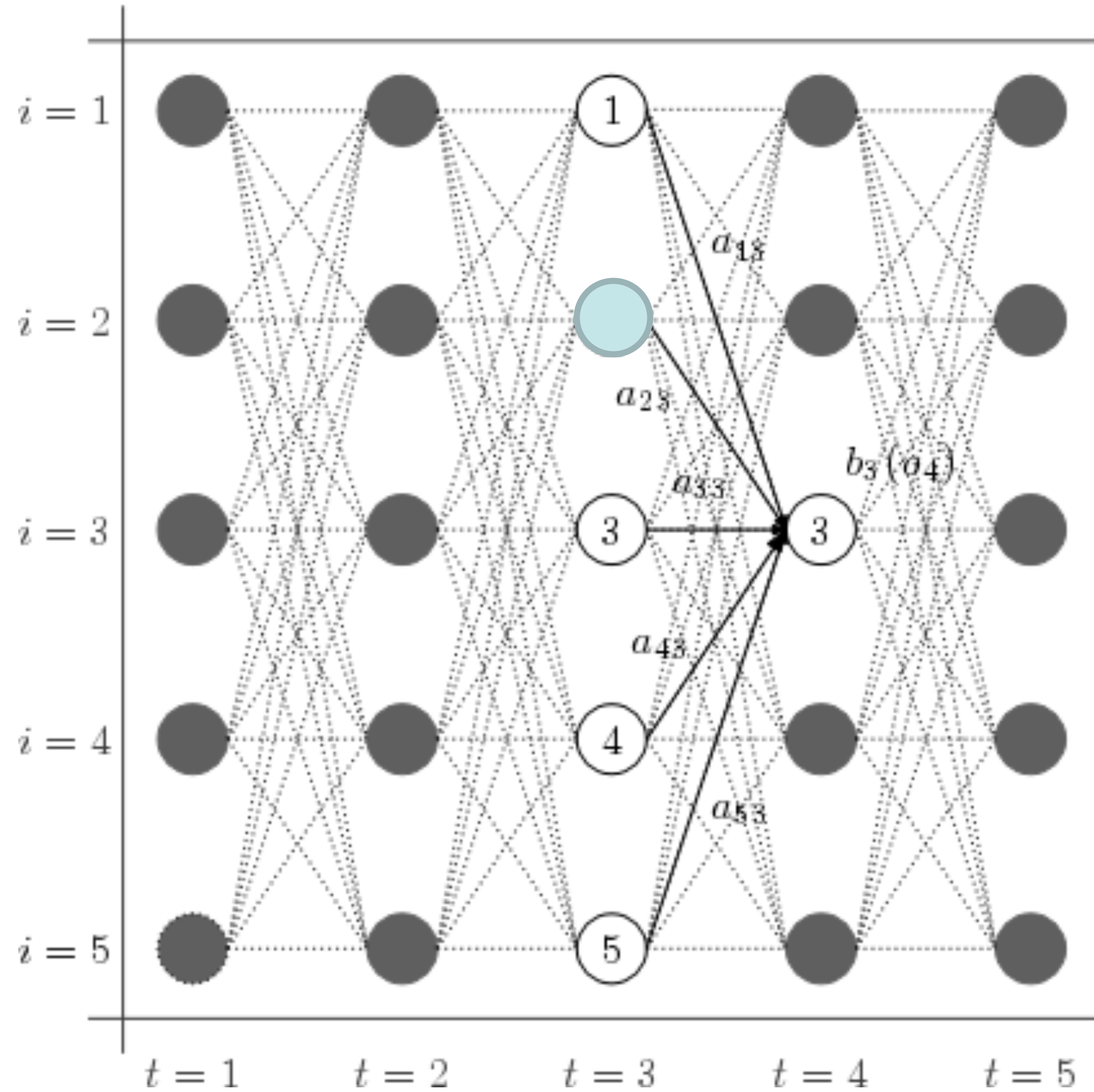
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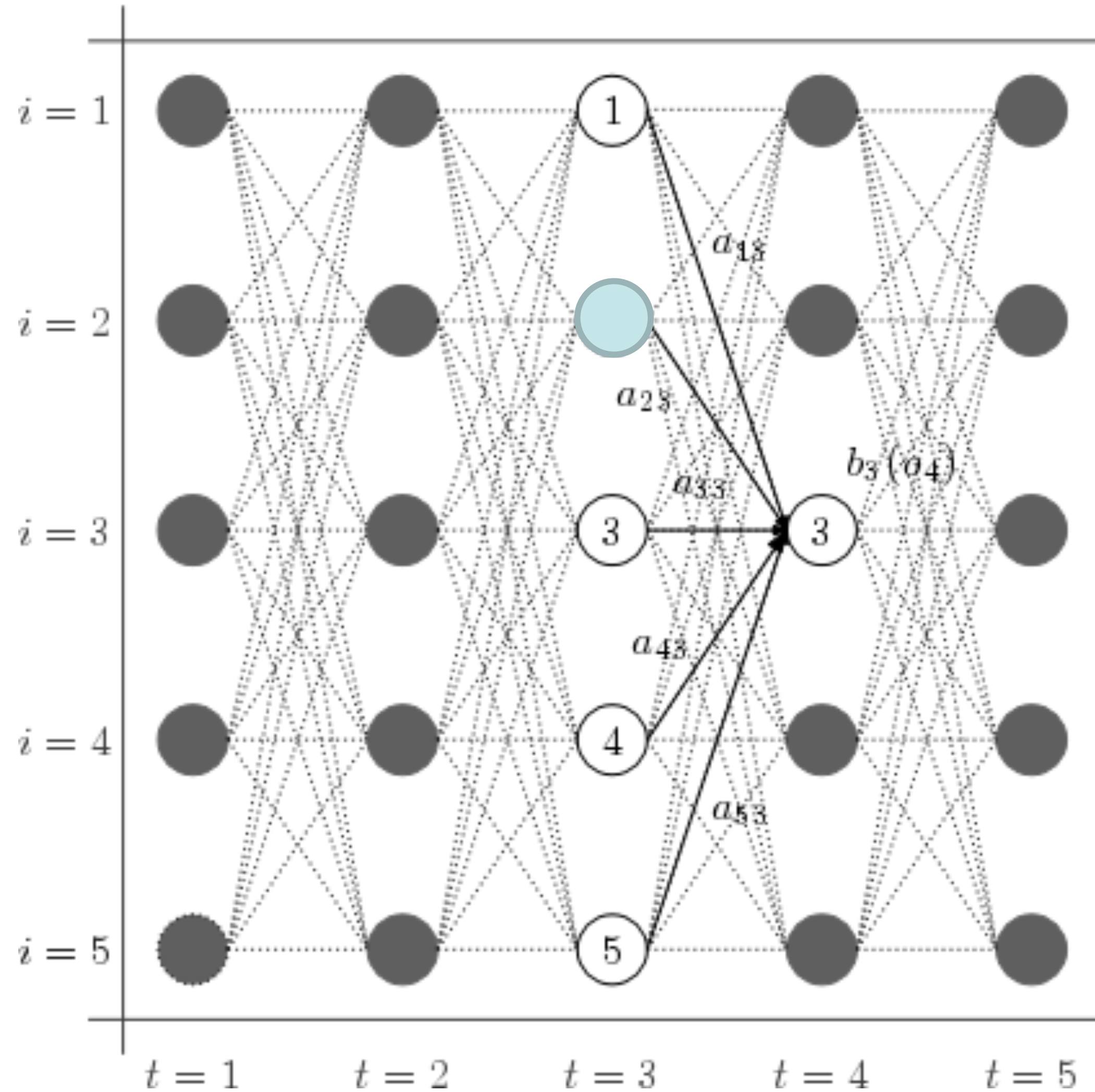
- ▶ Can compute marginals with dynamic programming as well using an algorithm called forward-backward

Forward-Backward Algorithm

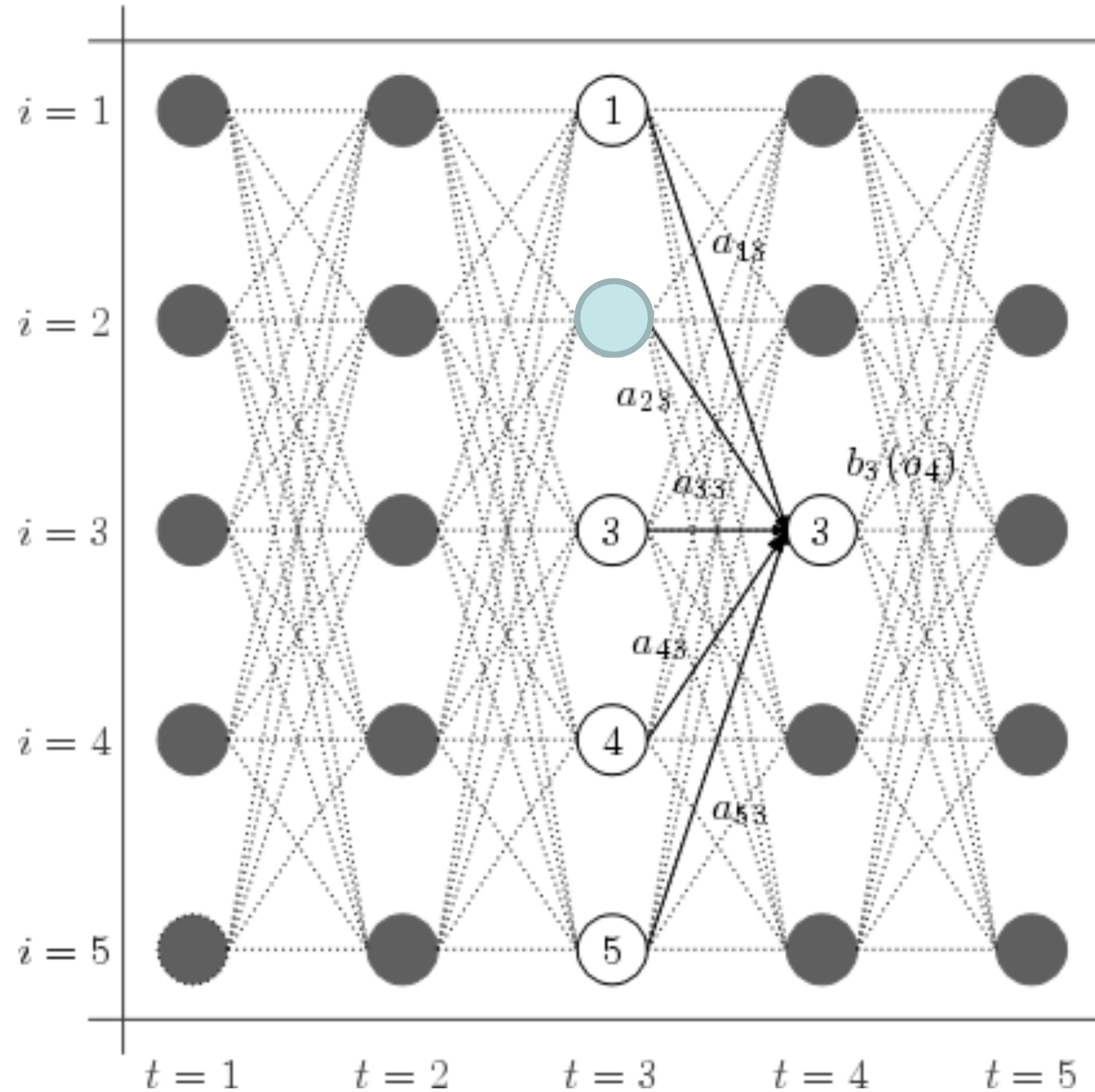


Forward-Backward Algorithm

$$P(y_3 = 2 | \mathbf{x}) =$$



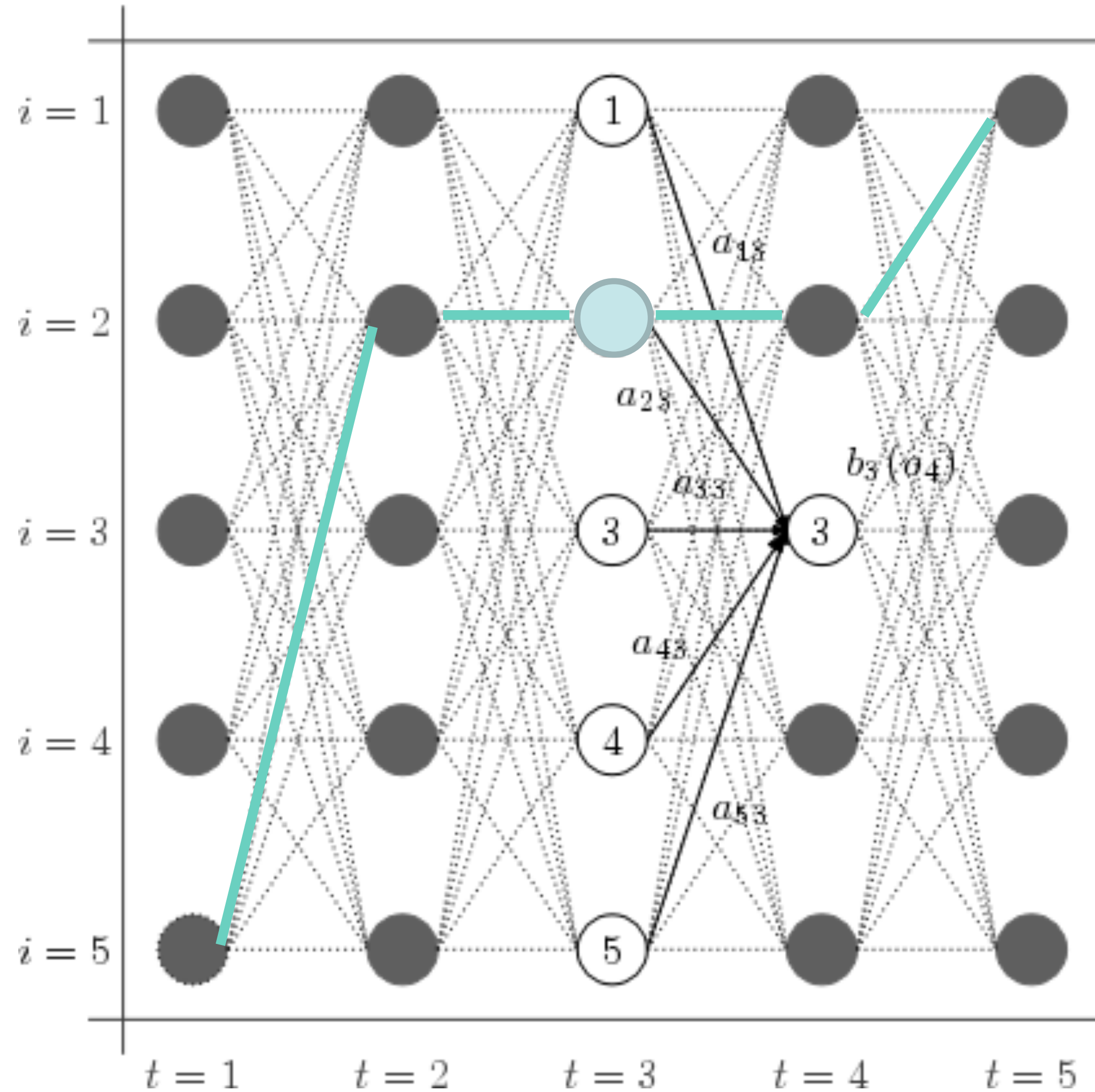
Forward-Backward Algorithm



$$P(y_3 = 2 | \mathbf{x}) =$$

$$\frac{\text{sum of all paths through state 2 at time 3}}{\text{sum of all paths}}$$

