Lecture 6: Neural Networks

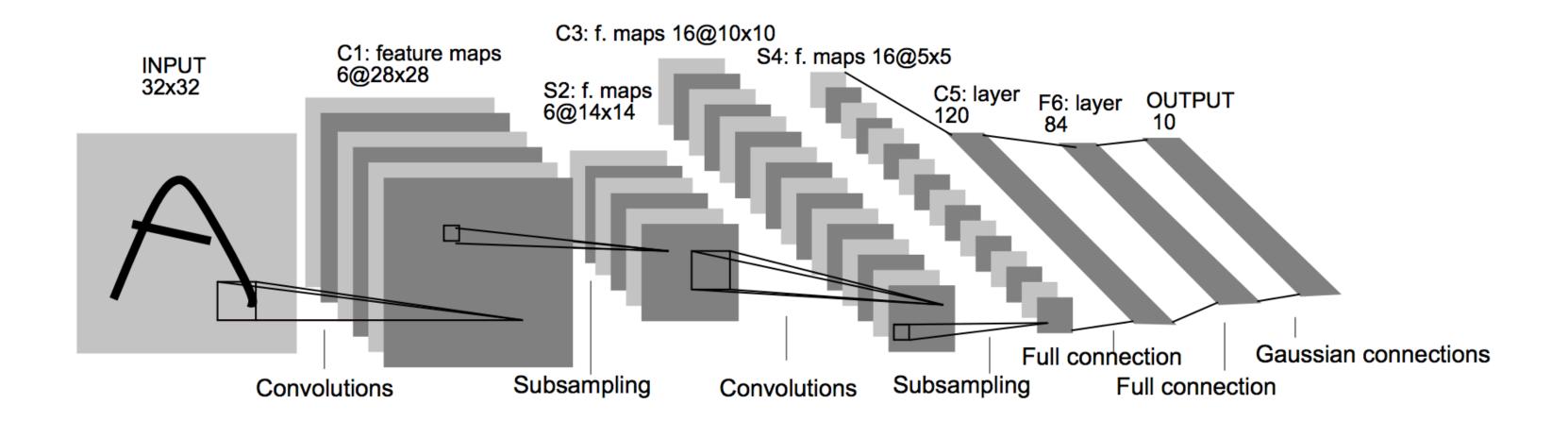
Alan Ritter

(many slides from Greg Durrett)

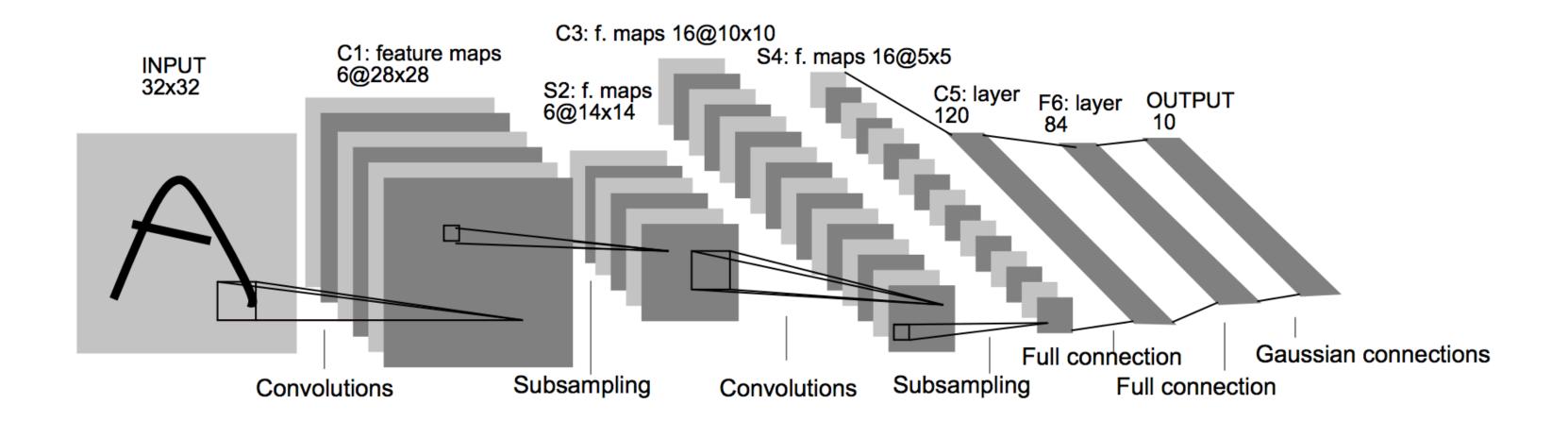
This Lecture

- Neural network history
- Neural network basics
- Feedforward neural networks + backpropagation
- Applications
- Implementing neural networks (if time)

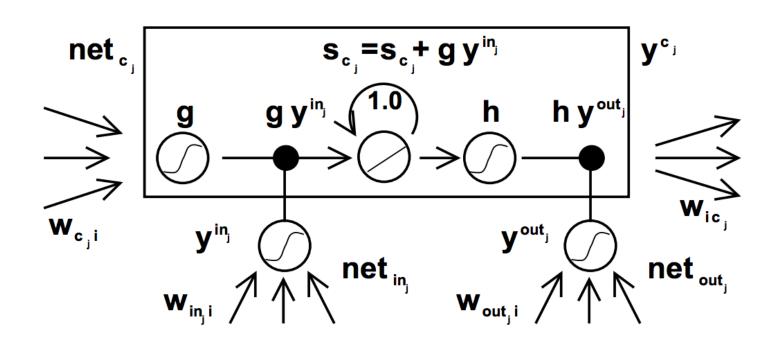
Convnets: applied to MNIST by LeCun in 1998



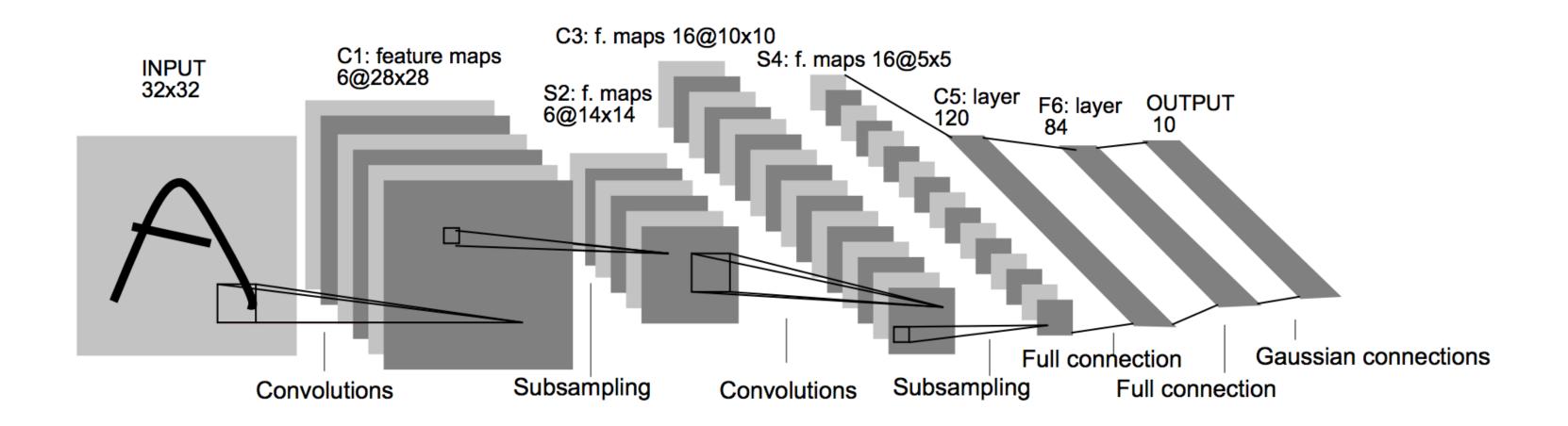
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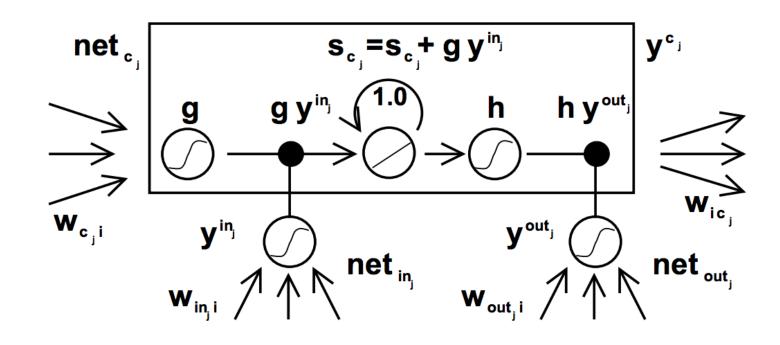
LSTMs: Hochreiter and Schmidhuber (1997)



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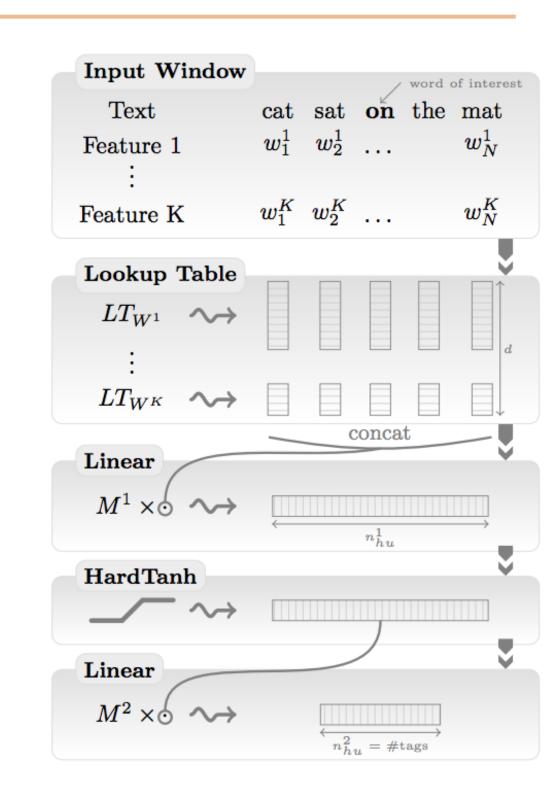


LSTMs: Hochreiter and Schmidhuber (1997)

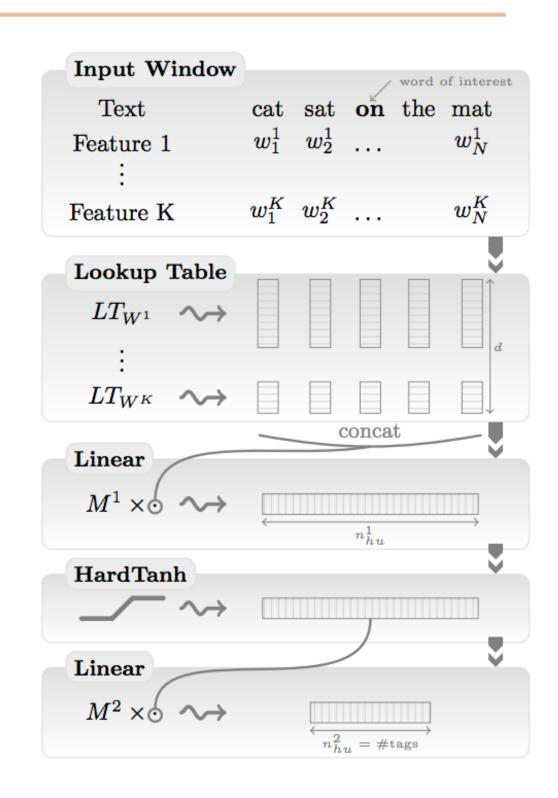


Henderson (2003): neural shift-reduce parser, not SOTA

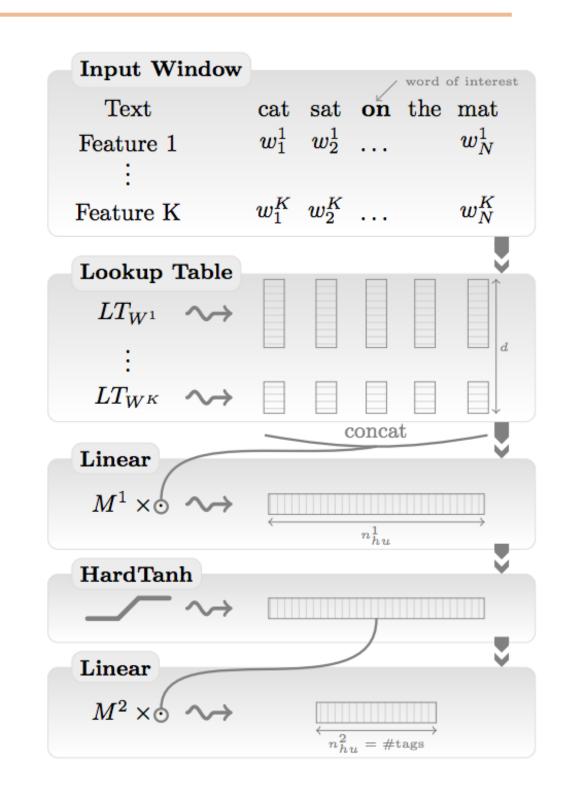
- Collobert and Weston 2011: "NLP (almost) from scratch"
 - Feedforward neural nets induce features for sequential CRFs ("neural CRF")
 - 2008 version was marred by bad experiments,
 claimed SOTA but wasn't, 2011 version tied SOTA

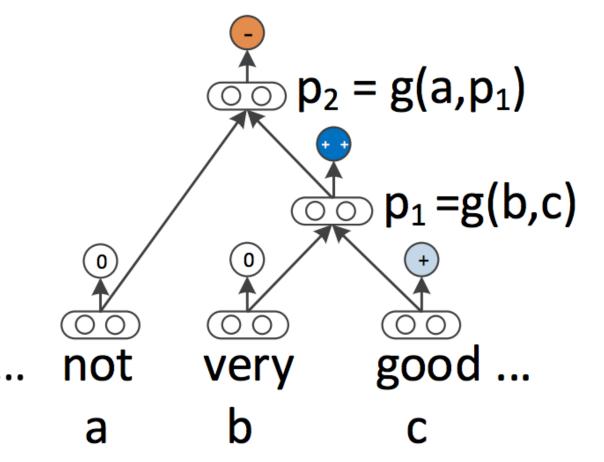


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- Socher 2011-2014: tree-structured RNNs working okay





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- Chen and Manning transition-based dependency parser (even feedforward networks work well for NLP?)
- 2015: explosion of neural nets for everything under the sun

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- ▶ Inputs: need word representations to have the right continuous semantics

Neural Net Basics

Linear classification: $\operatorname{argmax}_y w^\top f(x,y)$

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I[contains not & contains good]

 Let's see how we can use neural nets to learn a simple nonlinear function

Inputs

Output

- Let's see how we can use neural nets to learn a simple nonlinear function
- Inputs x_1, x_2 (generally $\mathbf{x} = (x_1, \dots, x_m)$)
- Output y $(\text{generally } \mathbf{y} = (y_1, \dots, y_n))$

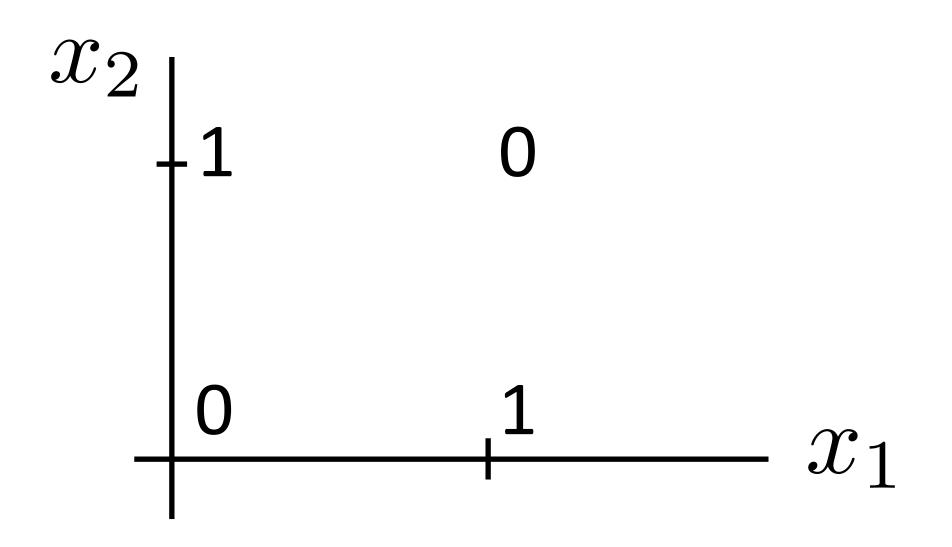
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x_1	x_2	$y = x_1 \text{ XOR } x_2$
0	0	
0	1	
1	0	
1	1	