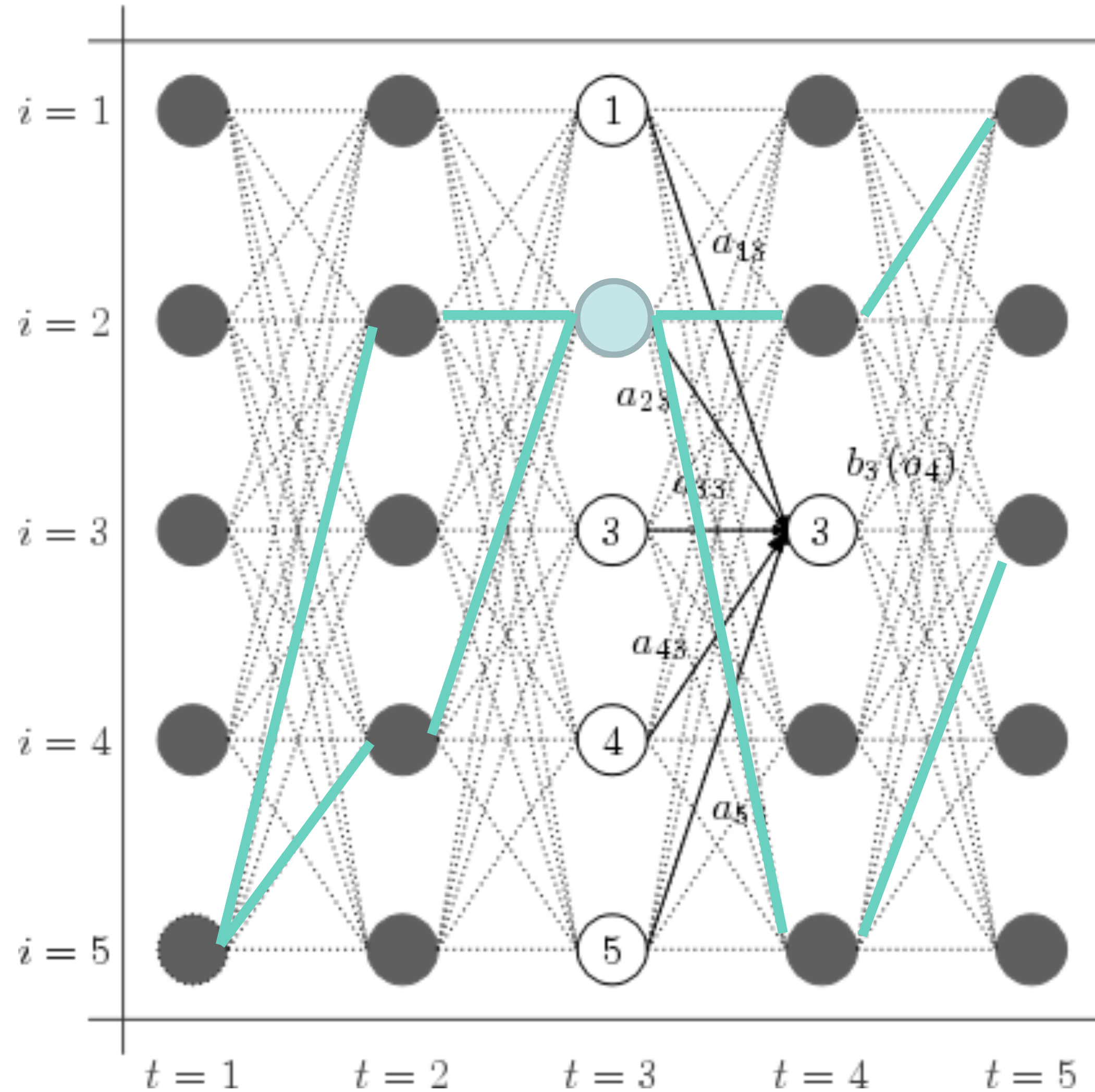
Forward-Backward Algorithm



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sum of all paths through state 2 at time 3
 sum of all paths

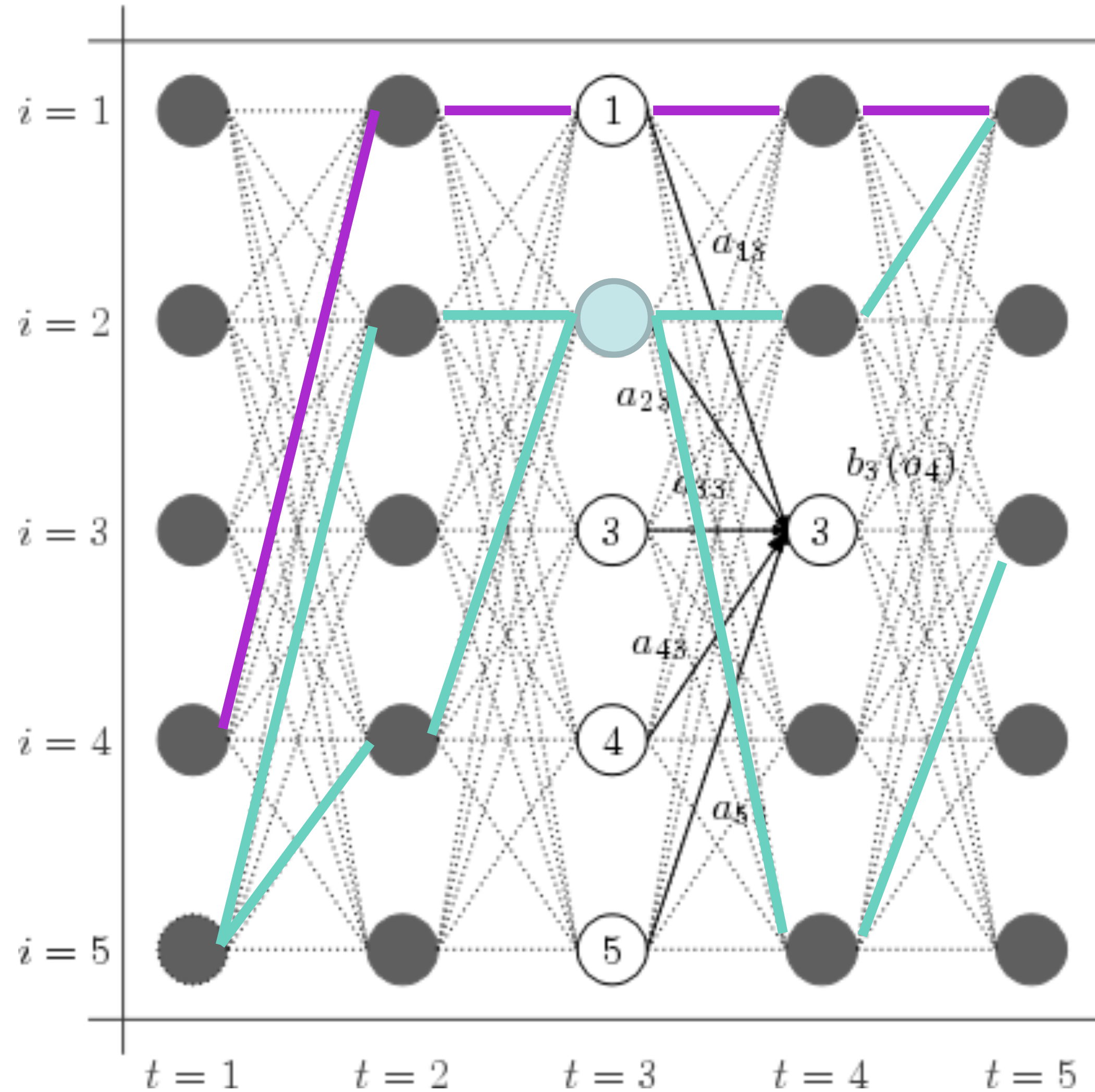
Forward-Backward Algorithm



$$P(y_3 = 2 | \mathbf{x}) =$$

$$\frac{\text{sum of all paths through state 2 at time 3}}{\text{sum of all paths}}$$

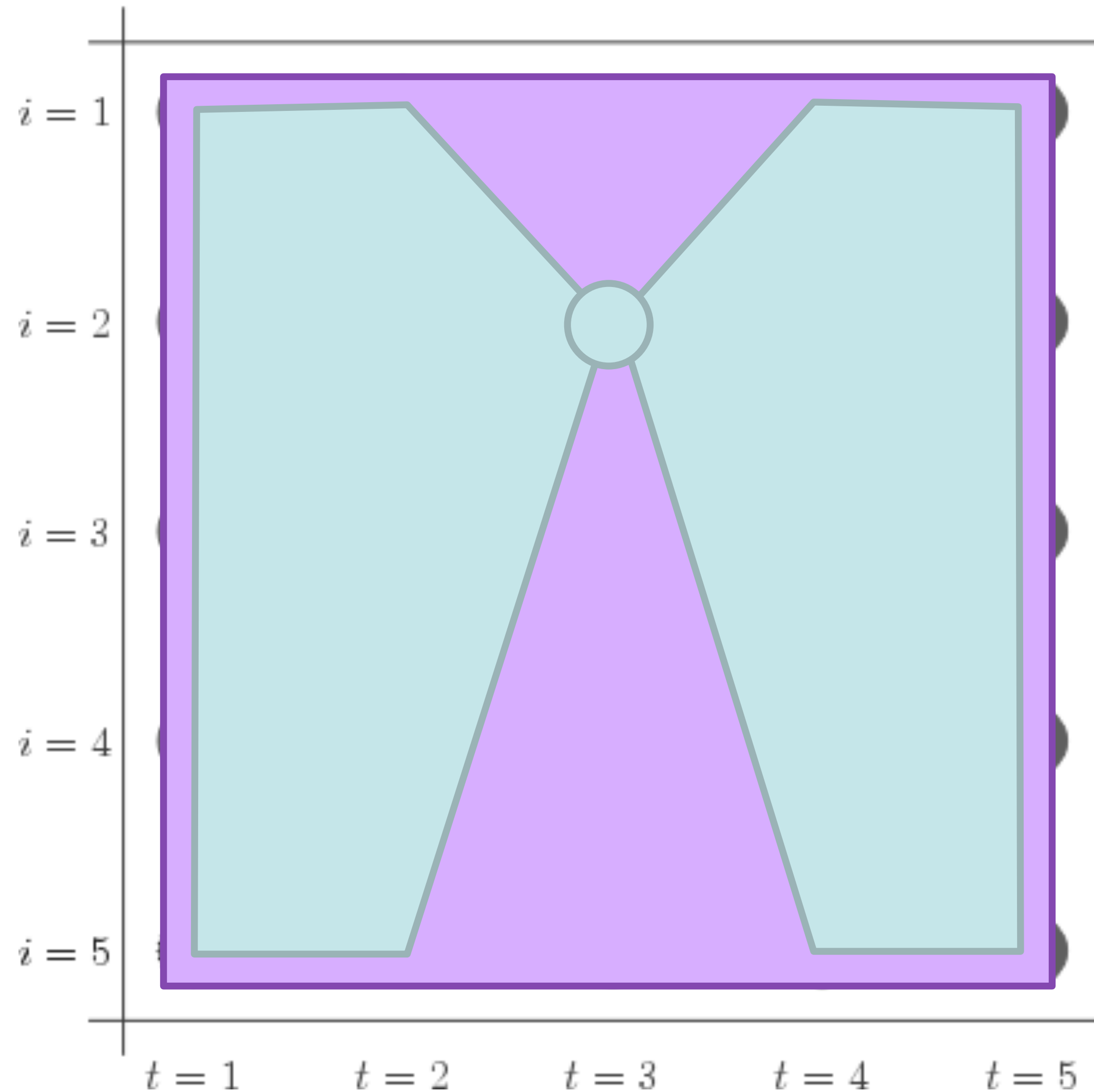
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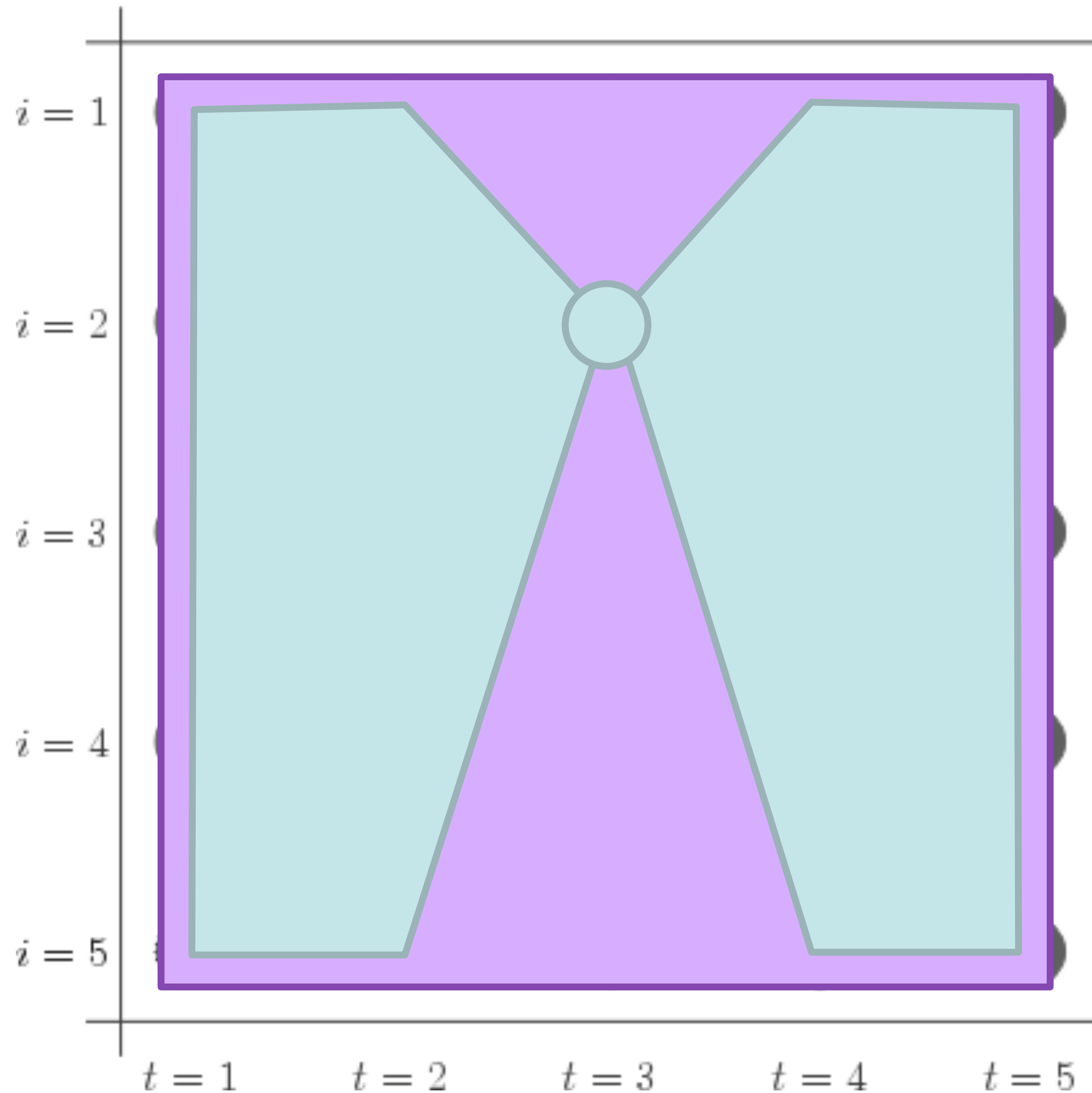
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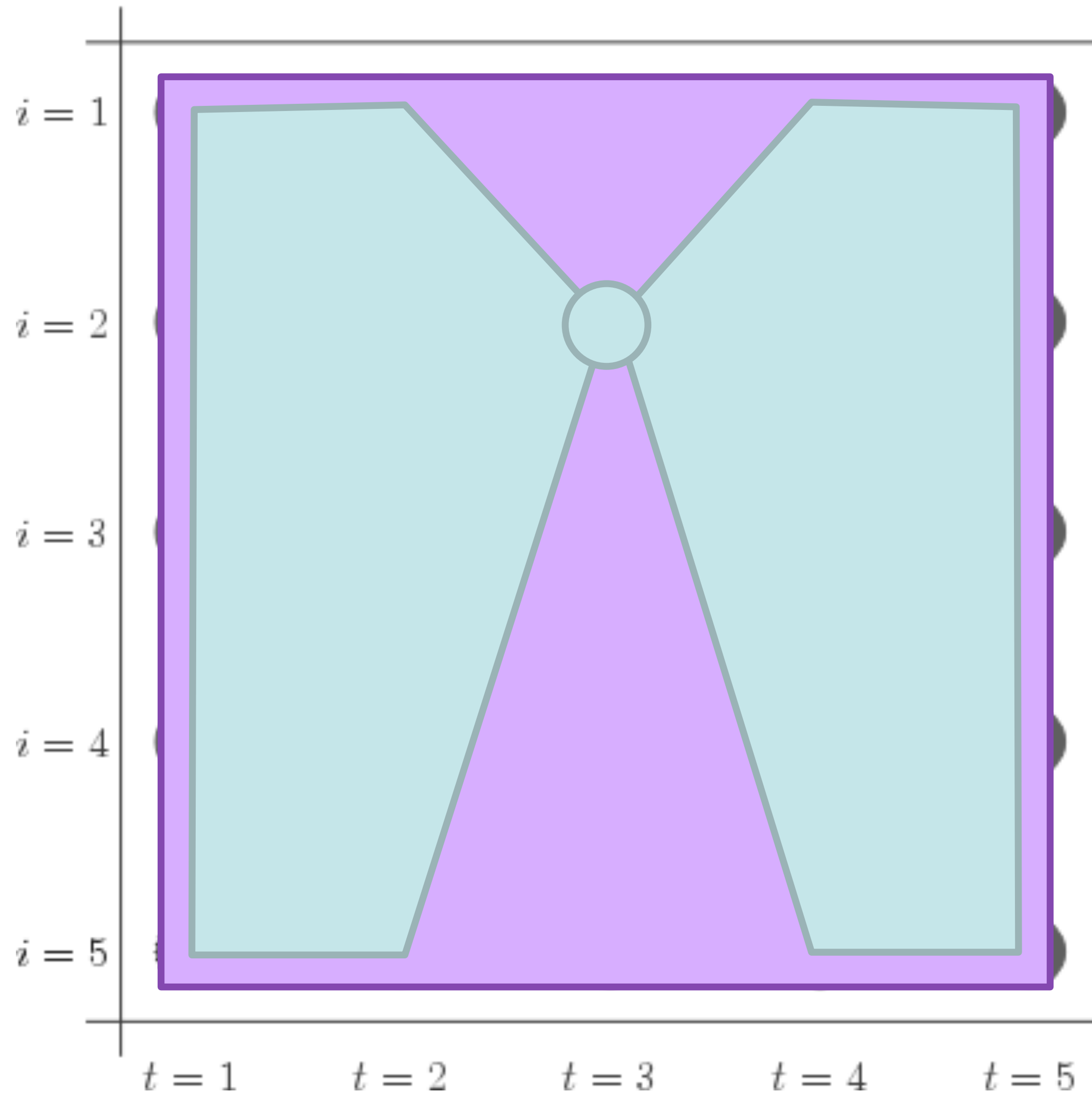


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sum of all paths

$$= \frac{\text{Diagram of forward pass through state 2 at time 3}}{\text{Diagram of full forward pass}}$$

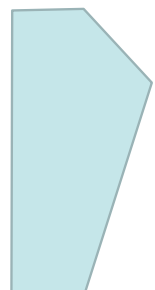
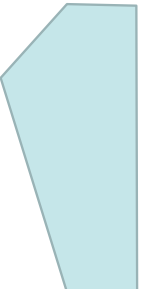
Forward-Backward Algorithm



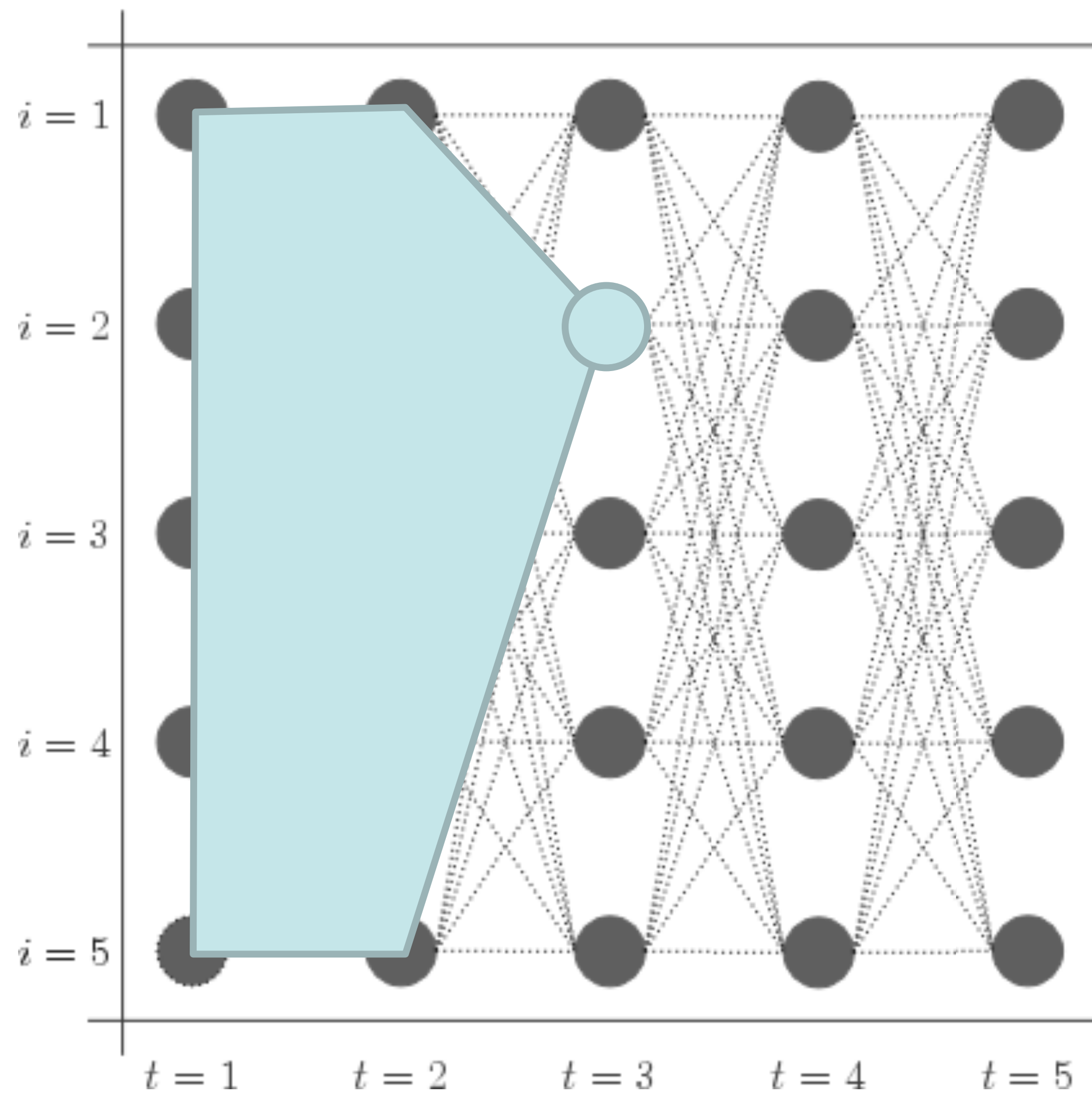
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sum of all paths through state 2 at time 3
 sum of all paths

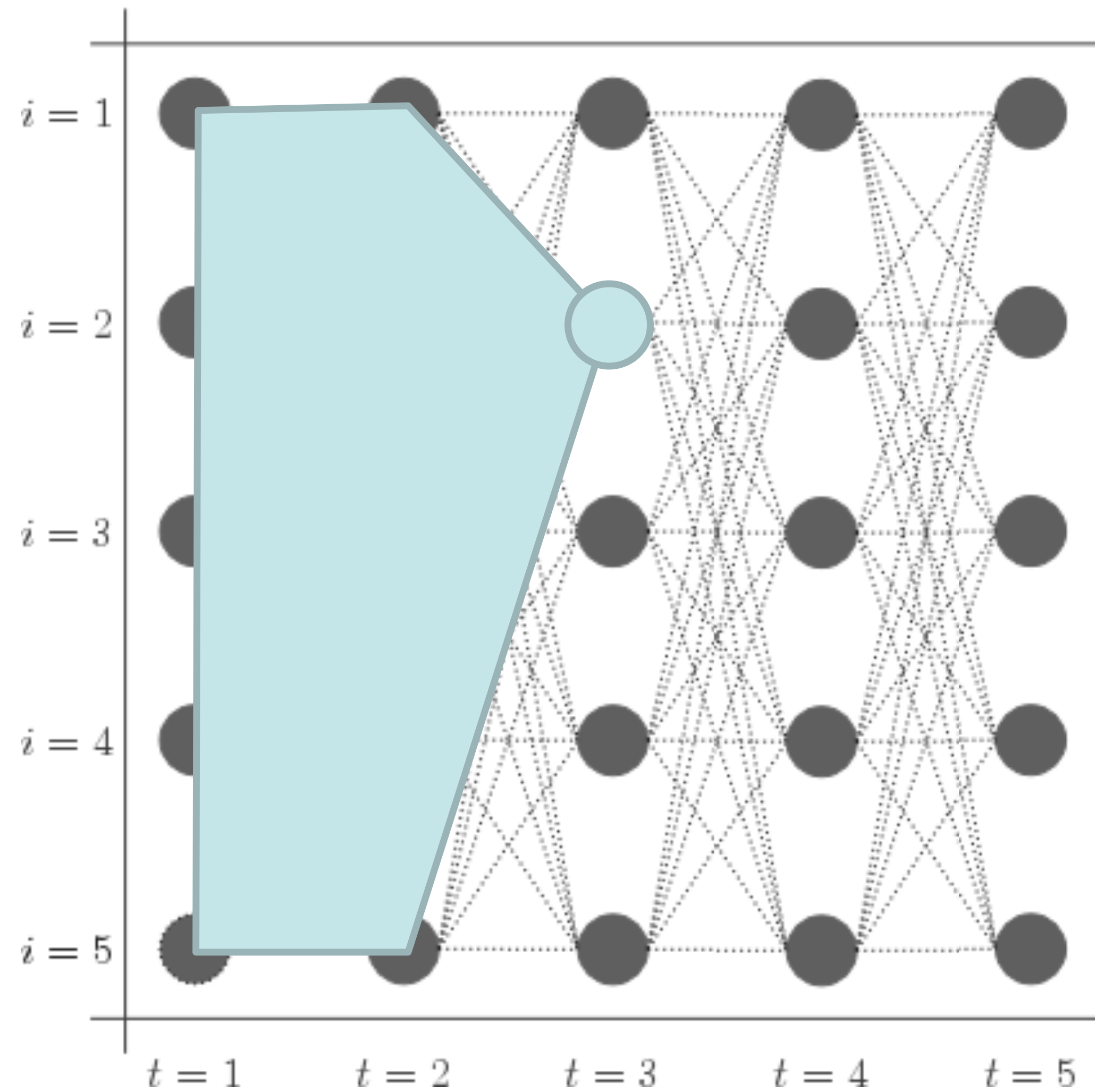
$$= \frac{\text{[Forward Pass Diagram]} \cdot \text{[Backward Pass Diagram]}}{\text{[Total Forward Pass Diagram]}}$$