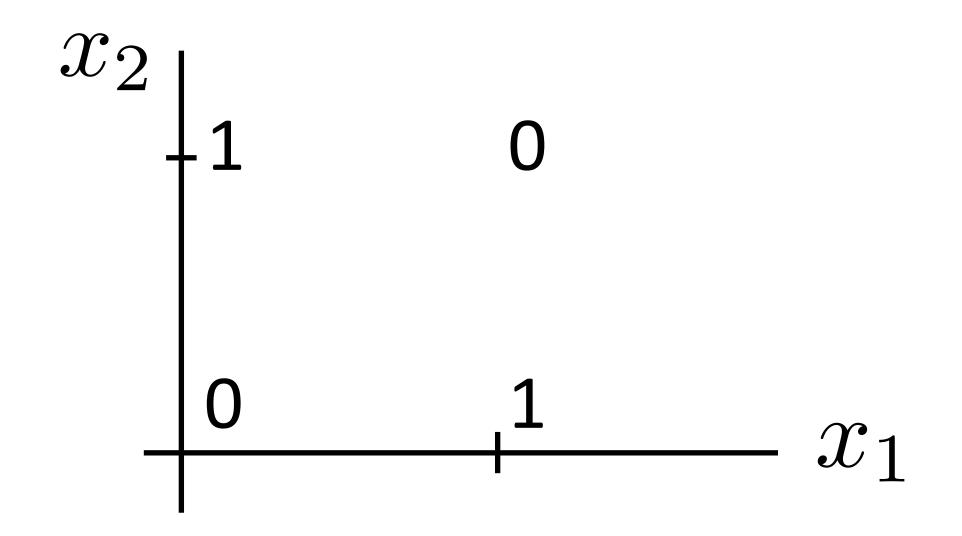
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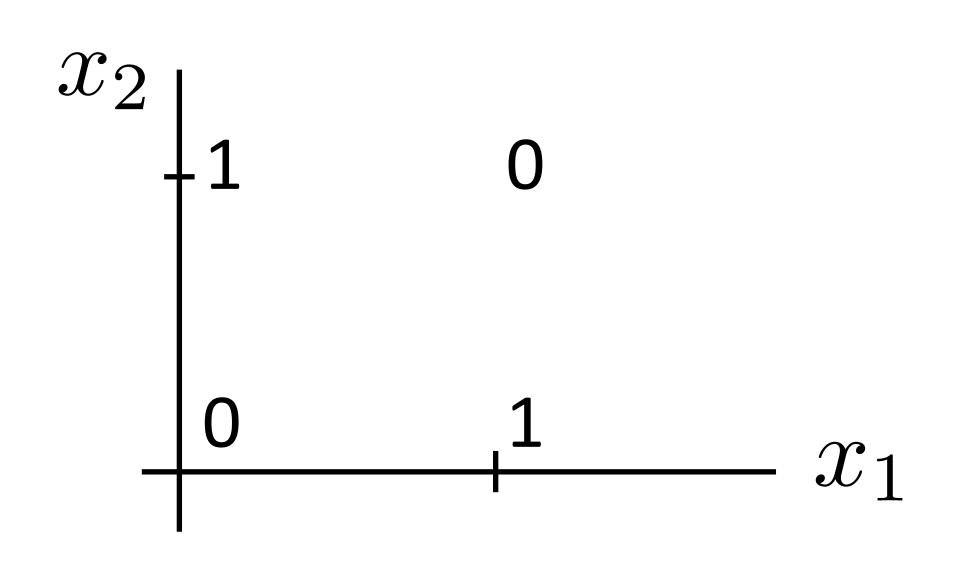
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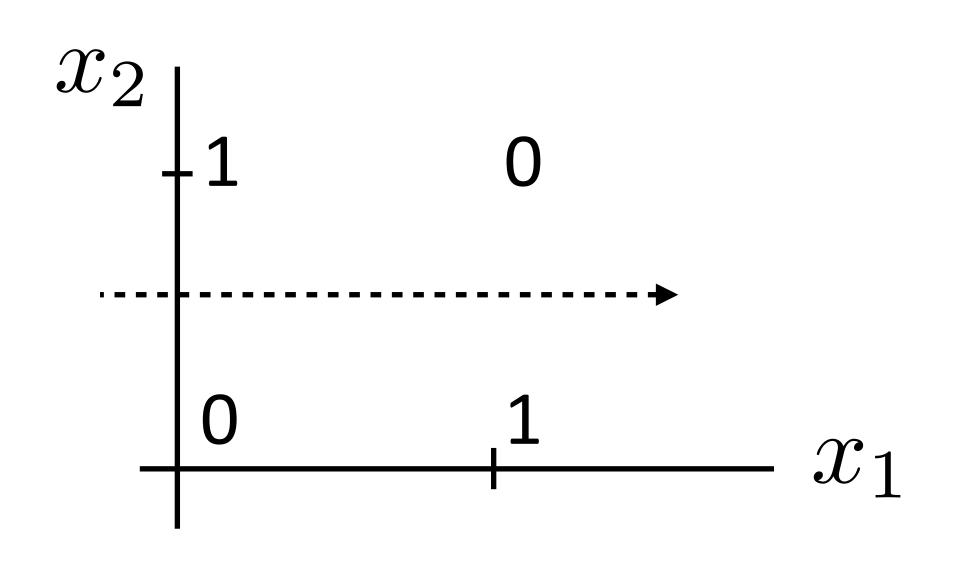


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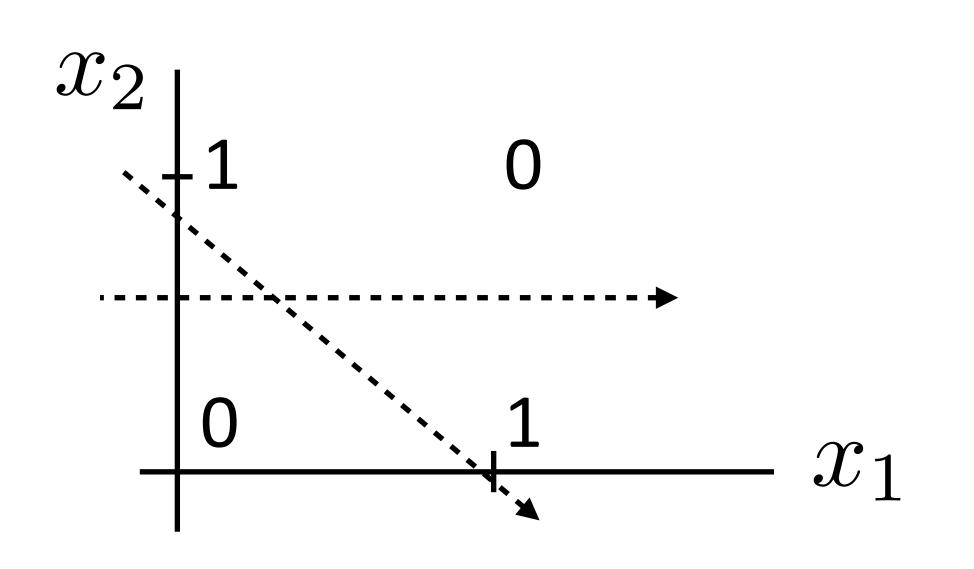
$$y = a_1 x_1 + a_2 x_2$$

	x_1	x_2	$x_1 \text{ XOR } x_2$
•	0	0	0
	0	1	1
	1	0	1
	1	1	0



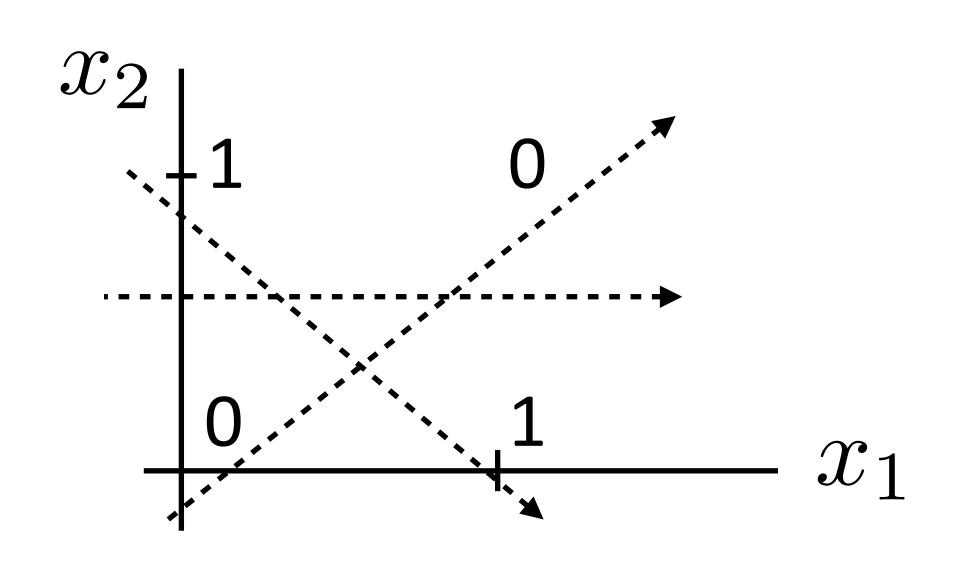
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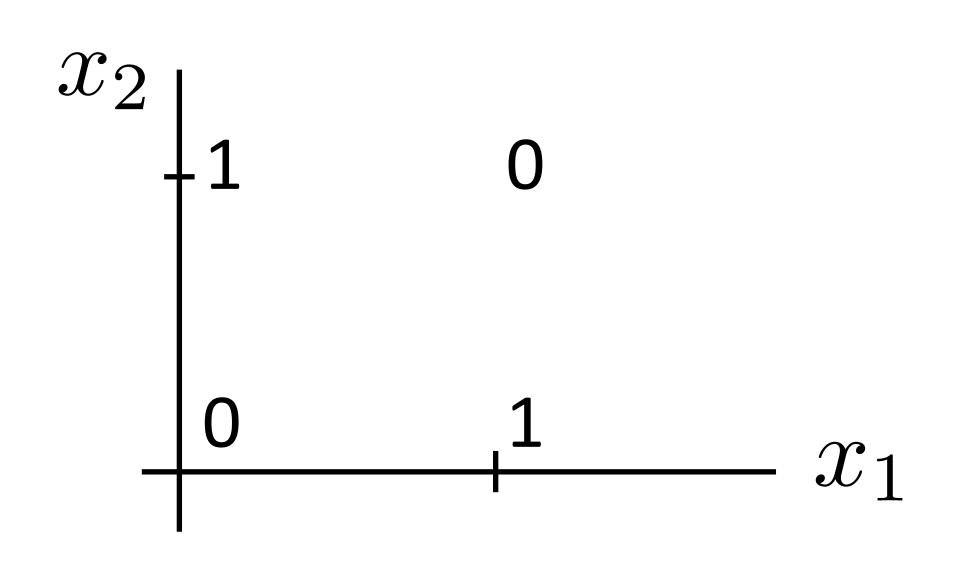
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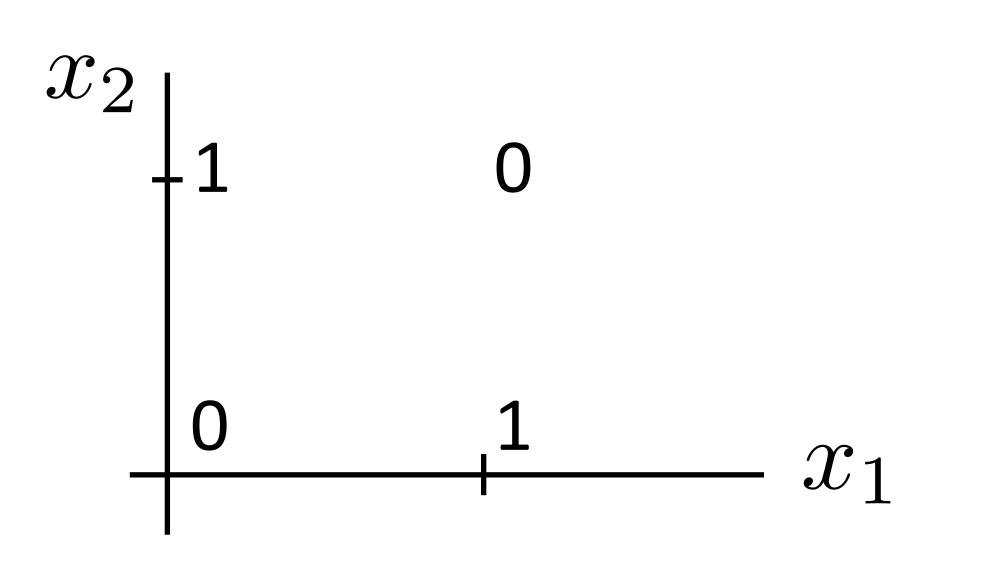
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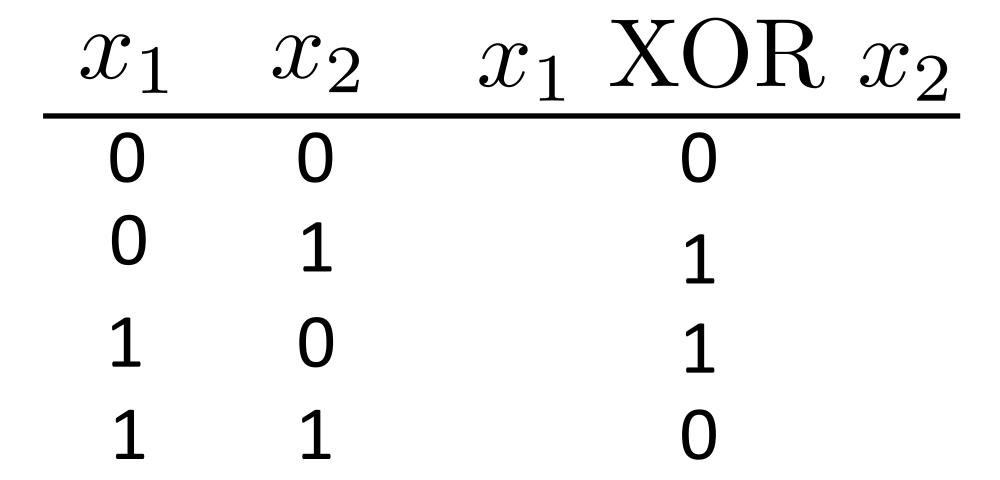


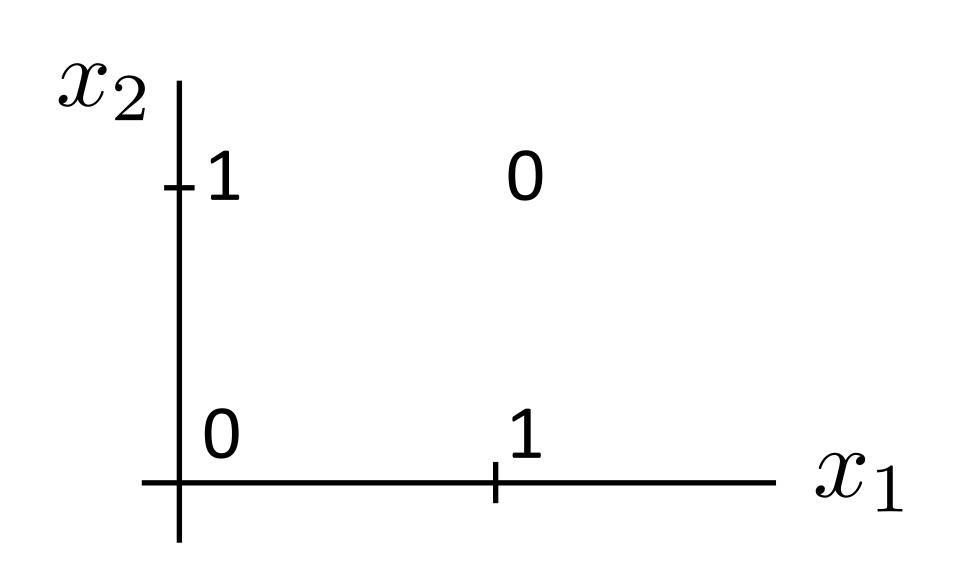
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	x_1	x_2	$x_1 \text{ XOR } x_2$
•	0	0	0
	0	1	1
	1	0	1
	1	1	0