- ▶ Easiest and most flexible to do one pass to compute  and one to compute 

Forward-Backward Algorithm

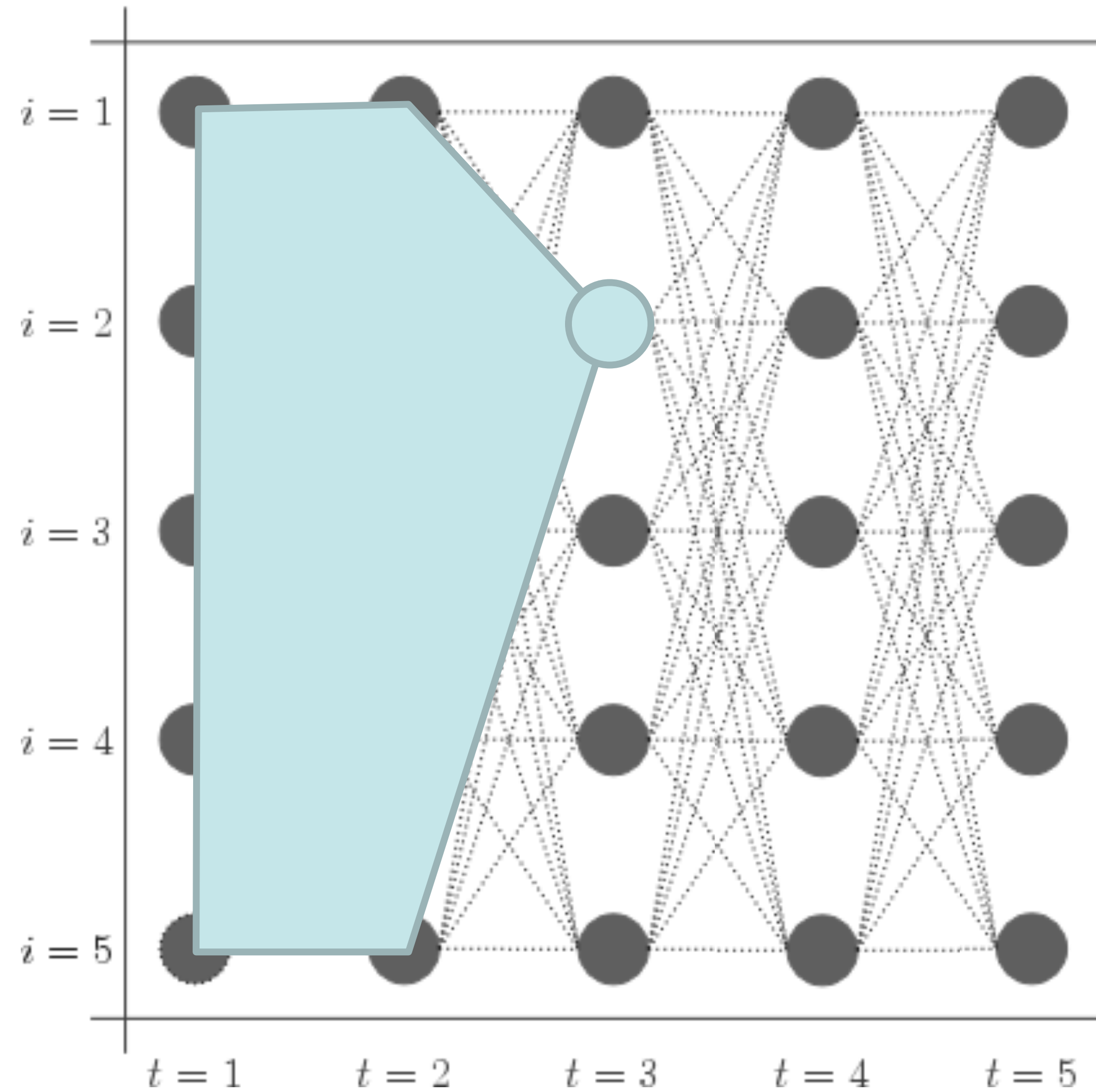


Forward-Backward Algorithm



► Initial:

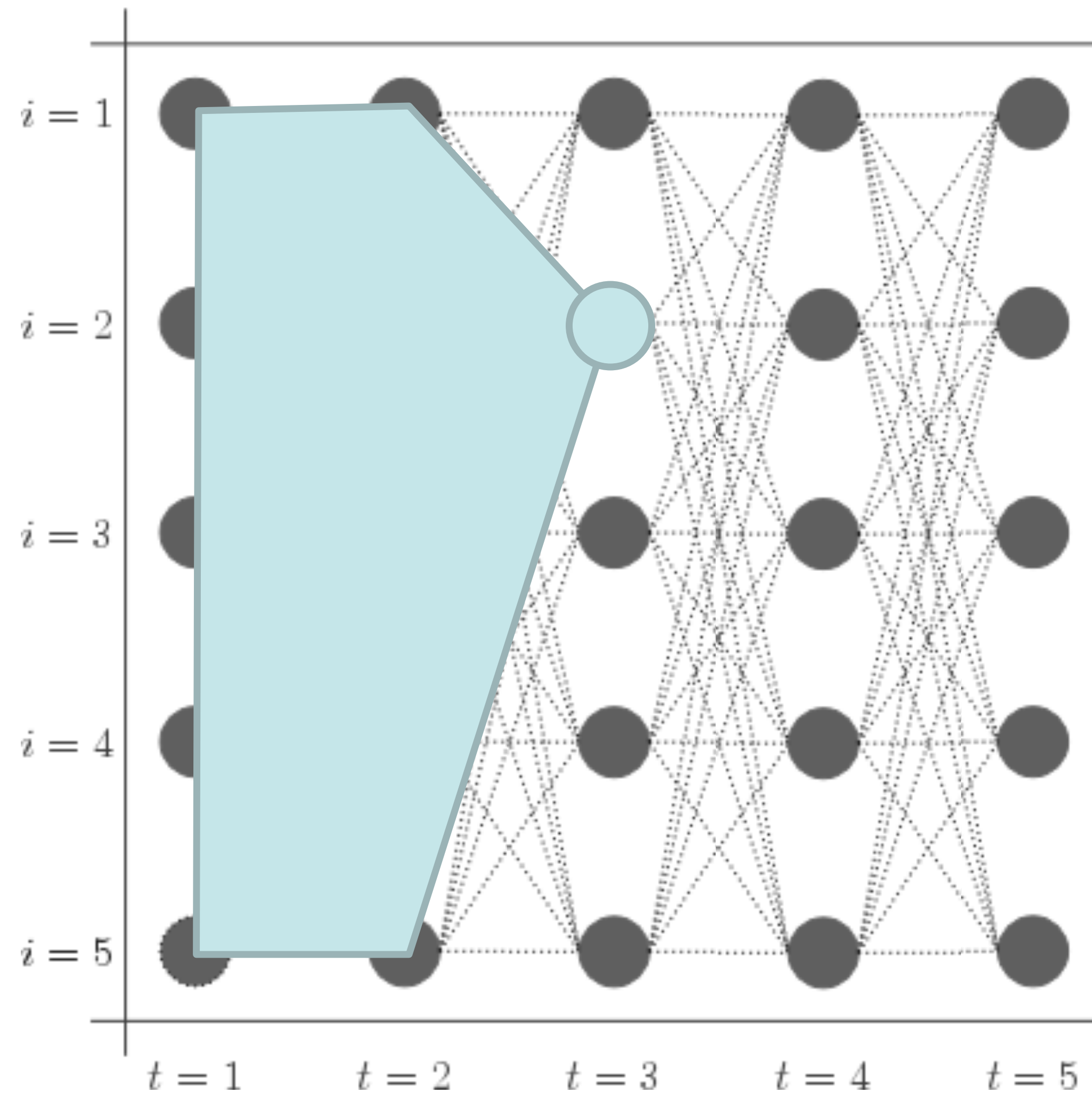
Forward-Backward Algorithm



► Initial:

$$\alpha_1(s) = P(s)P(x_1|s)$$

Forward-Backward Algorithm

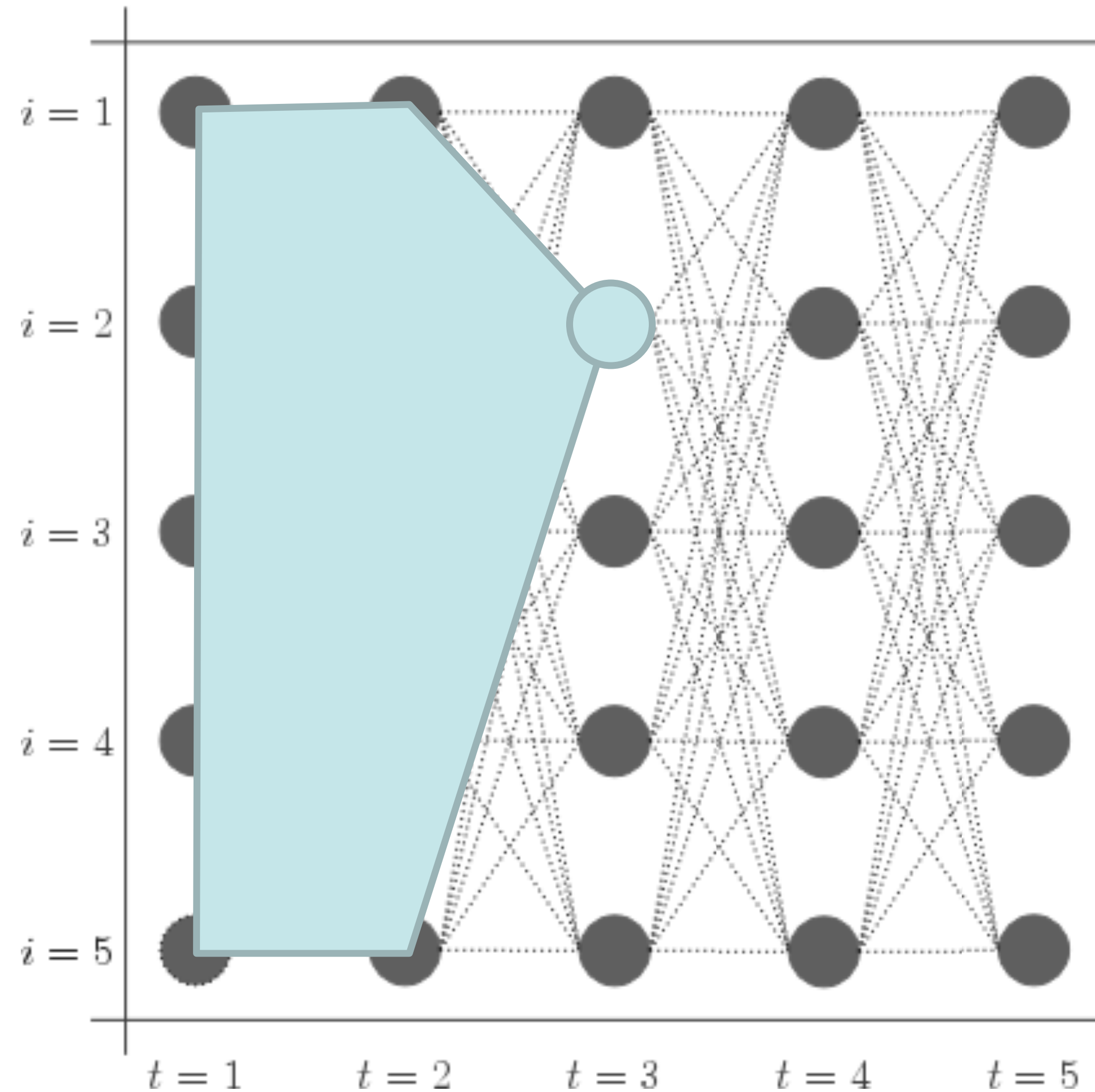


► Initial:

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► Recurrence:

Forward-Backward Algorithm



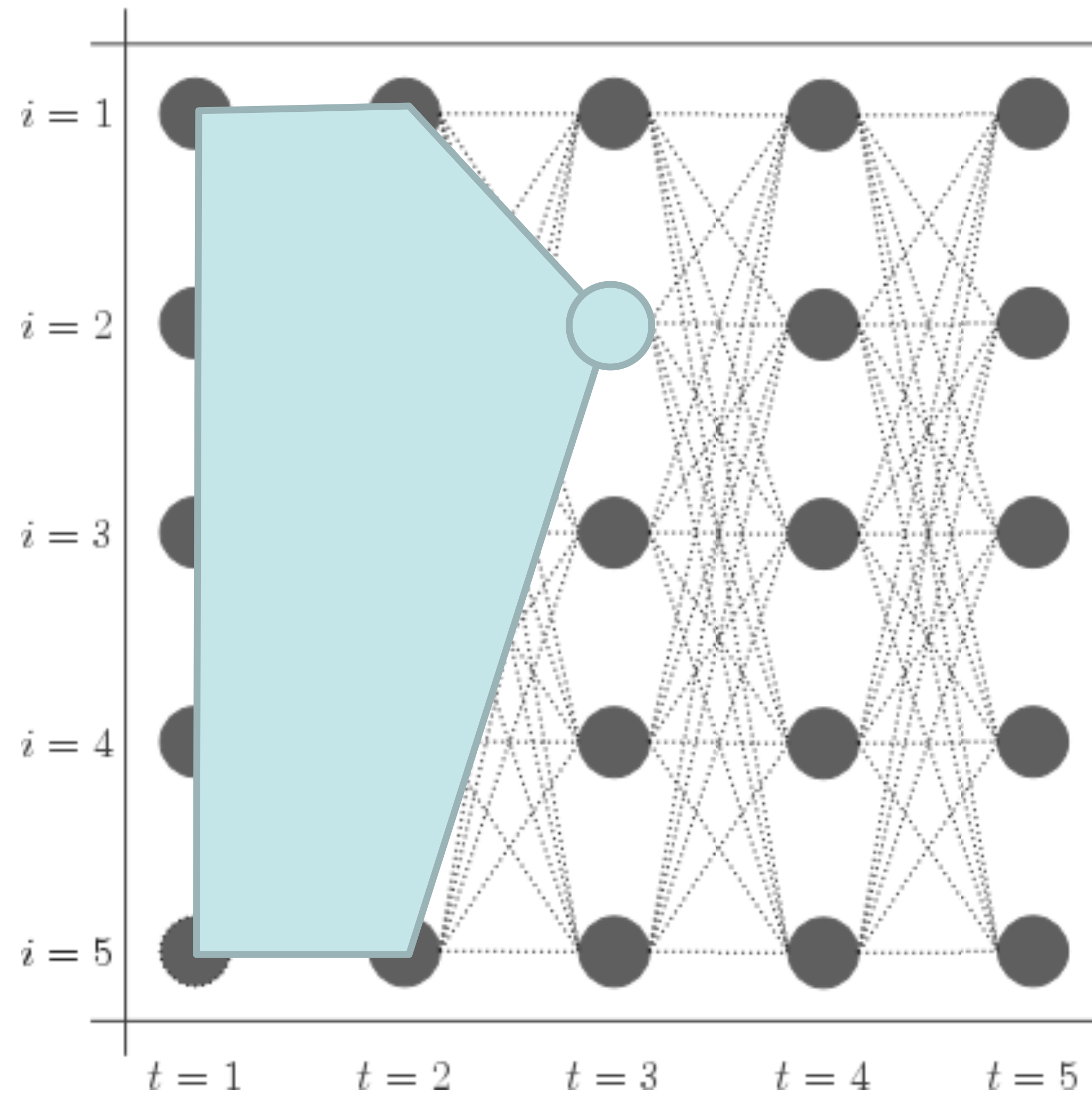
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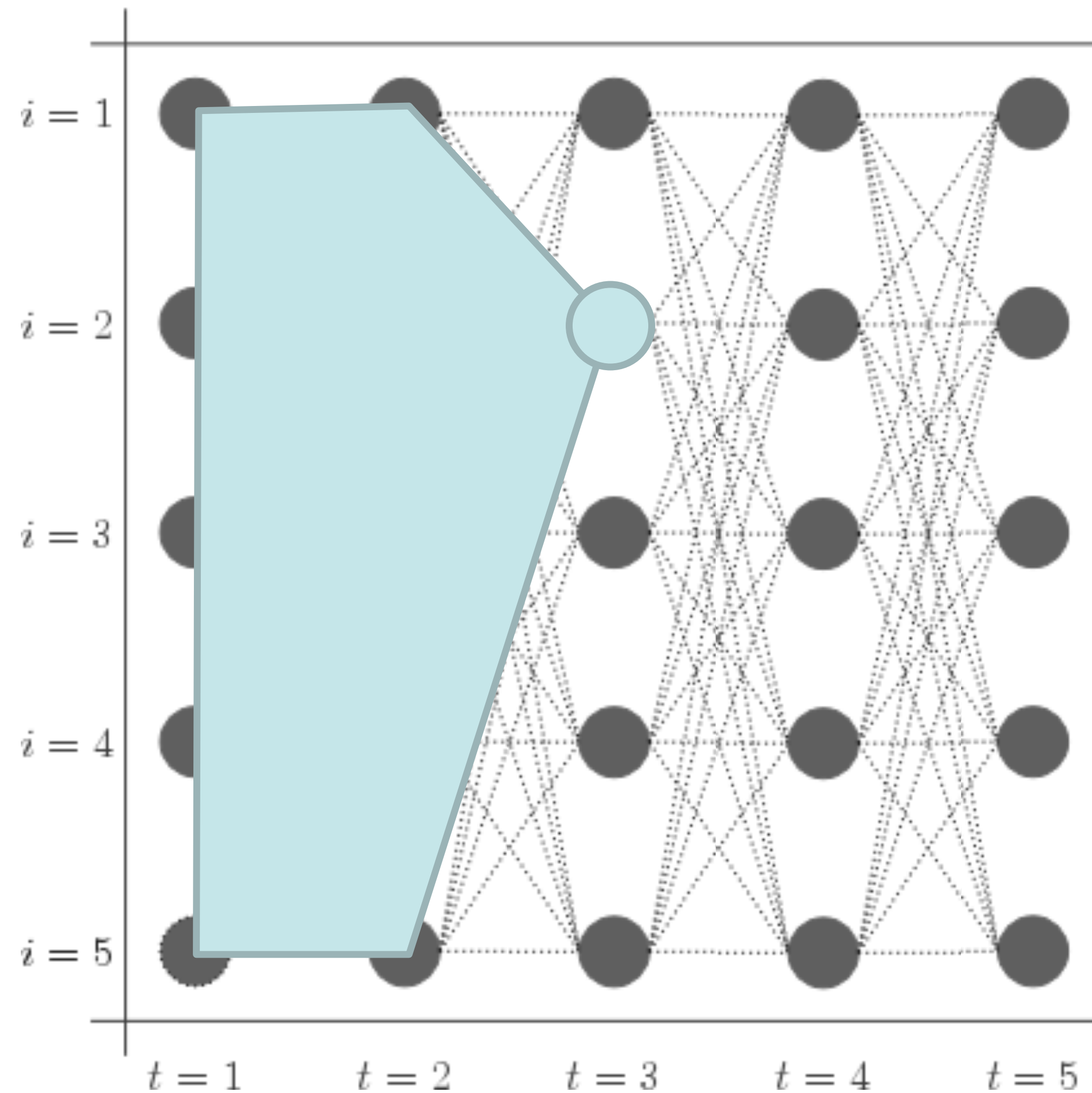
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- ▶ Same as Viterbi but summing instead of maxing!

Forward-Backward Algorithm



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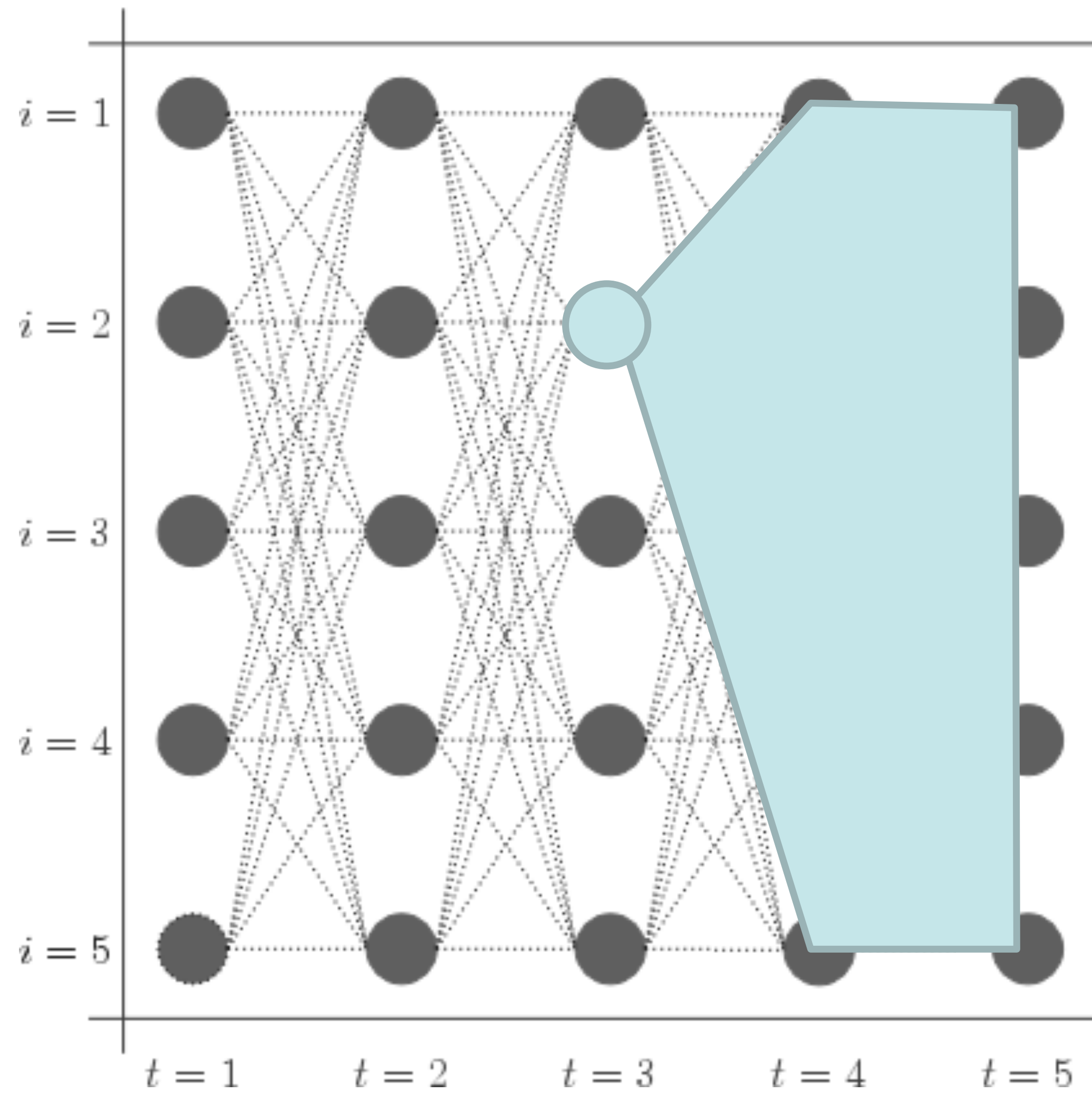
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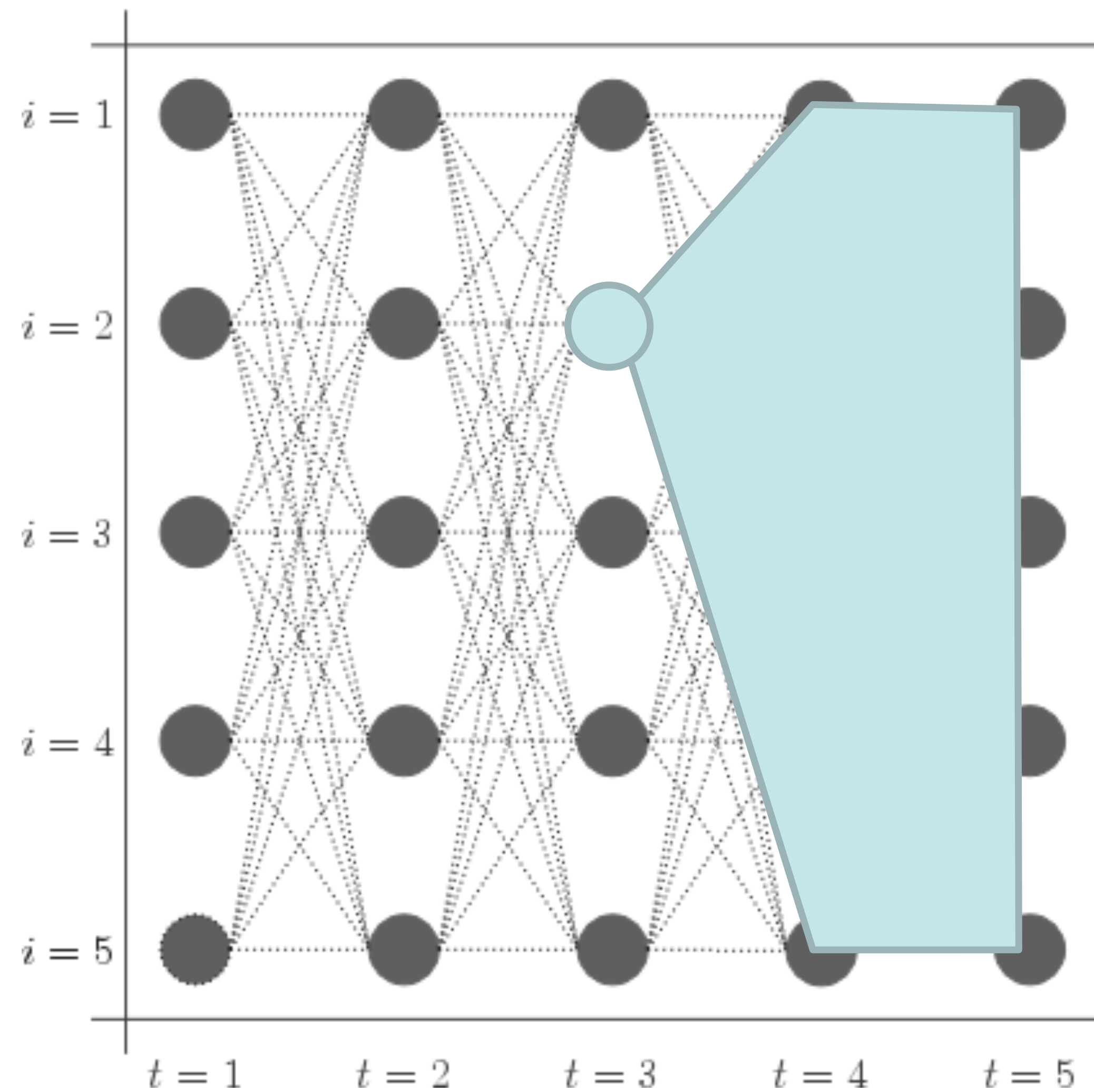
$$\alpha_t(s_t) = \sum_{s_{t-1}} \alpha_{t-1}(s_{t-1})P(s_t|s_{t-1})P(x_t|s_t)$$