$y = a_1 x_1$	$+a_2x_2$	



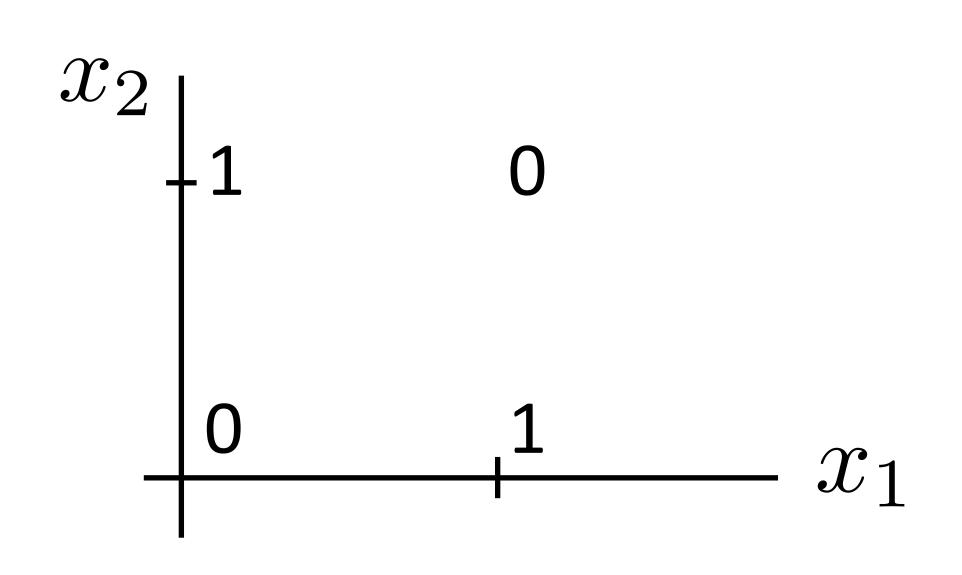




$$y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$$



	x_1	x_2	$x_1 \text{ XOR } x_2$
1	0	0	0
	0	1	1
	1	0	1
	1	1	0

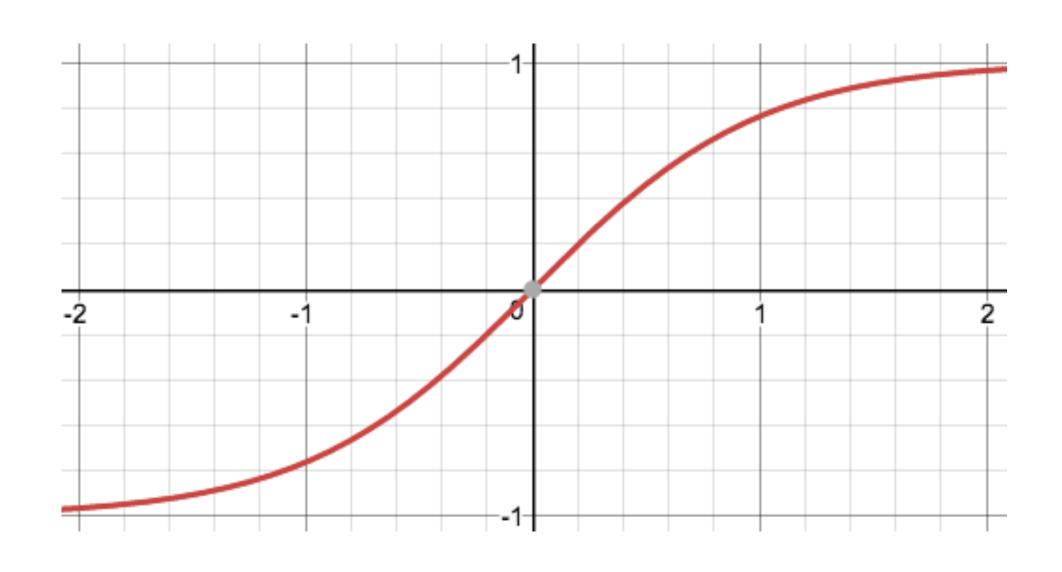


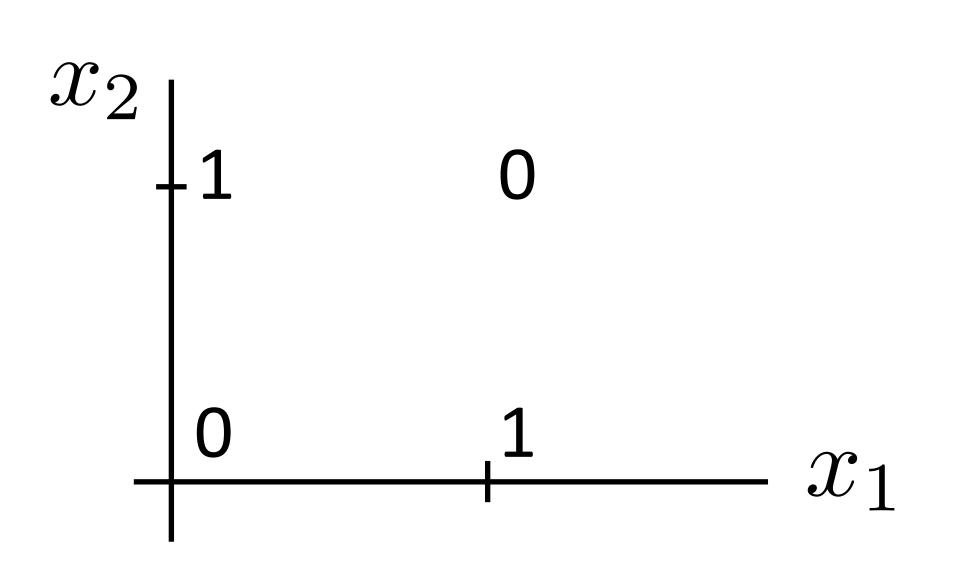
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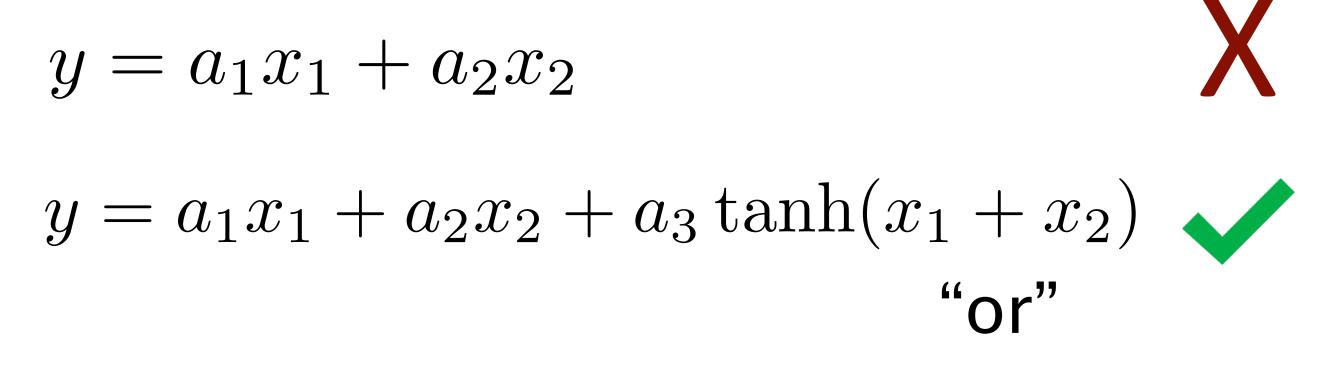


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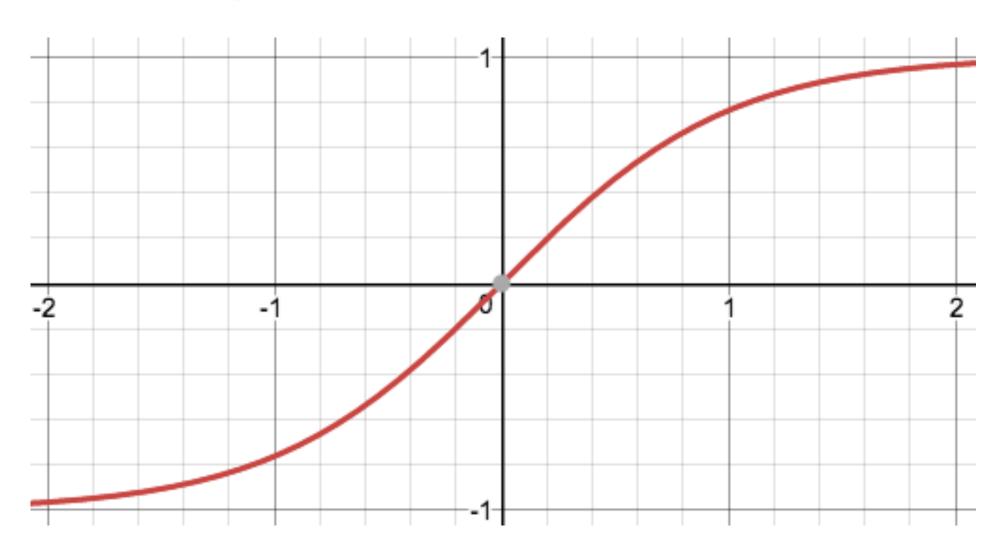


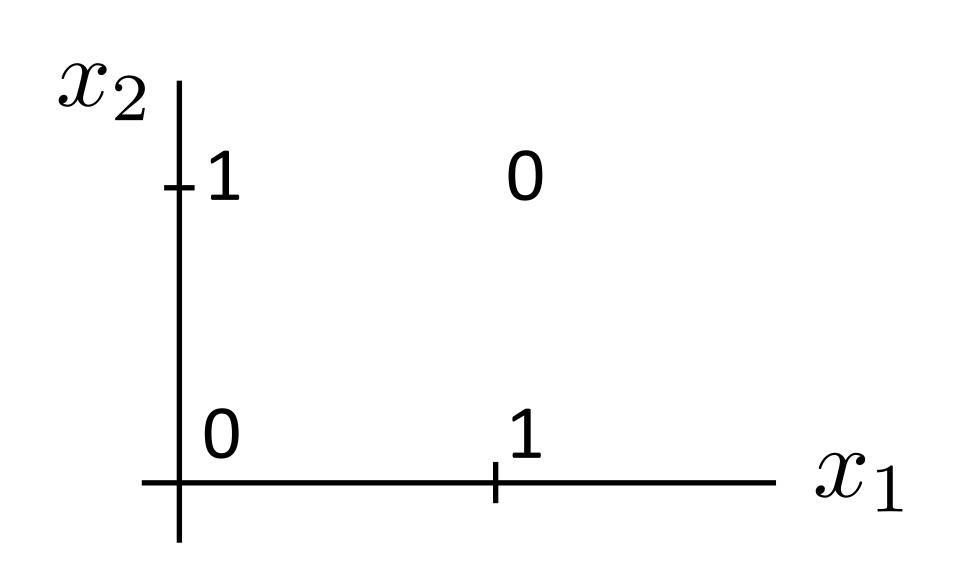


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(looks like action potential in neuron)



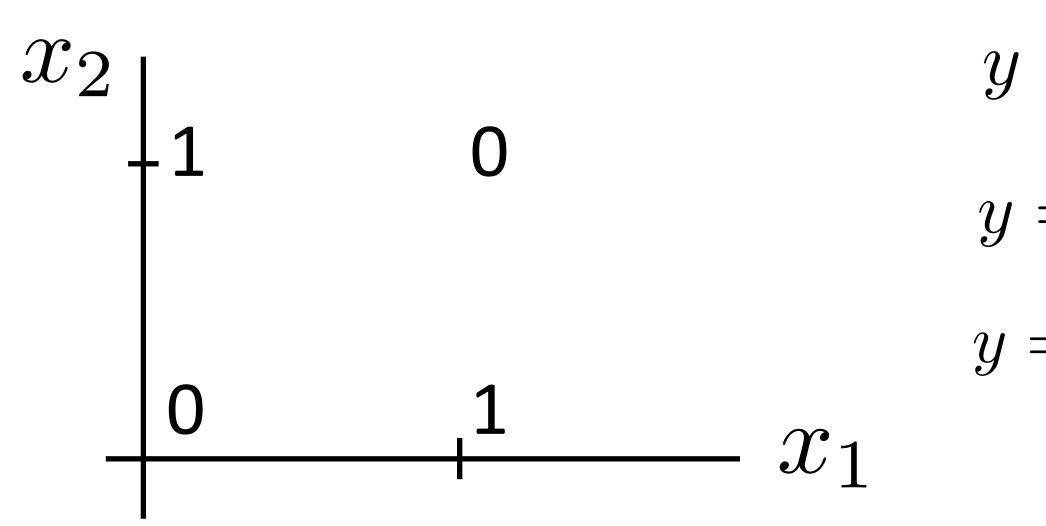




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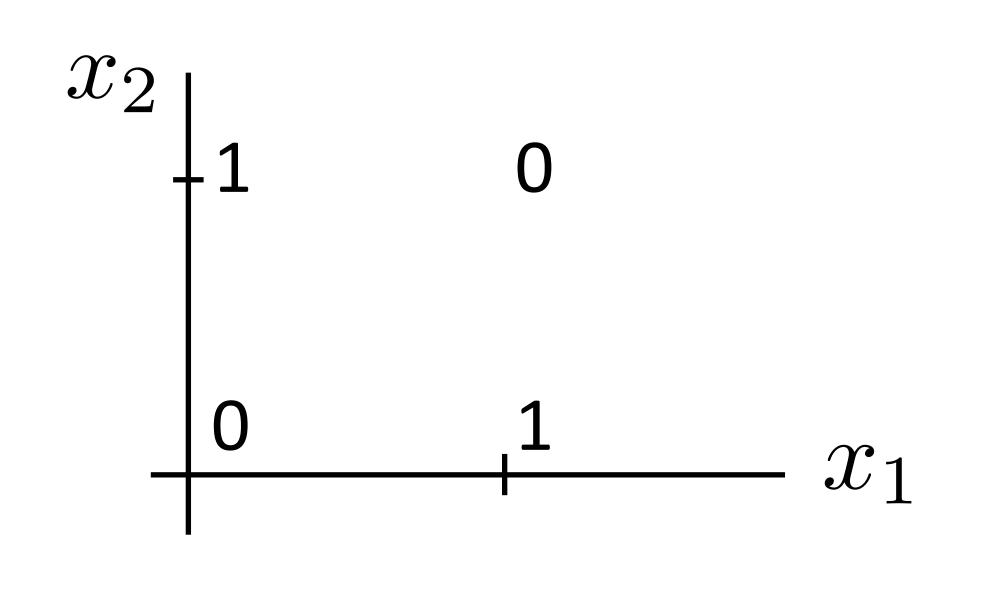


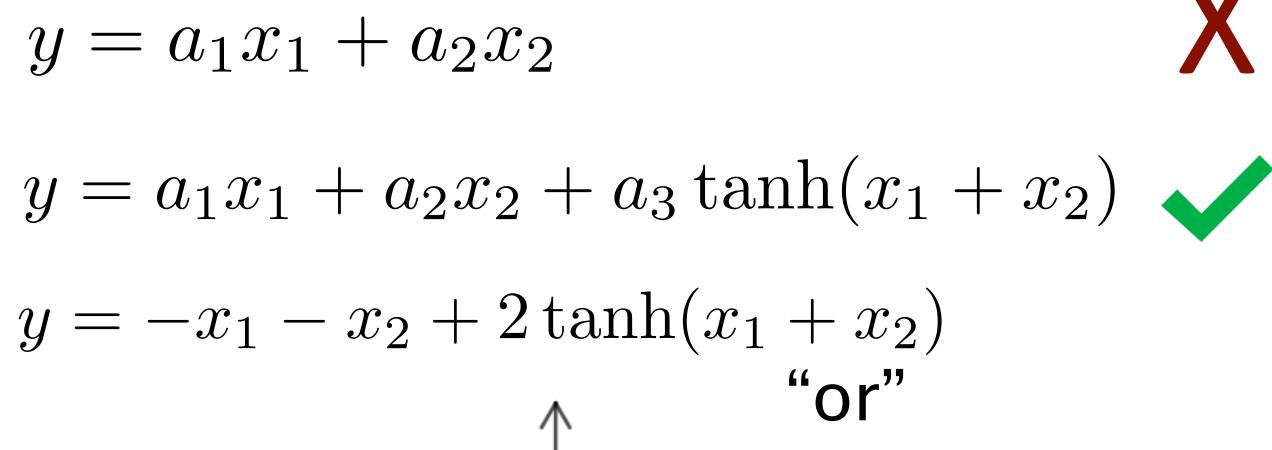
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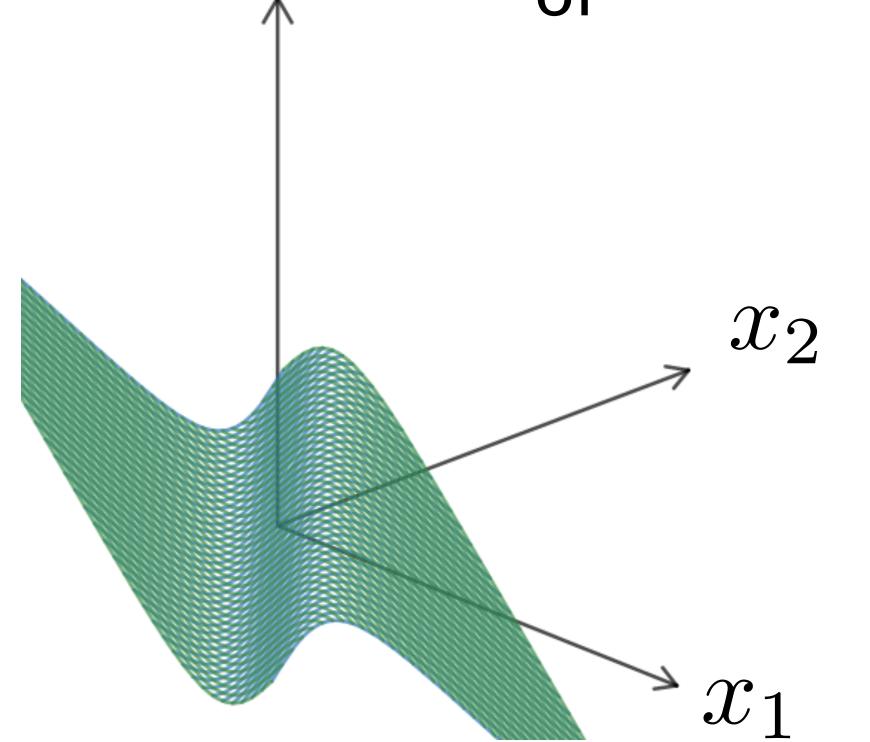