- ▶ Same as Viterbi but summing instead of maxing!
- ▶ These quantities get very small!
Store everything as log probabilities

Forward-Backward Algorithm

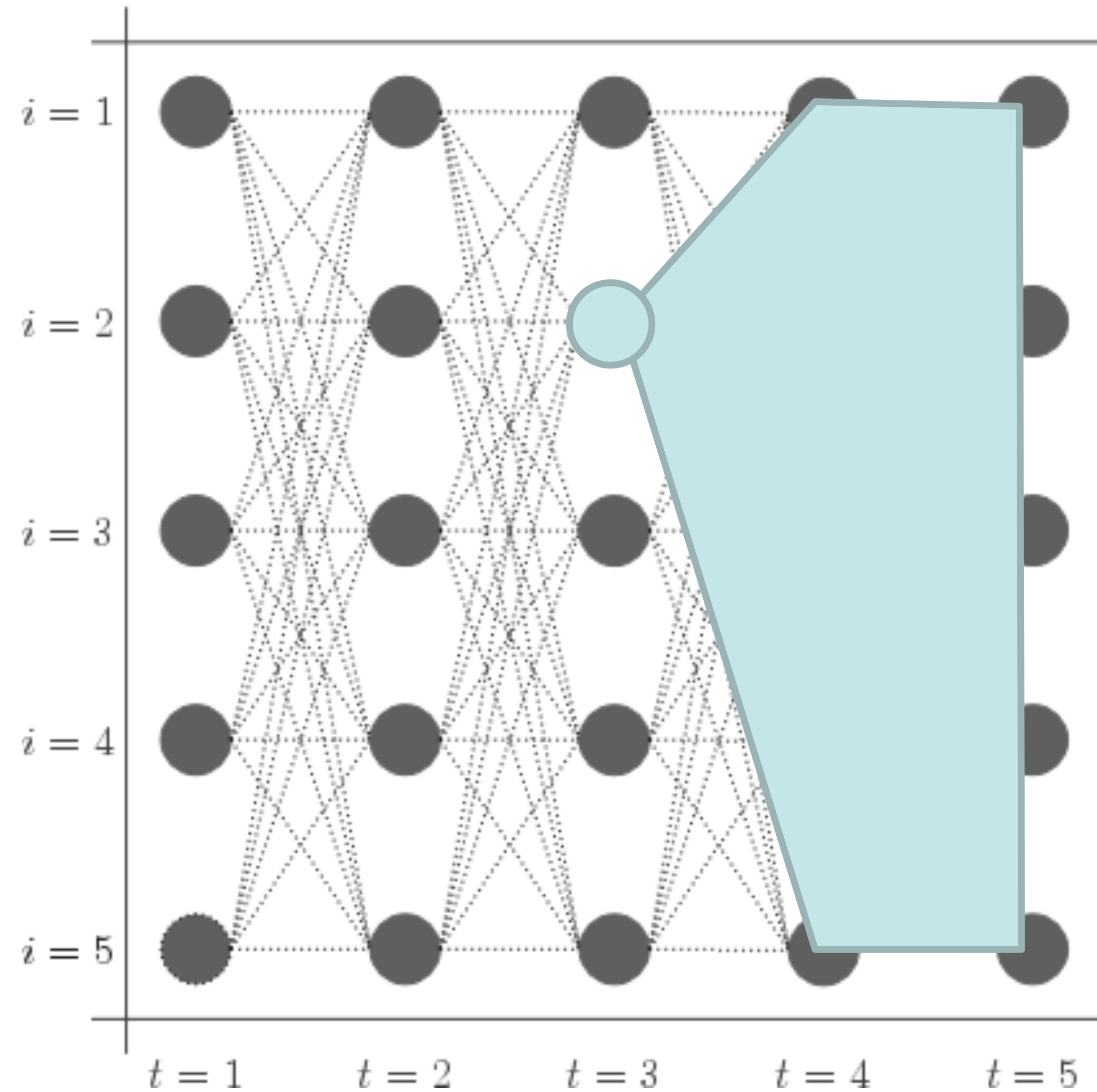


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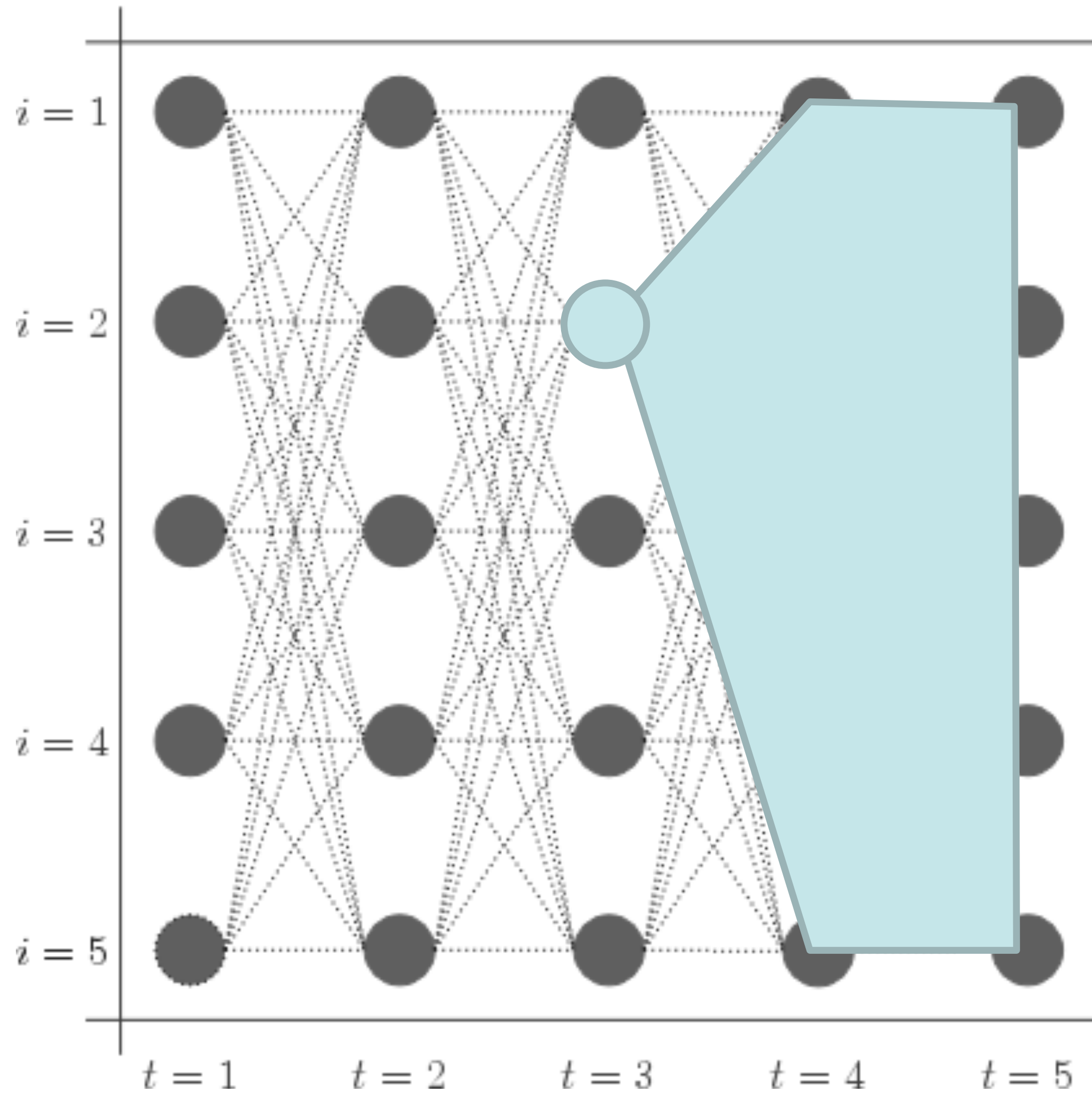
Forward-Backward Algorithm