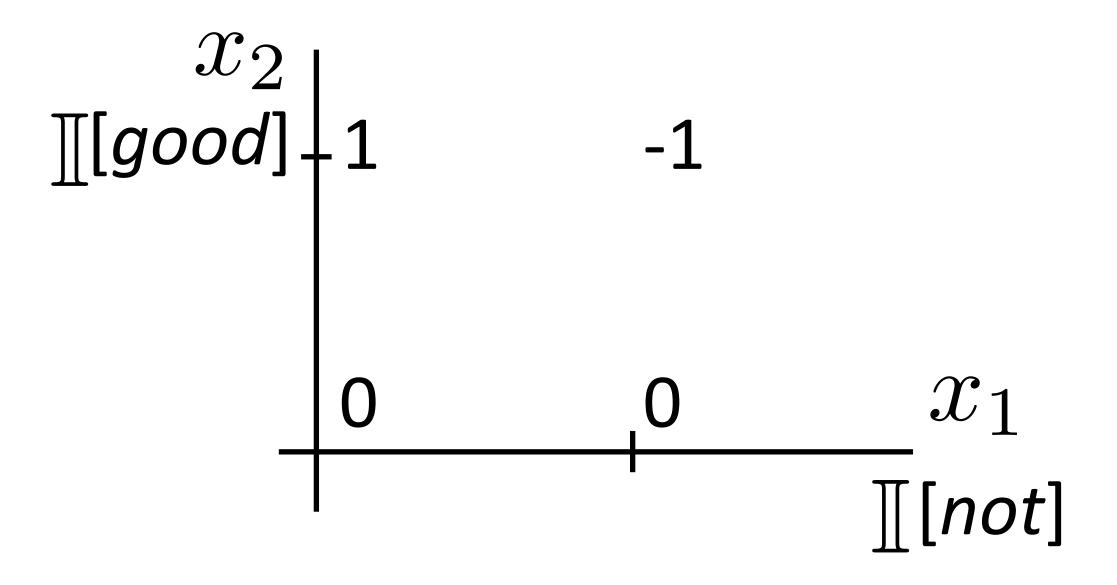
$$y = a_1x_1 + a_2x_2$$
 $y = a_1x_1 + a_2x_2 + a_3 \tanh(x_1 + x_2)$
 $y = -x_1 - x_2 + 2 \tanh(x_1 + x_2)$
"or"

x_1	x_2	$x_1 \text{ XOR } x_2$
0	0	0
0	1	1
1	0	1
1	1	0

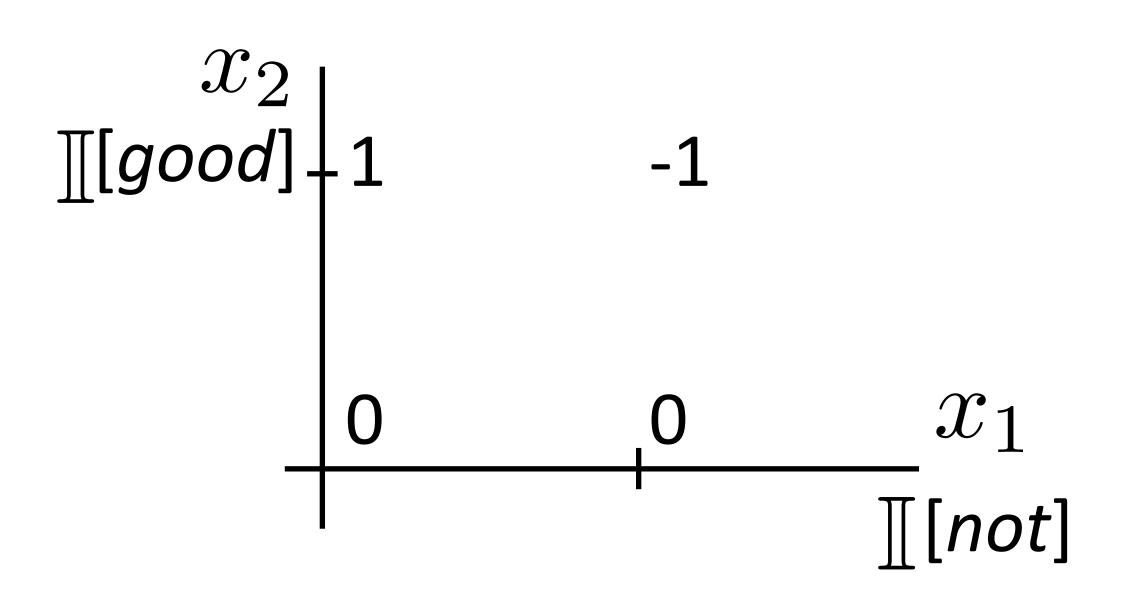




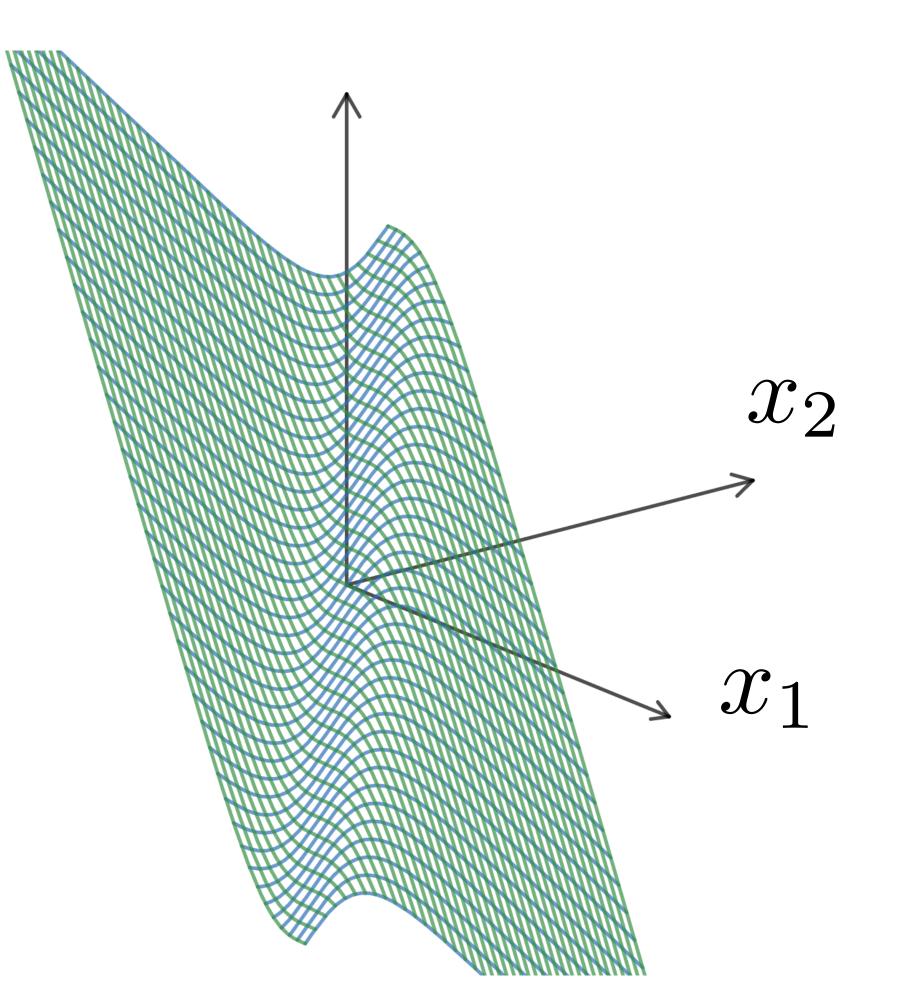




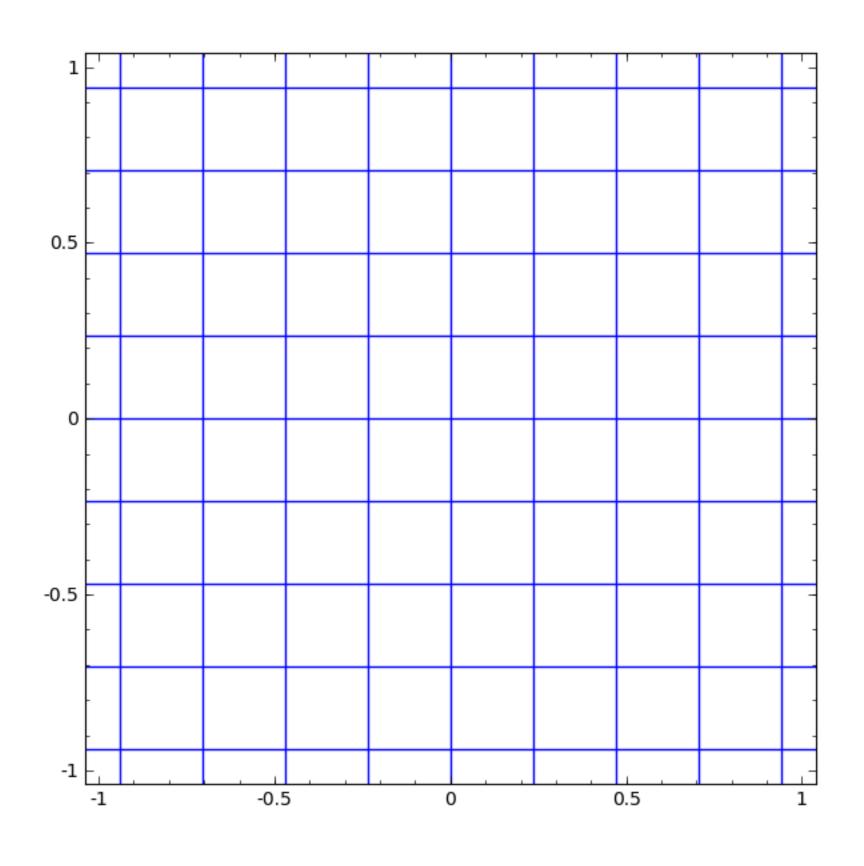
the movie was not all that good



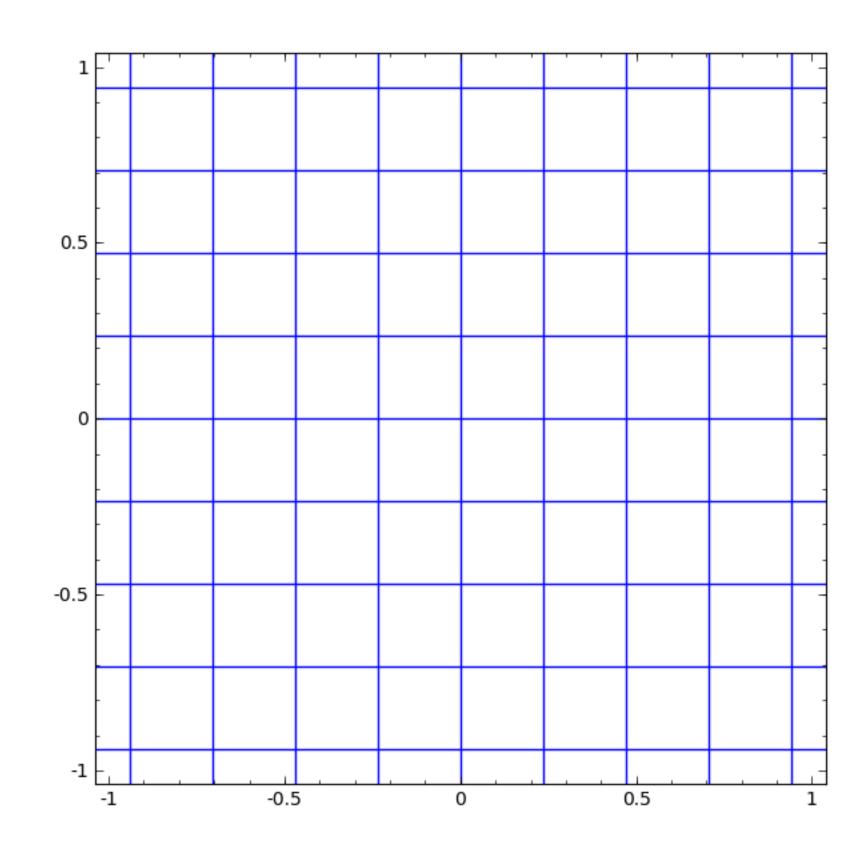
$$y = -2x_1 - x_2 + 2\tanh(x_1 + x_2)$$



the movie was not all that good



Linear model: $y = \mathbf{w} \cdot \mathbf{x} + b$

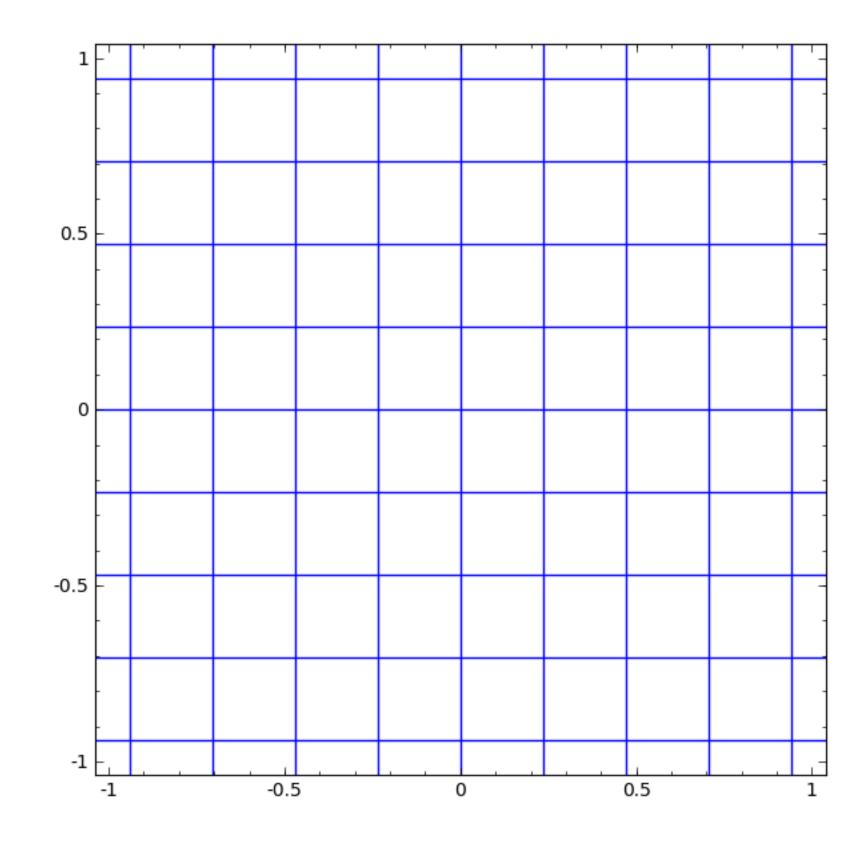


Linear model: $y = \mathbf{w} \cdot \mathbf{x} + b$

$$y = g(\mathbf{w} \cdot \mathbf{x} + b)$$

$$\mathbf{y} = g(\mathbf{W}\mathbf{x} + \mathbf{b})$$

Nonlinear transformation

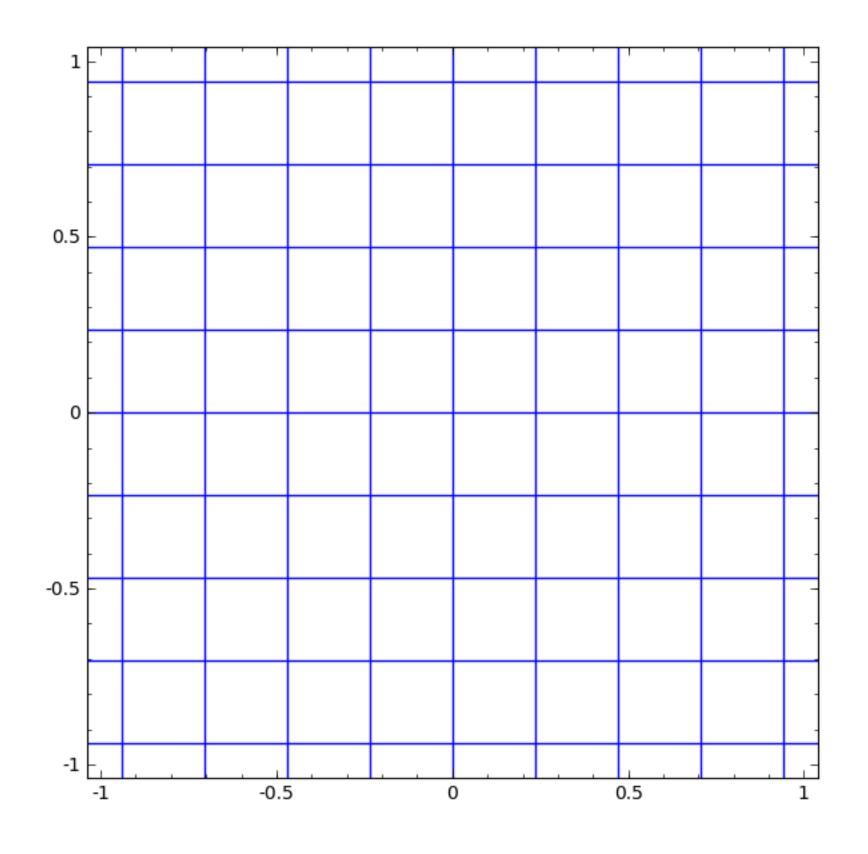


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Nonlinear Warp transformation space