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Forward-Backward Algorithm

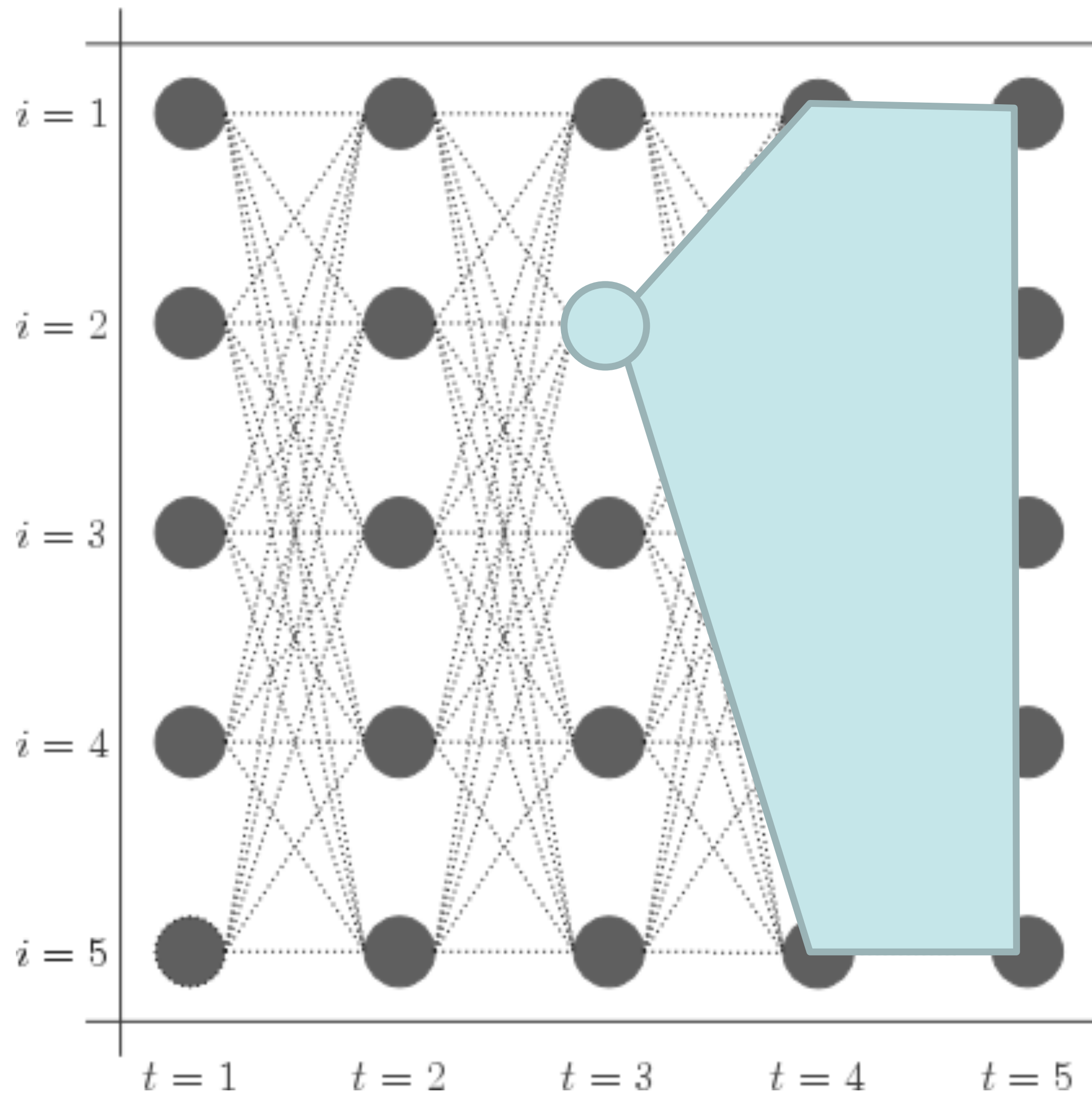


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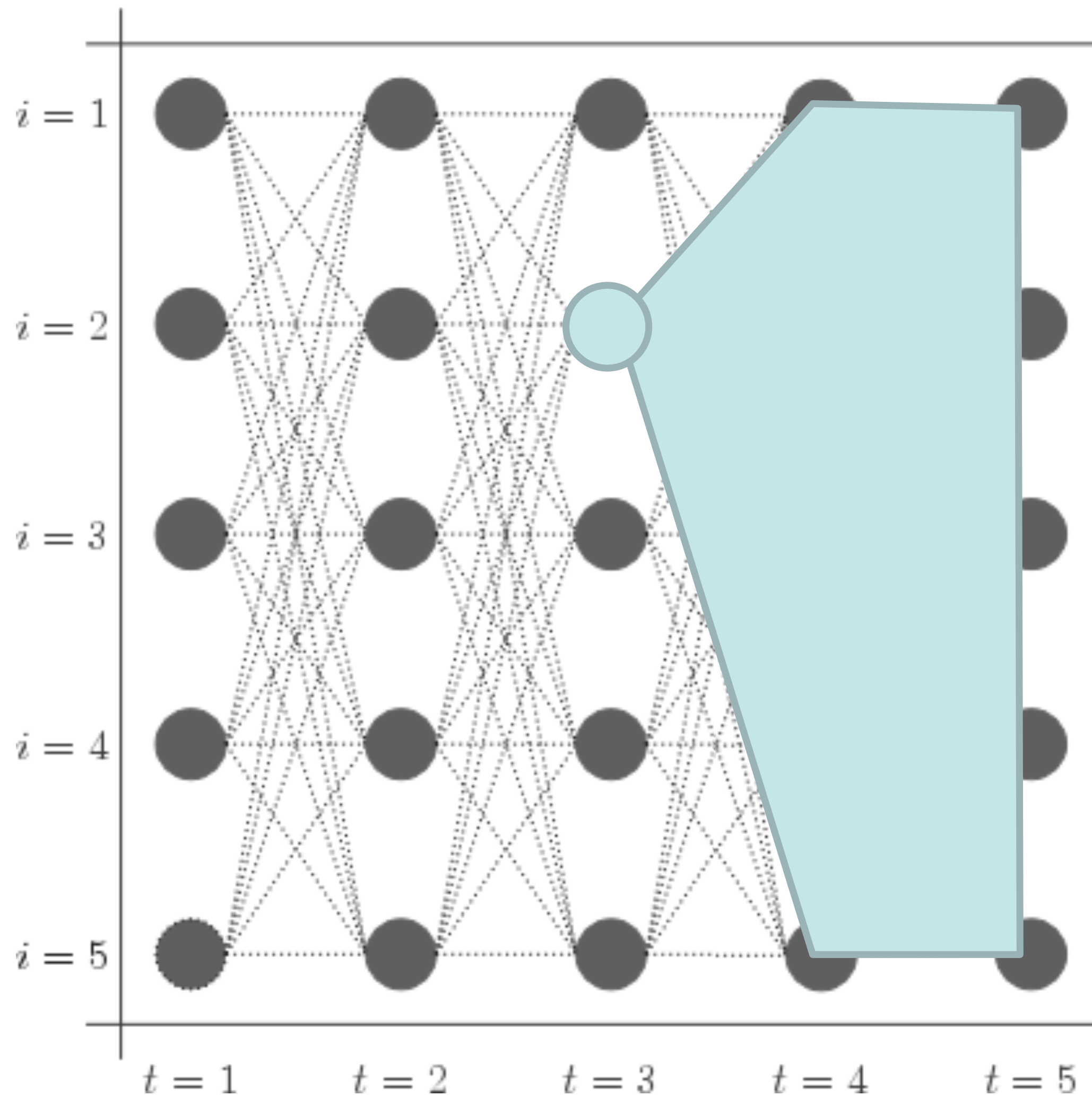
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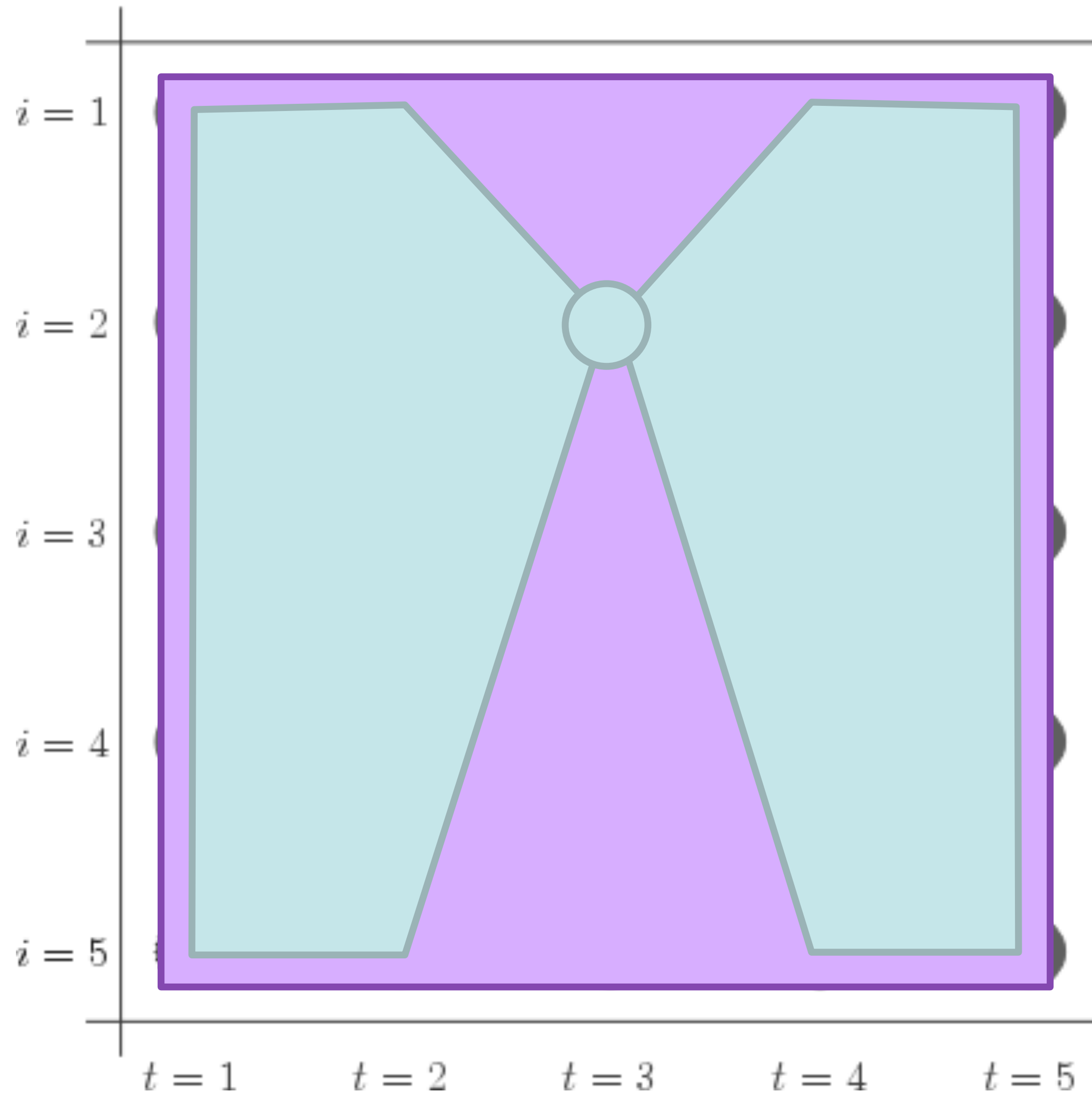
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Forward-Backward Algorithm



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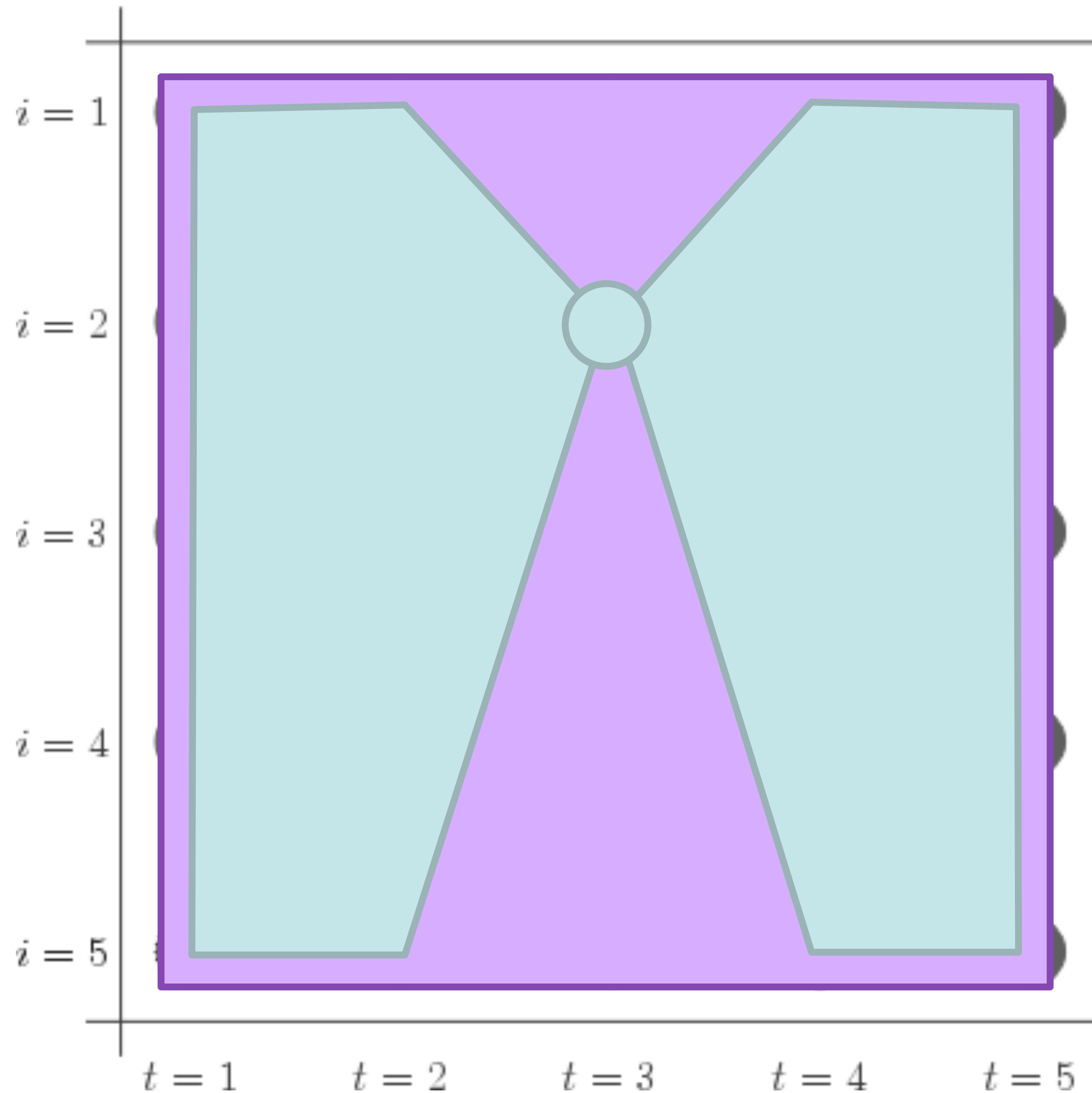
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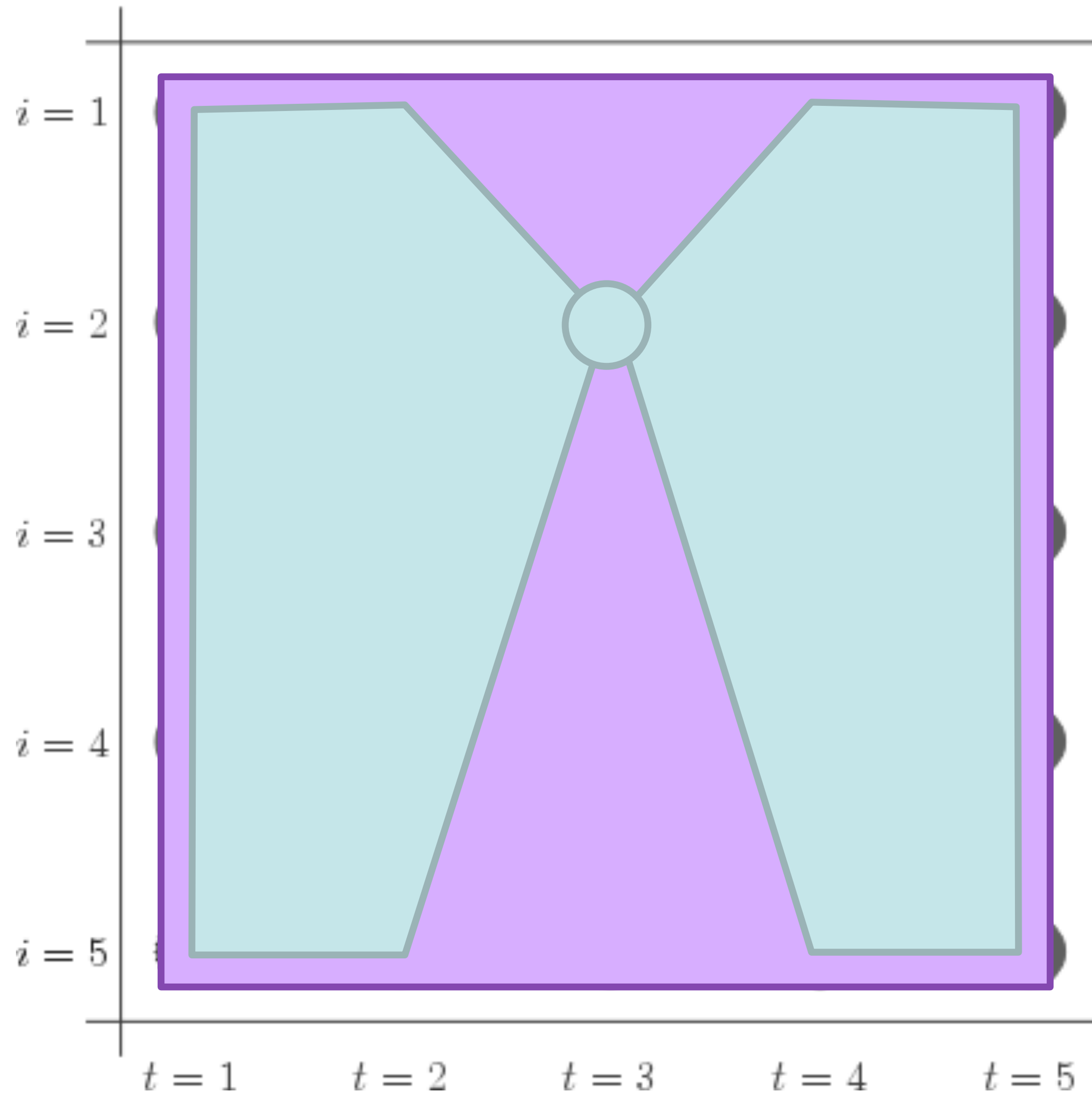
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$$= \frac{\text{[Diagram of path from } t=3 \text{ to } t=5 \text{ with circle at } (3,2)]}{\text{[Purple square]}}$$