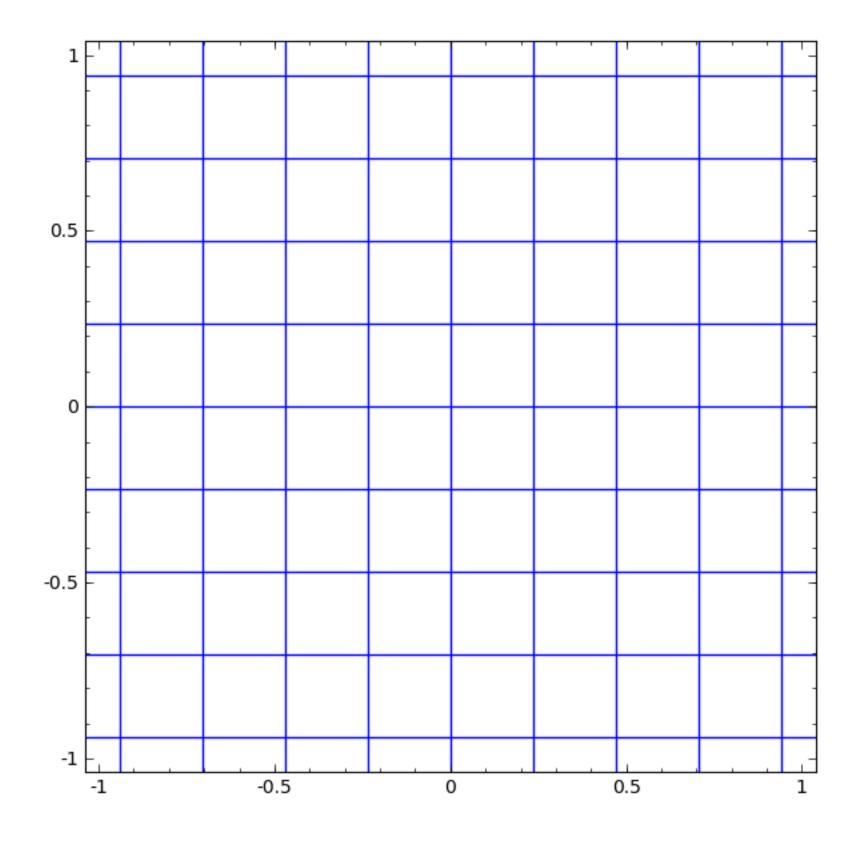


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Nonlinear Warp Shift transformation space

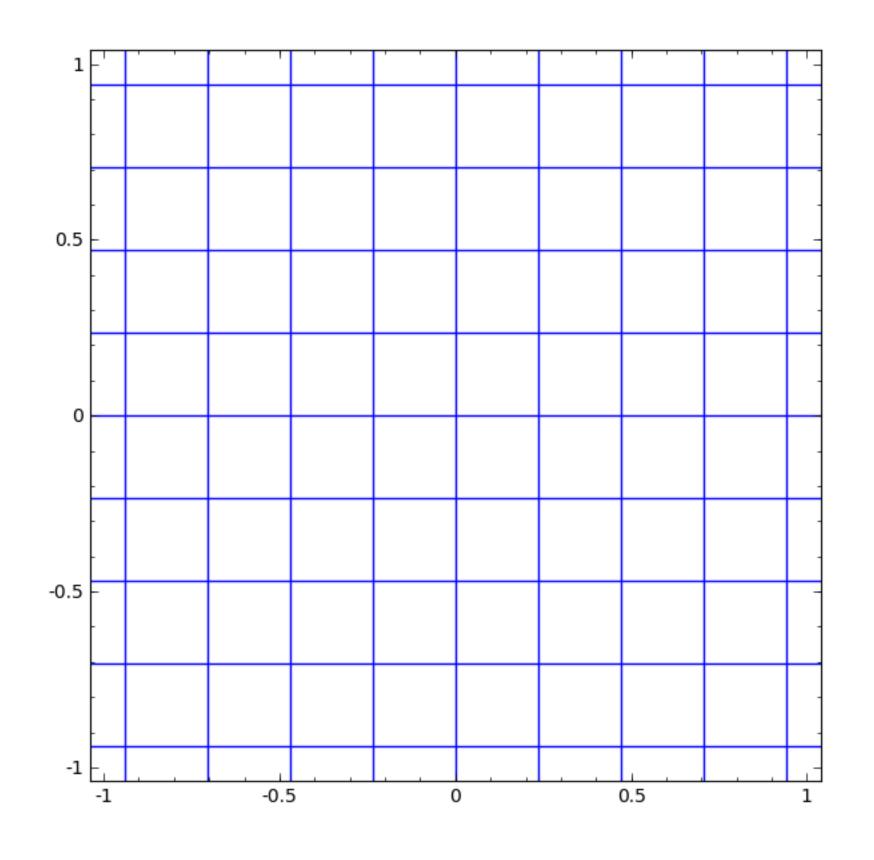


Linear model: $y = \mathbf{w} \cdot \mathbf{x} + b$

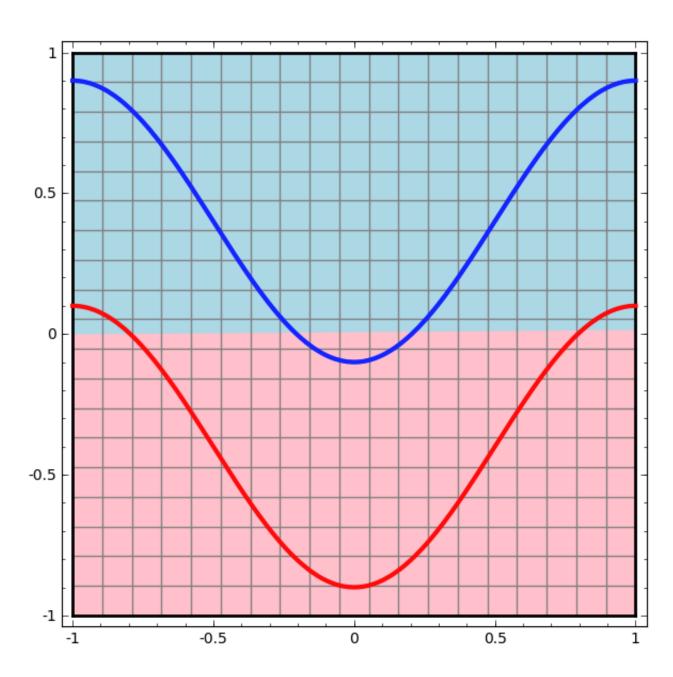
$$y = g(\mathbf{w} \cdot \mathbf{x} + b)$$

$$\mathbf{y} = g(\mathbf{W}\mathbf{x} + \mathbf{b})$$

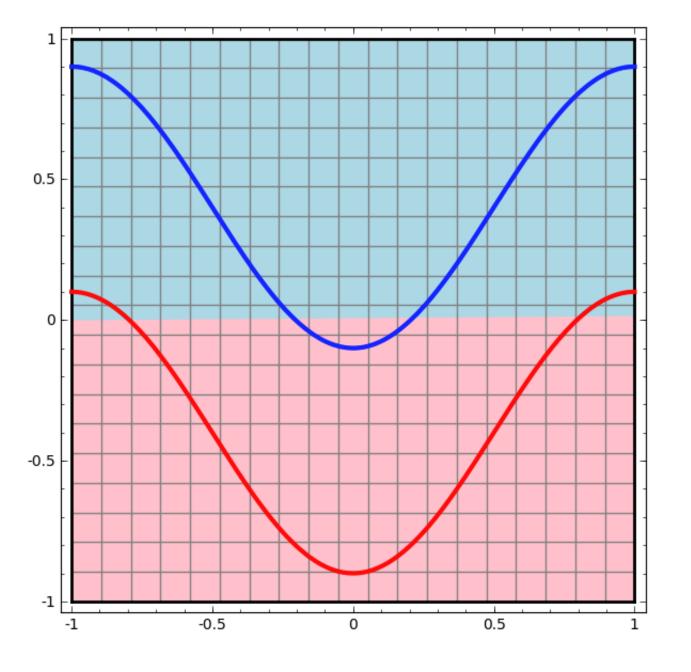
Nonlinear Warp Shift transformation space



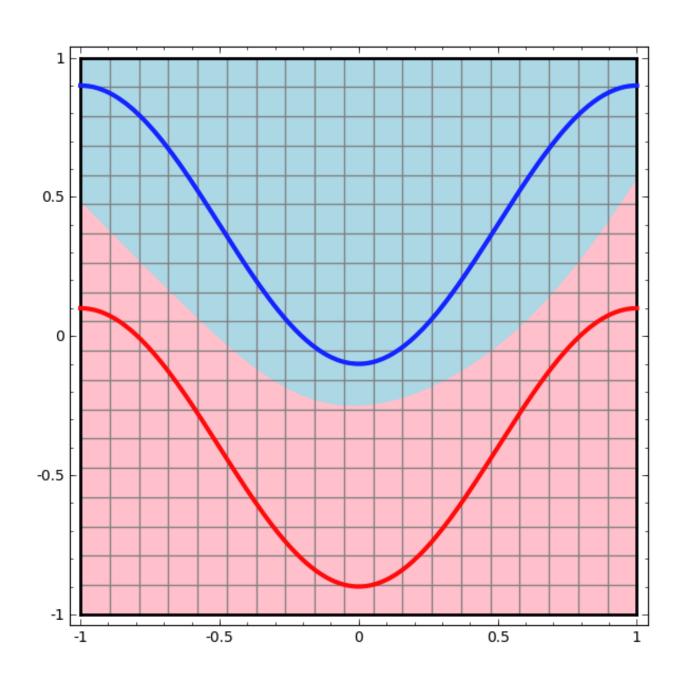
Linear classifier



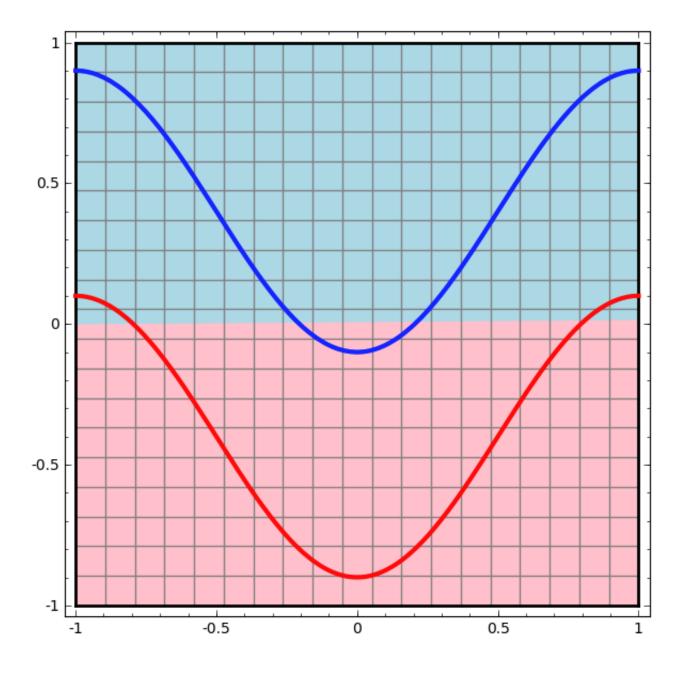
Linear classifier



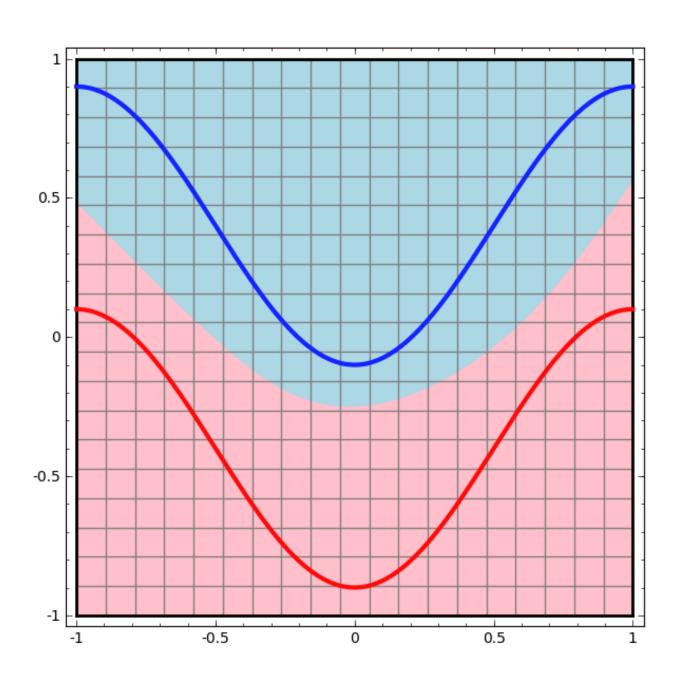
Neural network



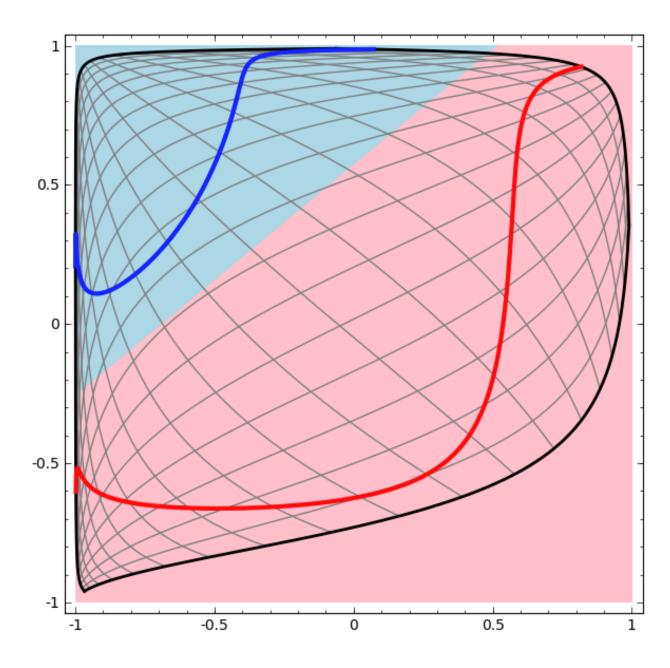
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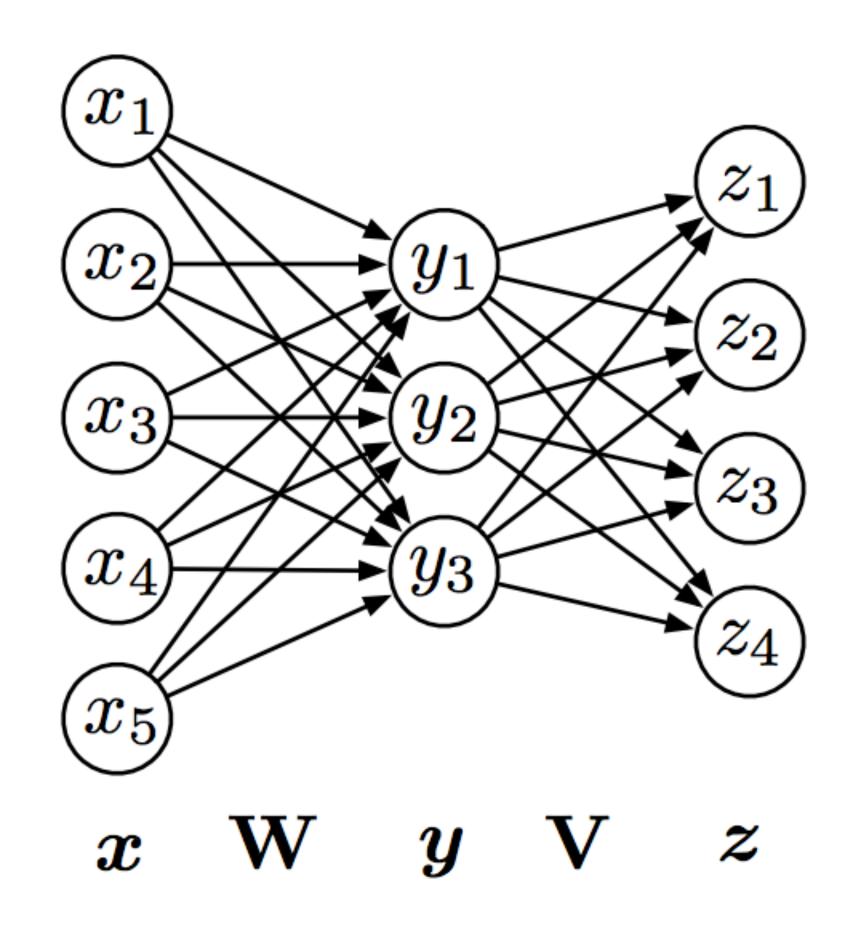
Neural network

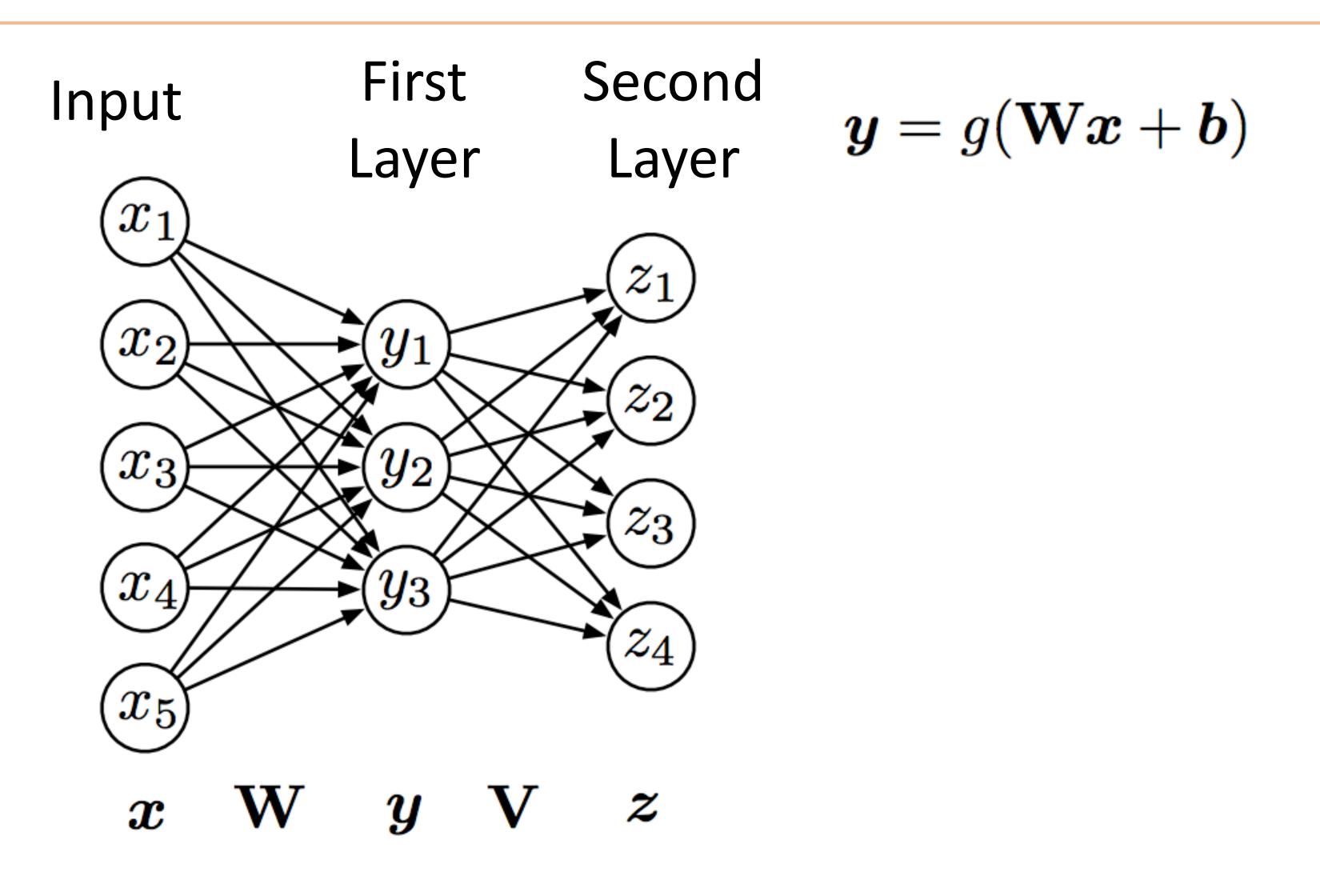


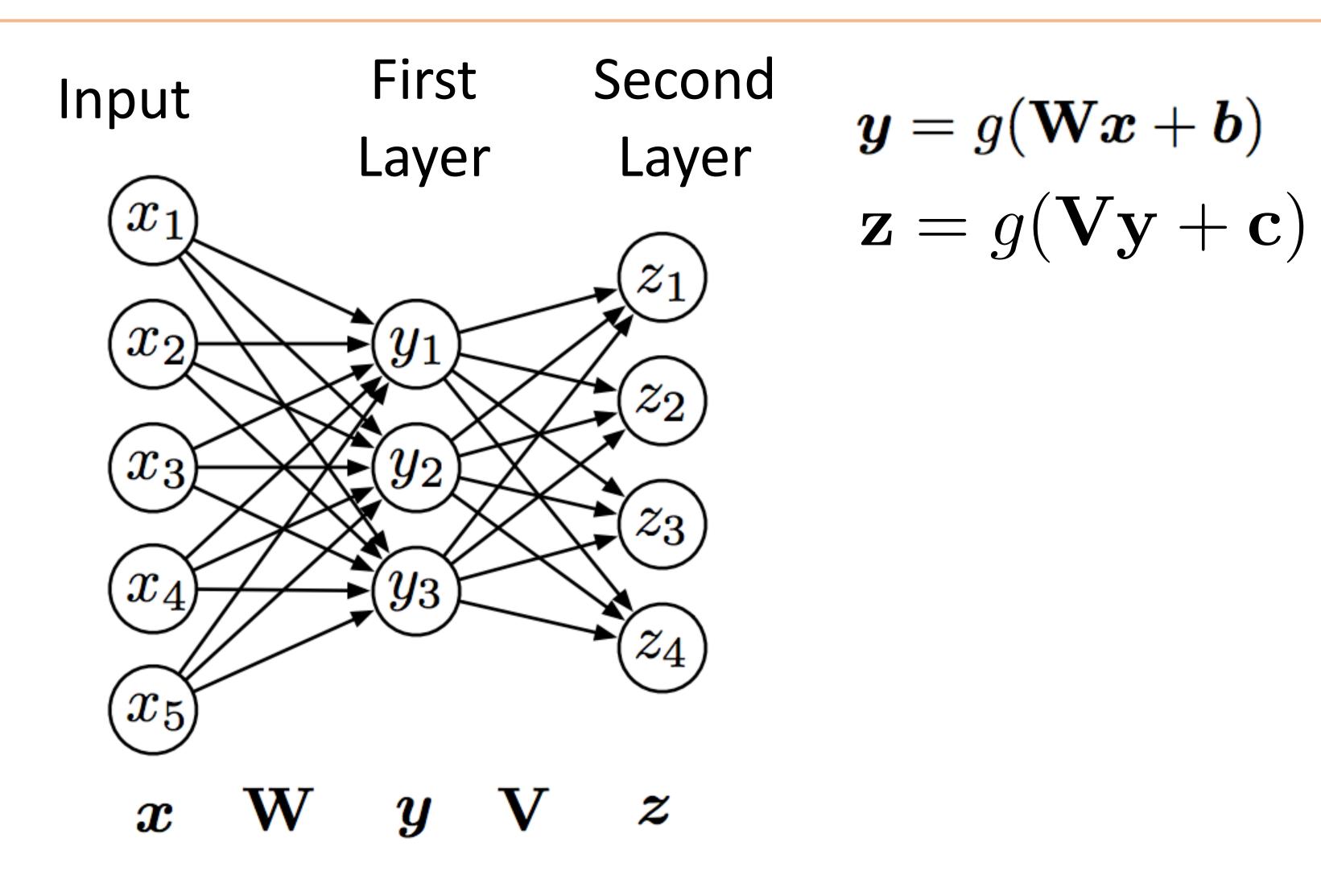
...possible because we transformed the space!

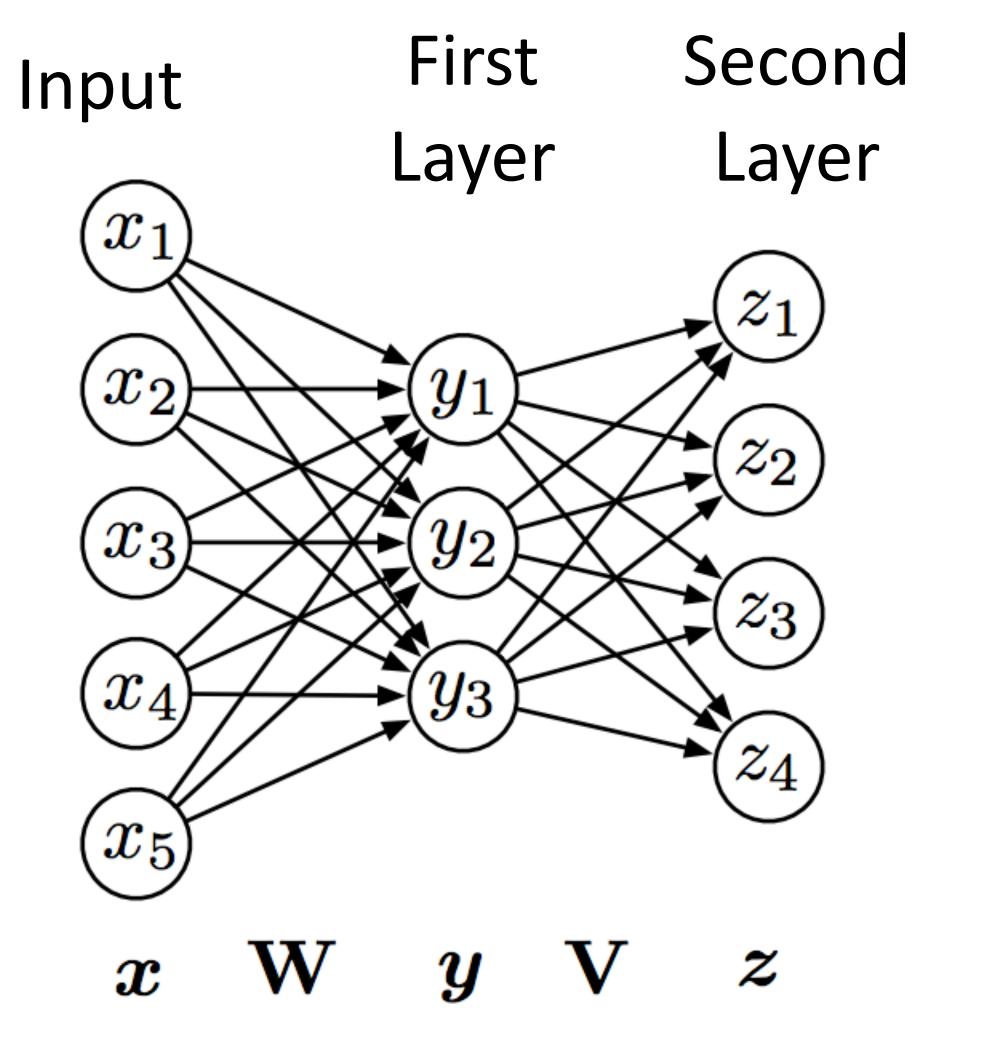


$$\boldsymbol{y} = g(\mathbf{W}\boldsymbol{x} + \boldsymbol{b})$$

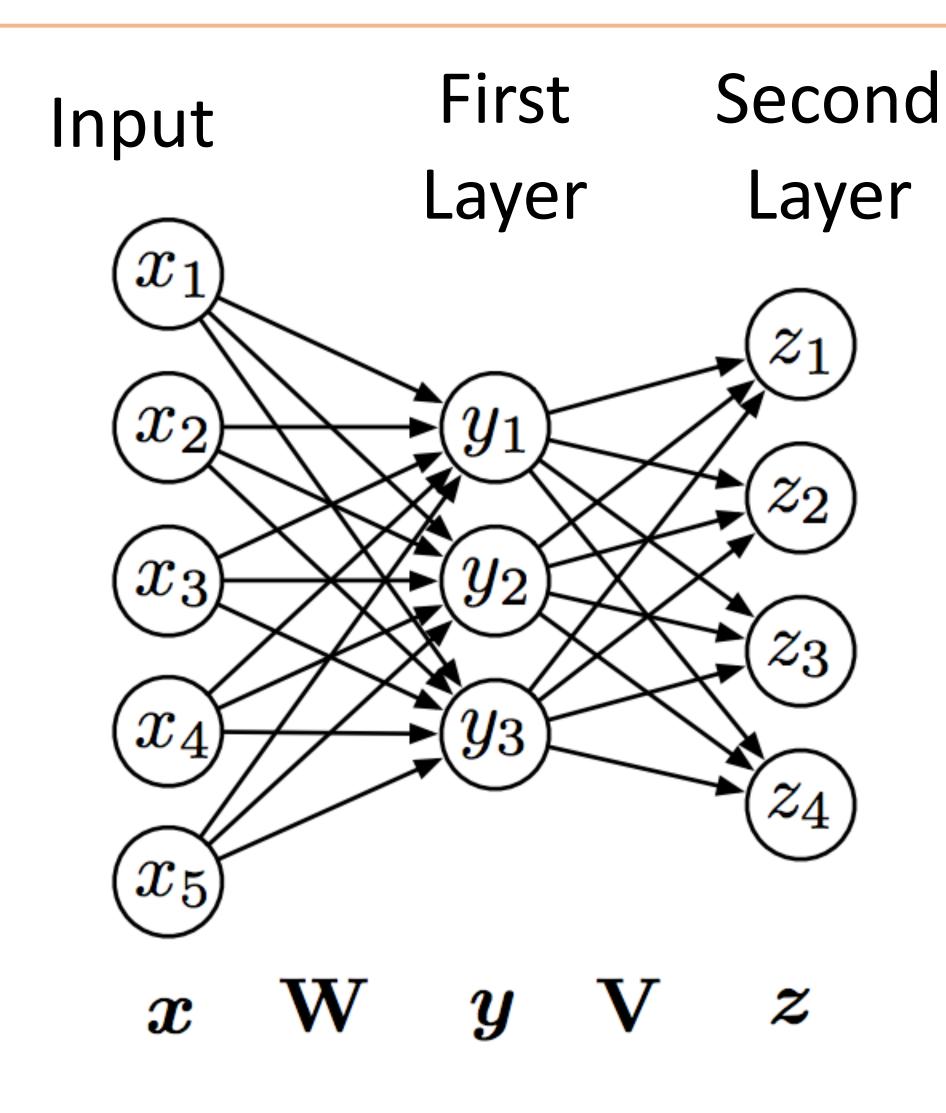






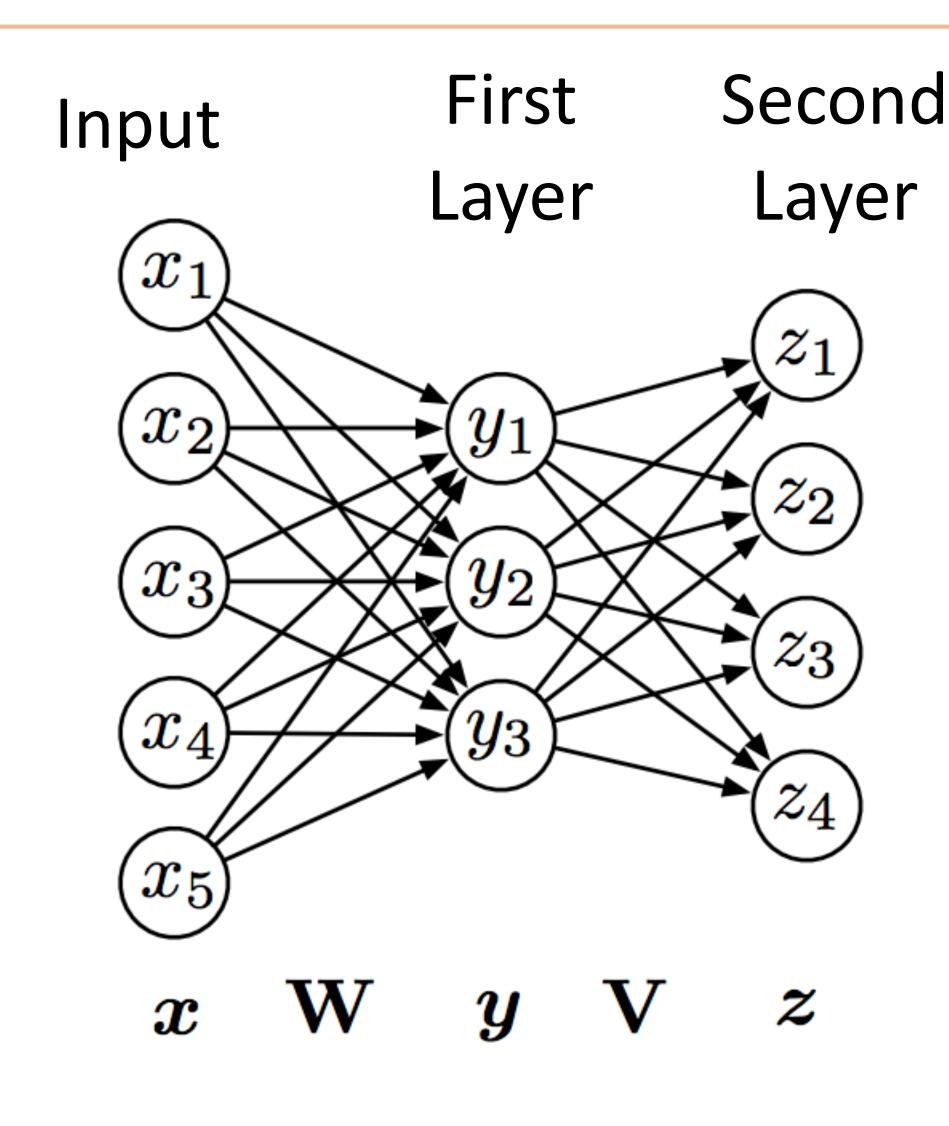


$$egin{aligned} oldsymbol{y} &= g(\mathbf{W}oldsymbol{x} + oldsymbol{b}) \ \mathbf{z} &= g(\mathbf{V}oldsymbol{y}(\mathbf{W}\mathbf{x} + \mathbf{b}) + \mathbf{c}) \ \end{aligned}$$
 output of first layer