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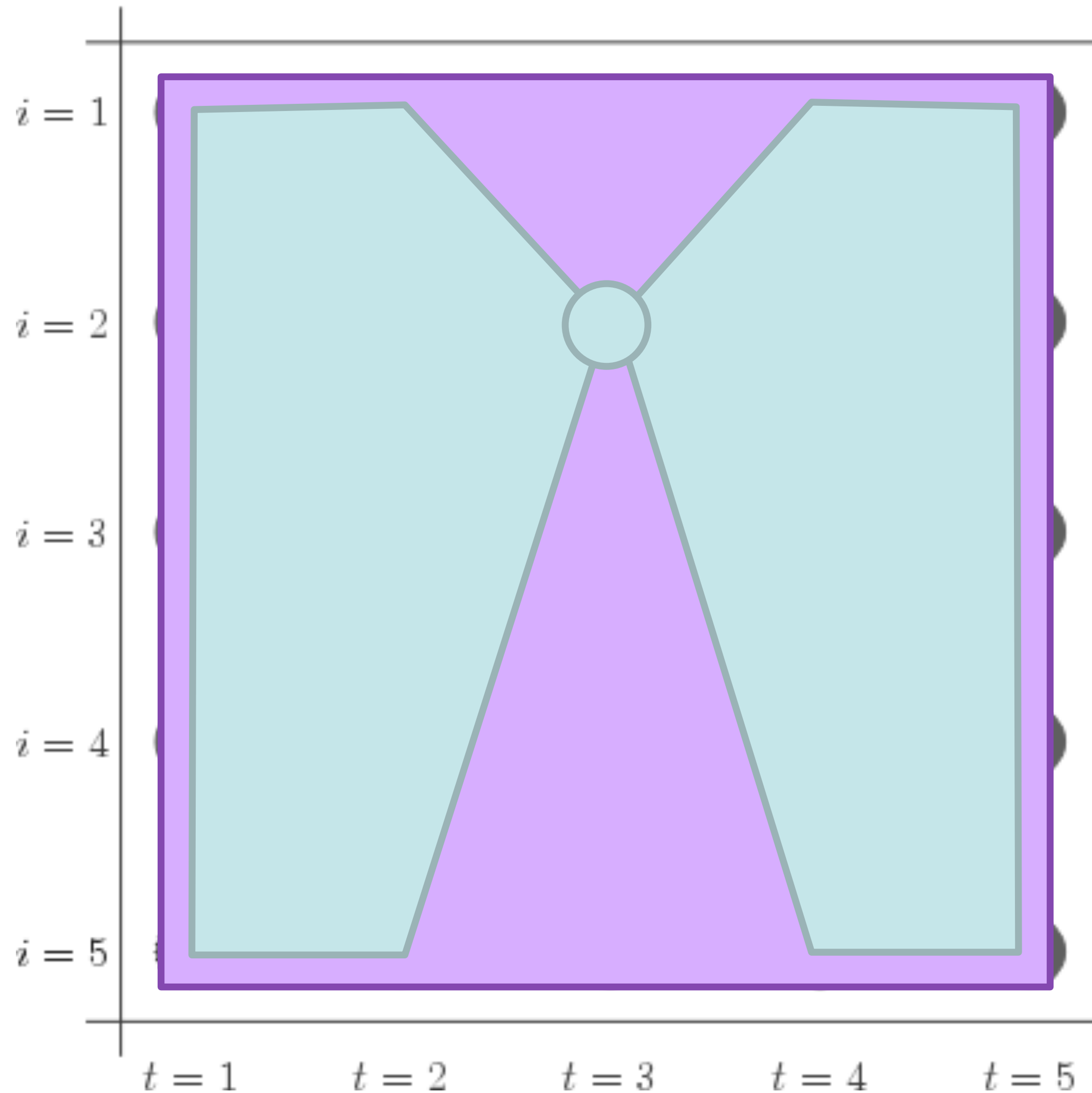
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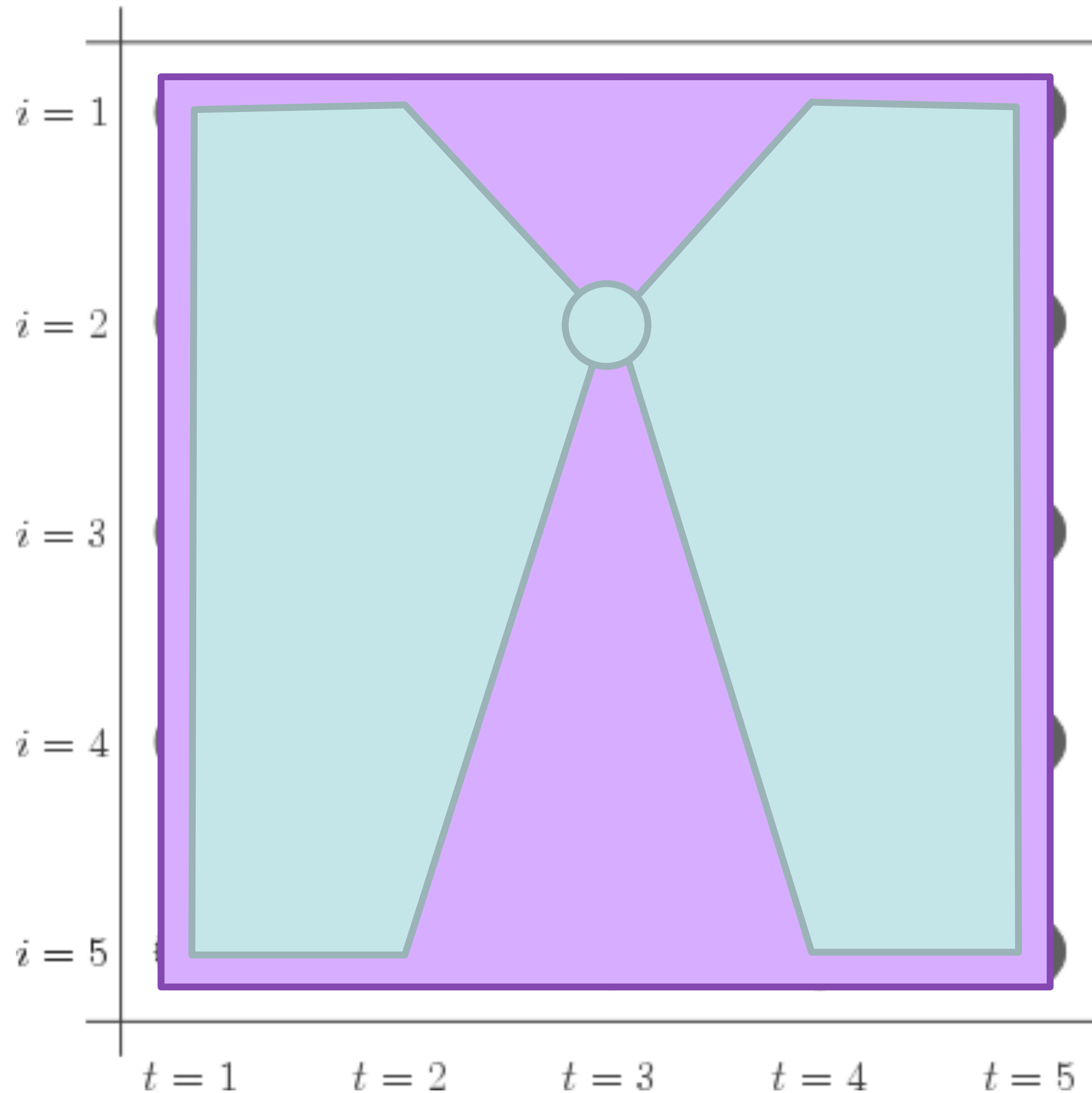
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- What is the denominator here? $P(\mathbf{x})$

HMM POS Tagging

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Fed raises interest rates 0.5 percent

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- ▶ Tradeoff between model capacity and data size — trigrams are a “sweet spot” for POS tagging

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- ▶ State-of-the-art (BiLSTM-CRFs): 97.5% / 89%+

Errors

| | JJ | NN | NNP | NNPS | RB | RP | IN | VB | VBD | VBN | VBP | Total |
|-------|-----|-----|-----|------|-----|-----|-----|-----|-----|-----|-----|-------|
| JJ | 0 | 177 | 56 | 0 | 61 | 2 | 5 | 10 | 15 | 108 | 0 | 488 |
| NN | 244 | 0 | 103 | 0 | 12 | 1 | 1 | 29 | 5 | 6 | 19 | 525 |
| NNP | 107 | 106 | 0 | 132 | 5 | 0 | 7 | 5 | 1 | 2 | 0 | 427 |
| NNPS | 1 | 0 | 110 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 142 |
| RB | 72 | 21 | 7 | 0 | 0 | 16 | 138 | 1 | 0 | 0 | 0 | 295 |
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| IN | 11 | 0 | 1 | 0 | 169 | 103 | 0 | 1 | 0 | 0 | 0 | 323 |
| VB | 17 | 64 | 9 | 0 | 2 | 0 | 1 | 0 | 4 | 7 | 85 | 189 |
| VBD | 10 | 5 | 3 | 0 | 0 | 0 | 0 | 3 | 0 | 143 | 2 | 166 |
| VBN | 101 | 3 | 3 | 0 | 0 | 0 | 0 | 3 | 108 | 0 | 1 | 221 |
| VBP | 5 | 34 | 3 | 1 | 1 | 0 | 2 | 49 | 6 | 3 | 0 | 104 |
| Total | 626 | 536 | 348 | 144 | 317 | 122 | 279 | 102 | 140 | 269 | 108 | 3651 |

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JJ/**NN** NN

official knowledge

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JJ/**NN** NN

official knowledge

(NN NN: *tax cut, art gallery, ...*)

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JJ/**NN** NN
official knowledge

VBD RP/**IN** DT NN
made up the story

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JJ/**NN** NN
official knowledge

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RB VBD/**VBN** NNS
recently sold shares

(NN NN: *tax cut, art gallery, ...*)

Remaining Errors

Manning 2011 “Part-of-Speech Tagging from 97% to 100%: Is It Time for Some Linguistics?”

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VBD / VBP? (past or present?)

They set up absurd situations, detached from reality

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adjective or verbal participle? JJ / VBN?

*a \$ 10 million fourth-quarter charge against **discontinued** operations*

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Other Languages

| Language | Source | # Tags | O/O | U/U | O/U |
|------------|--|--------|------|------|------|
| Arabic | PADT/CoNLL07 (Hajič et al., 2004) | 21 | 96.1 | 96.9 | 97.0 |
| Basque | Basque3LB/CoNLL07 (Aduriz et al., 2003) | 64 | 89.3 | 93.7 | 93.7 |
| Bulgarian | BTB/CoNLL06 (Simov et al., 2002) | 54 | 95.7 | 97.5 | 97.8 |
| Catalan | CESS-ECE/CoNLL07 (Martí et al., 2007) | 54 | 98.5 | 98.2 | 98.8 |
| Chinese | Penn ChineseTreebank 6.0 (Palmer et al., 2007) | 34 | 91.7 | 93.4 | 94.1 |
| Chinese | Sinica/CoNLL07 (Chen et al., 2003) | 294 | 87.5 | 91.8 | 92.6 |
| Czech | PDT/CoNLL07 (Böhmová et al., 2003) | 63 | 99.1 | 99.1 | 99.1 |
| Danish | DDT/CoNLL06 (Kromann et al., 2003) | 25 | 96.2 | 96.4 | 96.9 |
| Dutch | Alpino/CoNLL06 (Van der Beek et al., 2002) | 12 | 93.0 | 95.0 | 95.0 |
| English | PennTreebank (Marcus et al., 1993) | 45 | 96.7 | 96.8 | 97.7 |
| French | FrenchTreebank (Abeillé et al., 2003) | 30 | 96.6 | 96.7 | 97.3 |
| German | Tiger/CoNLL06 (Brants et al., 2002) | 54 | 97.9 | 98.1 | 98.8 |
| German | Negra (Skut et al., 1997) | 54 | 96.9 | 97.9 | 98.6 |
| Greek | GDT/CoNLL07 (Prokopidis et al., 2005) | 38 | 97.2 | 97.5 | 97.8 |
| Hungarian | Szeged/CoNLL07 (Csendes et al., 2005) | 43 | 94.5 | 95.6 | 95.8 |
| Italian | ISST/CoNLL07 (Montemagni et al., 2003) | 28 | 94.9 | 95.8 | 95.8 |
| Japanese | Verbmobil/CoNLL06 (Kawata and Bartels, 2000) | 80 | 98.3 | 98.0 | 99.1 |
| Japanese | Kyoto4.0 (Kurohashi and Nagao, 1997) | 42 | 97.4 | 98.7 | 99.3 |
| Korean | Sejong (http://www.sejong.or.kr) | 187 | 96.5 | 97.5 | 98.4 |
| Portuguese | Floresta Sintá(c)tica/CoNLL06 (Afonso et al., 2002) | 22 | 96.9 | 96.8 | 97.4 |
| Russian | SynTagRus-RNC (Boguslavsky et al., 2002) | 11 | 96.8 | 96.8 | 96.8 |
| Slovene | SDT/CoNLL06 (Džeroski et al., 2006) | 29 | 94.7 | 94.6 | 95.3 |
| Spanish | Ancora-Cast3LB/CoNLL06 (Civit and Martí, 2004) | 47 | 96.3 | 96.3 | 96.9 |
| Swedish | Talbanken05/CoNLL06 (Nivre et al., 2006) | 41 | 93.6 | 94.7 | 95.1 |
| Turkish | METU-Sabancı/CoNLL07 (Ofłazer et al., 2003) | 31 | 87.5 | 89.1 | 90.2 |

Next Time

Next Time

- ▶ CRFs: feature-based discriminative models

Next Time

- ▶ CRFs: feature-based discriminative models
- ▶ Structured SVM for sequences

Next Time

- ▶ CRFs: feature-based discriminative models
- ▶ Structured SVM for sequences
- ▶ Named entity recognition