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"Feedforward" computation (not recurrent)

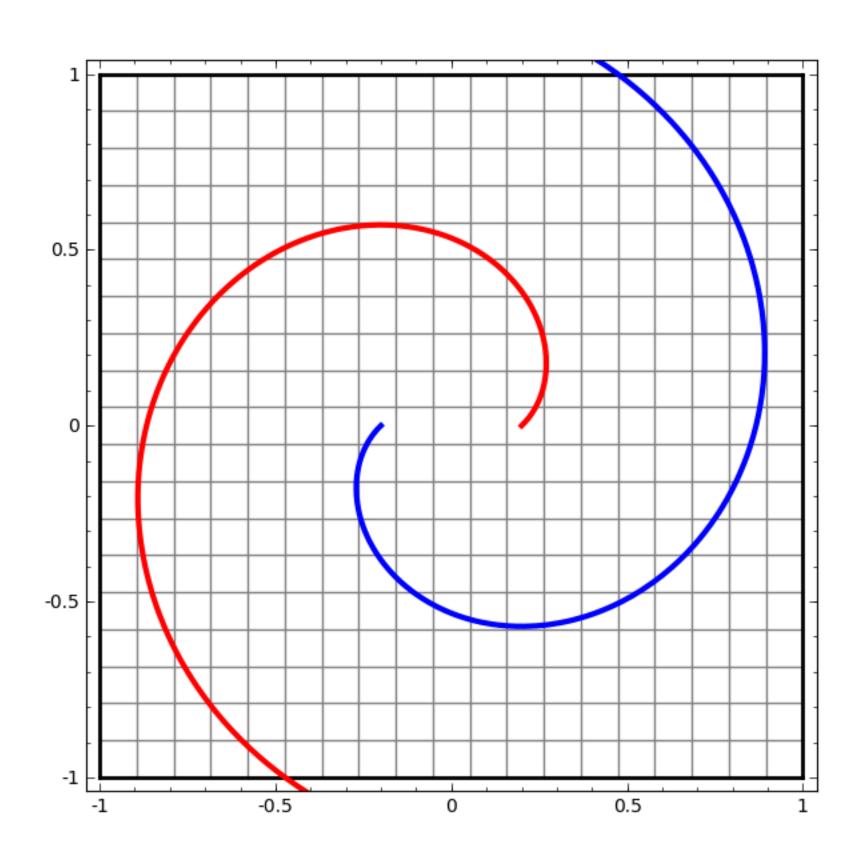


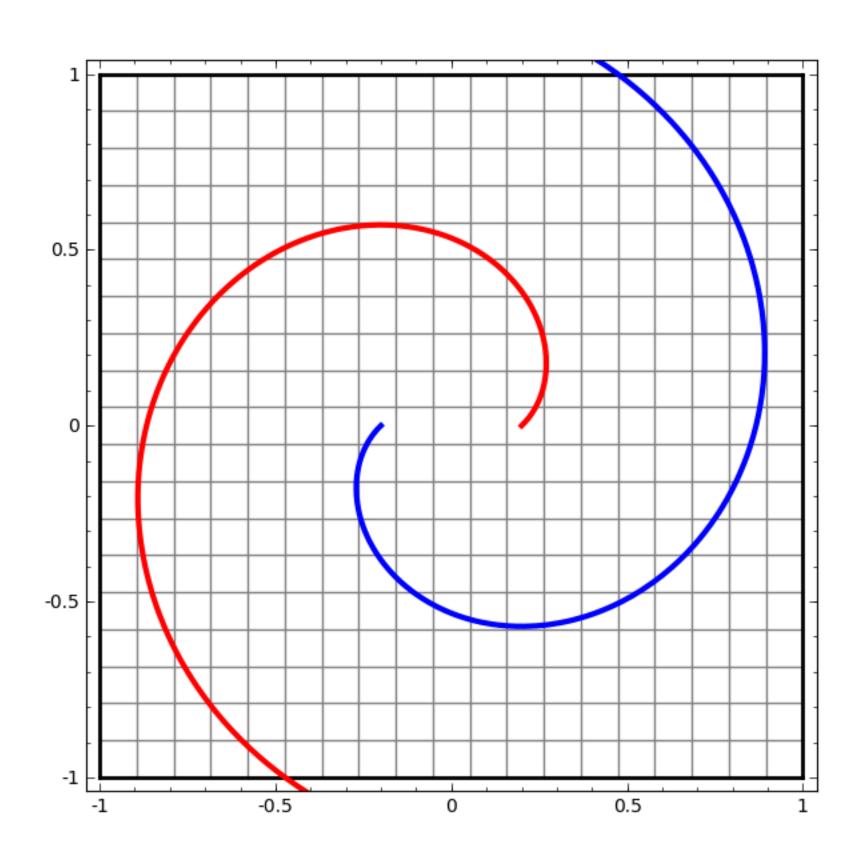
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 output of first layer

"Feedforward" computation (not recurrent)

Check: what happens if no nonlinearity? More powerful than basic linear models?

$$z = V(Wx + b) + c$$





Feedforward Networks, Backpropagation

$$P(y|\mathbf{x}) = \frac{\exp(w^{\top} f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top} f(\mathbf{x}, y'))}$$

Single scalar probability

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Single scalar probability

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}\left([w^{\top} f(\mathbf{x}, y)]_{y \in \mathcal{Y}}\right)$$

$$P(y|\mathbf{x}) = \frac{\exp(w^{\top} f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top} f(\mathbf{x}, y'))}$$

Single scalar probability

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax} \left([w^{\top} f(\mathbf{x}, y)]_{y \in \mathcal{Y}} \right)$$
$$\operatorname{softmax}(p)_i = \frac{\exp(p_i)}{\sum_{i'} \exp(p_{i'})}$$

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- Single scalar probability
- Compute scores for all possible labels at once (returns vector)

$$P(y|\mathbf{x}) = \frac{\exp(w^{\top} f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top} f(\mathbf{x}, y'))}$$

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$$\operatorname{softmax}(p)_i = \frac{\exp(p_i)}{\sum_{i'} \exp(p_{i'})}$$

- Single scalar probability
- Compute scores for all possible labels at once (returns vector)
- $\operatorname{softmax}(p)_i = \frac{\exp(p_i)}{\sum_{i'} \exp(p_{i'})}$ softmax: exps and normalizes a given vector given vector

$$P(y|\mathbf{x}) = \frac{\exp(w^{\top} f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top} f(\mathbf{x}, y'))}$$

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}\left([w^{\top} f(\mathbf{x}, y)]_{y \in \mathcal{Y}}\right)$$

$$\operatorname{softmax}(p)_i = \frac{\exp(p_i)}{\sum_{i'} \exp(p_{i'})}$$

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wf(\mathbf{x}))$$

- Single scalar probability
- Compute scores for all possible labels at once (returns vector)
- softmax: exps and normalizes a given vector
- Weight vector per class;W is [num classes x num feats]

$$P(y|\mathbf{x}) = \frac{\exp(w^{\top} f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top} f(\mathbf{x}, y'))}$$

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}\left([w^{\top} f(\mathbf{x}, y)]_{y \in \mathcal{Y}}\right)$$

$$\operatorname{softmax}(p)_i = \frac{\exp(p_i)}{\sum_{i'} \exp(p_{i'})}$$

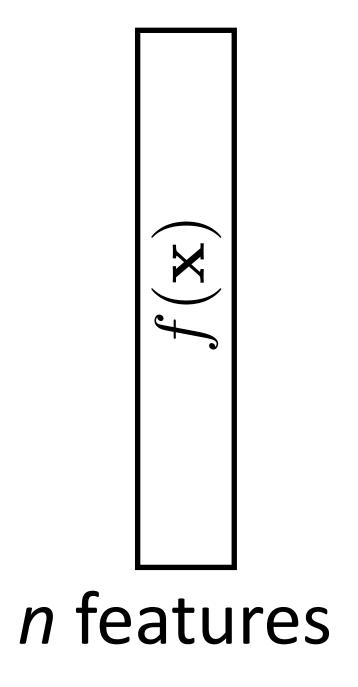
$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wf(\mathbf{x}))$$

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$$

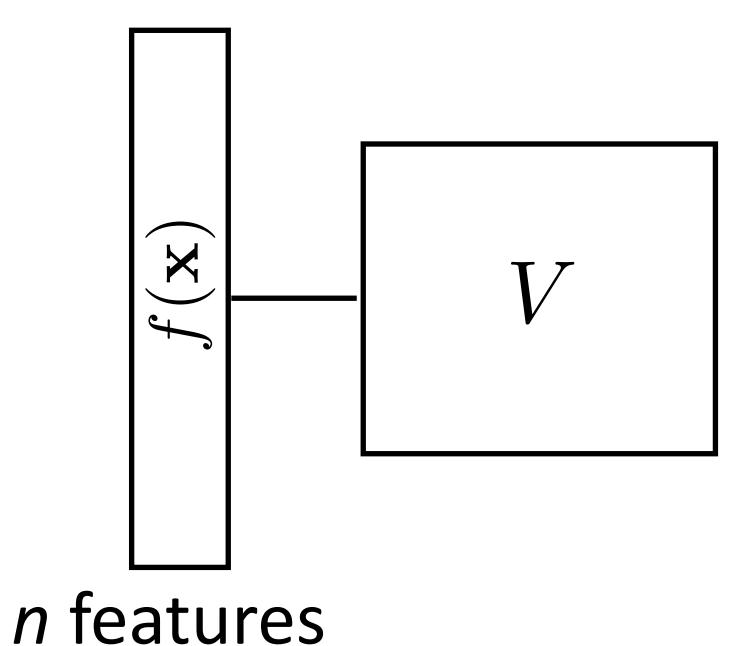
- Single scalar probability
- Compute scores for all possible labels at once (returns vector)
- softmax: exps and normalizes a given vector
- Weight vector per class;W is [num classes x num feats]
- Now one hidden layer

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$$

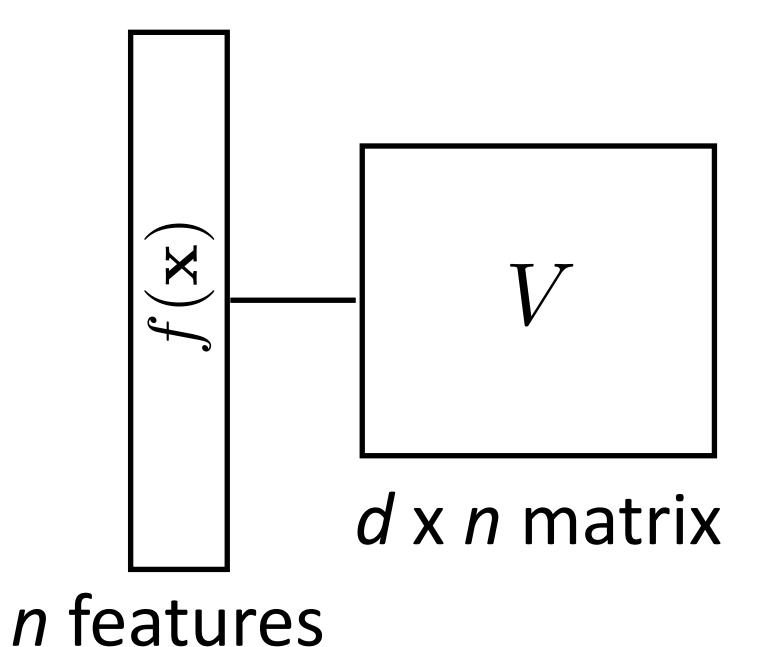
$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$$



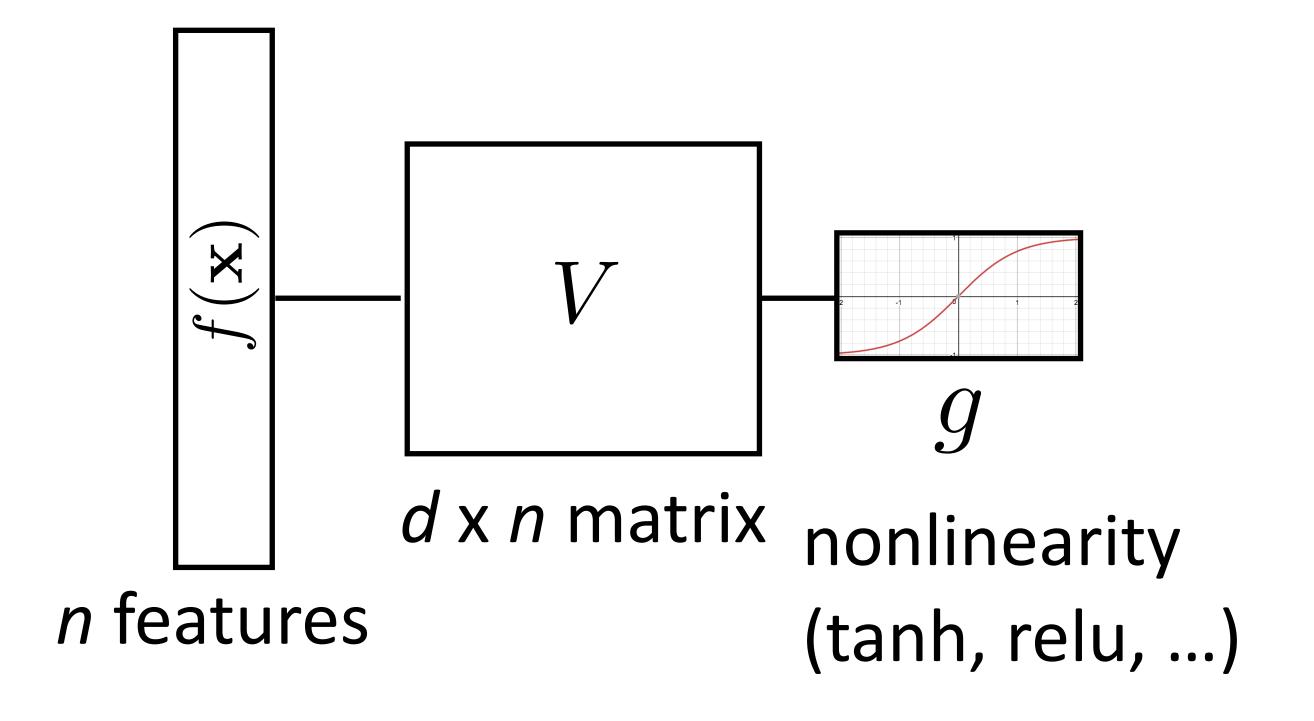
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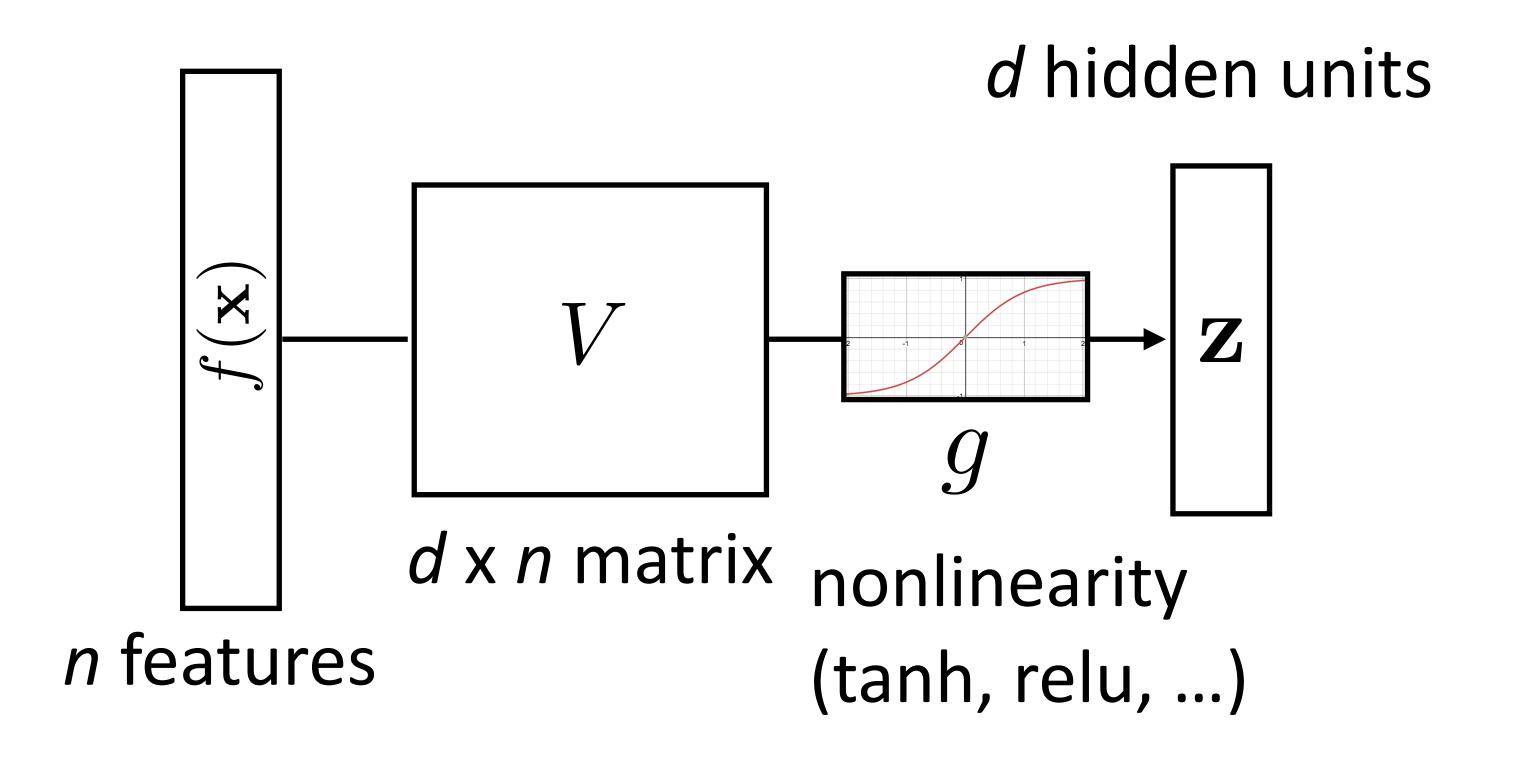
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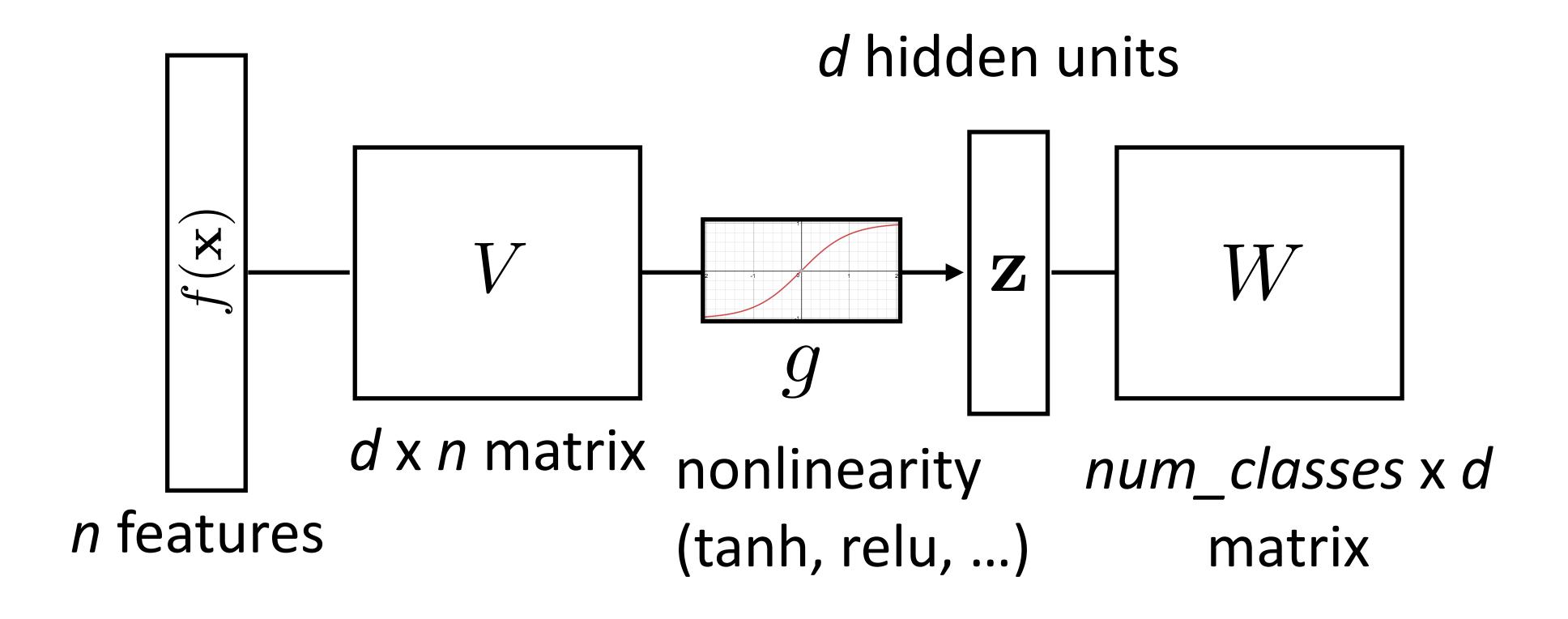
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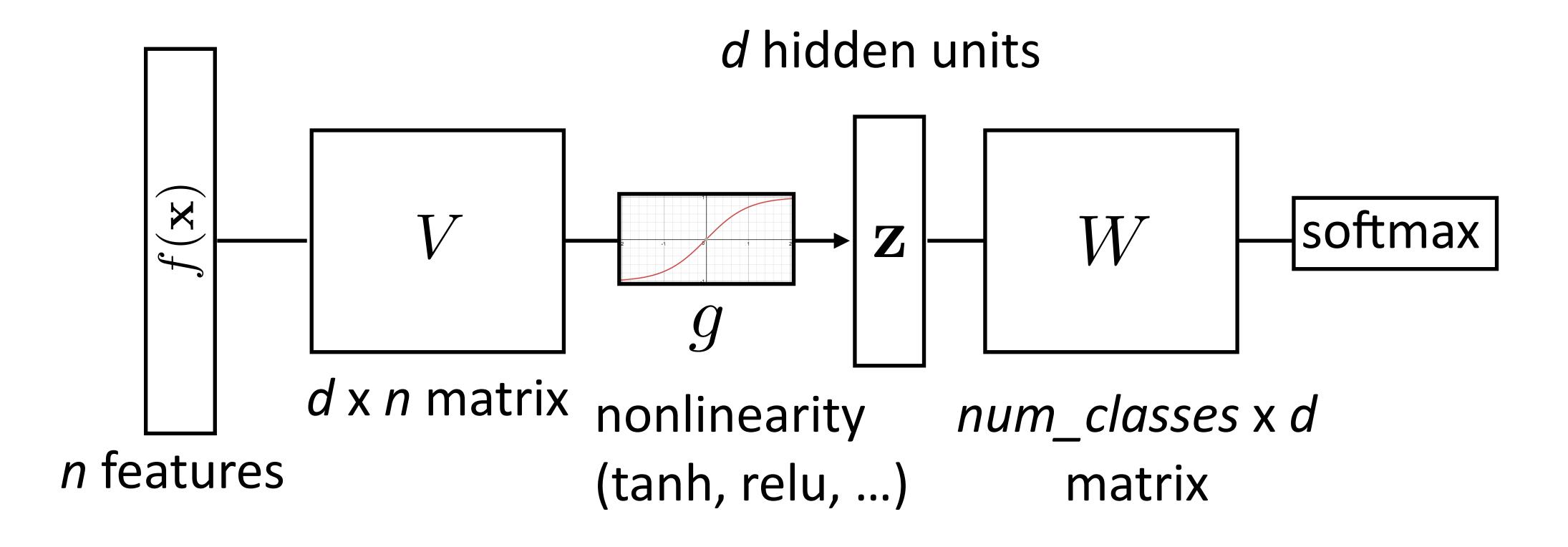
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$$d \text{ hidden units}$$

$$probs$$

$$d \text{ x } n \text{ matrix}$$

$$d \text{ nonlinearity}$$

$$num_classes \text{ x } d$$

$$n \text{ features}$$

$$num_classes \text{ x } d$$

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(W\mathbf{z})$$
 $\mathbf{z} = g(Vf(\mathbf{x}))$

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Maximize log likelihood of training data

$$\mathcal{L}(\mathbf{x}, i^*) = \log P(y = i^* | \mathbf{x}) = \log (\operatorname{softmax}(W\mathbf{z}) \cdot e_{i^*})$$

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- e_i : 1 in the *i*th row, zero elsewhere. Dot by this = select *i*th index

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Gradient with respect to W

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Gradient with respect to W

$$\frac{\partial}{\partial W_{ij}} \mathcal{L}(\mathbf{x}, i^*) = \begin{cases} \mathbf{z}_j - P(y = i | \mathbf{x}) \mathbf{z}_j & \text{if } i = i^* \\ -P(y = i | \mathbf{x}) \mathbf{z}_j & \text{otherwise} \end{cases}$$

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V j

 $\mathbf{z}_j - P(y = i|\mathbf{x})\mathbf{z}_j$

 $-P(y=i|\mathbf{x})\mathbf{z}_{j}$

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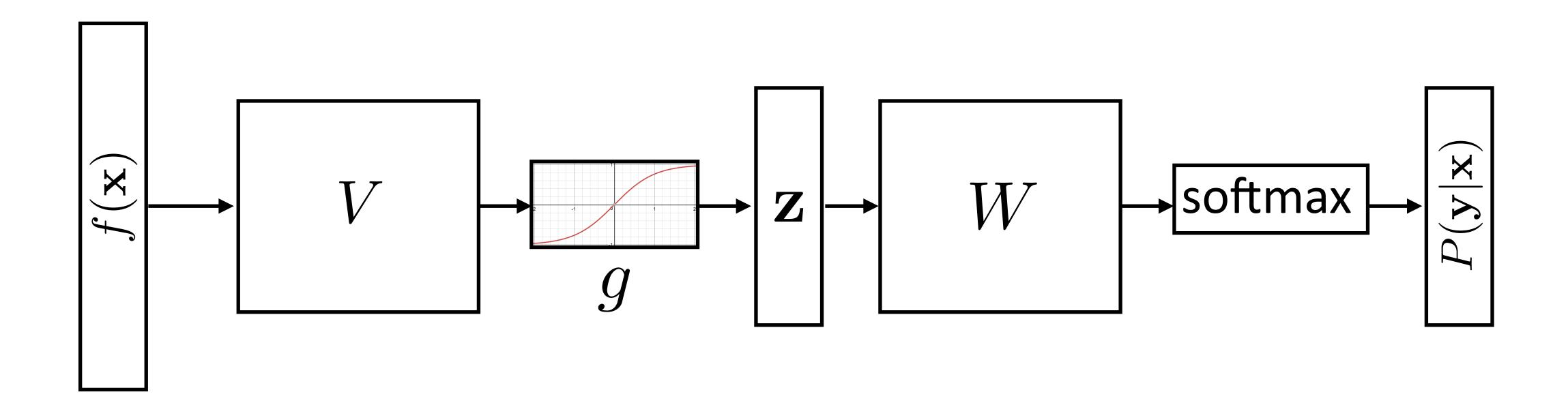
N j

 $\mathbf{z}_j - P(y = i|\mathbf{x})\mathbf{z}_j$

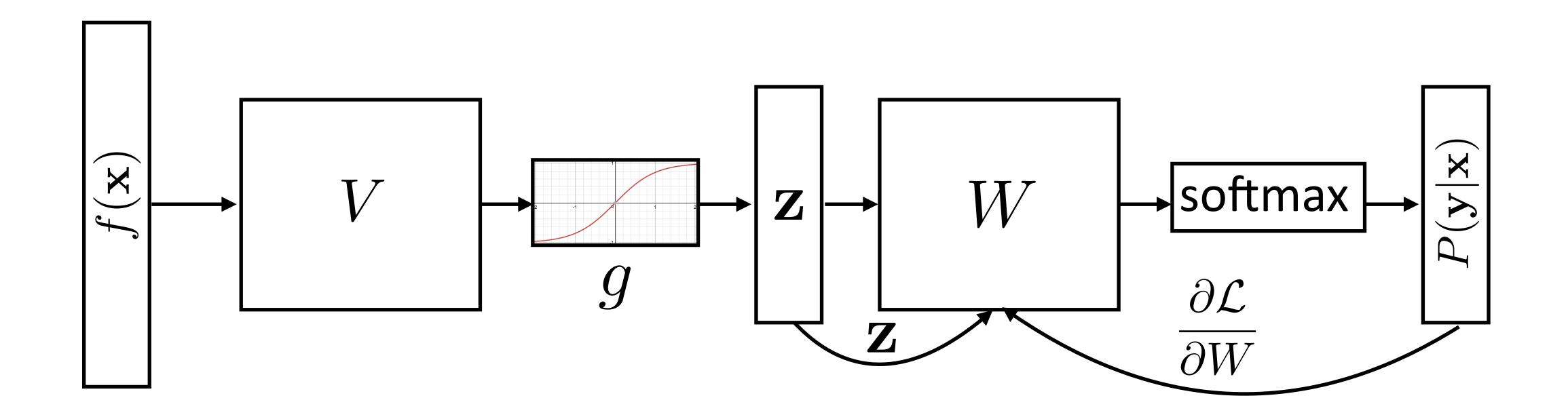
 $-P(y=i|\mathbf{x})\mathbf{z}_j$

Looks like logistic regression with z as the features!

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$$



$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$$



$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{j} \exp(W\mathbf{z}) \cdot e_{j} \qquad \mathbf{z} = g(Vf(\mathbf{x}))$$
 Activations at hidden layer

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{j} \exp(W\mathbf{z}) \cdot e_{j}$$

 $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial V_{ij}} = \frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial V_{ij}}$$

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{j} \exp(W\mathbf{z}) \cdot e_{j}$$

Gradient with respect to V: apply the chain rule

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$$\text{dim = num_classes}$$

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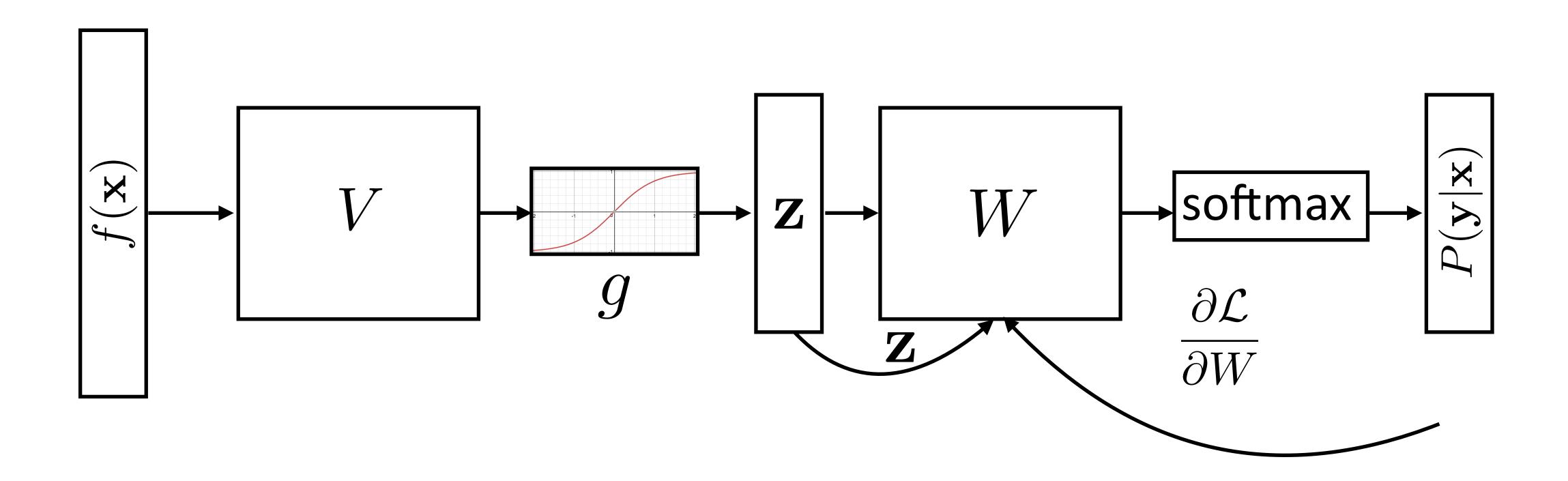
$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial V_{ij}} = \frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial V_{ij}}$$
[some math...]

$$err(root) = e_{i^*} - P(\mathbf{y}|\mathbf{x})$$

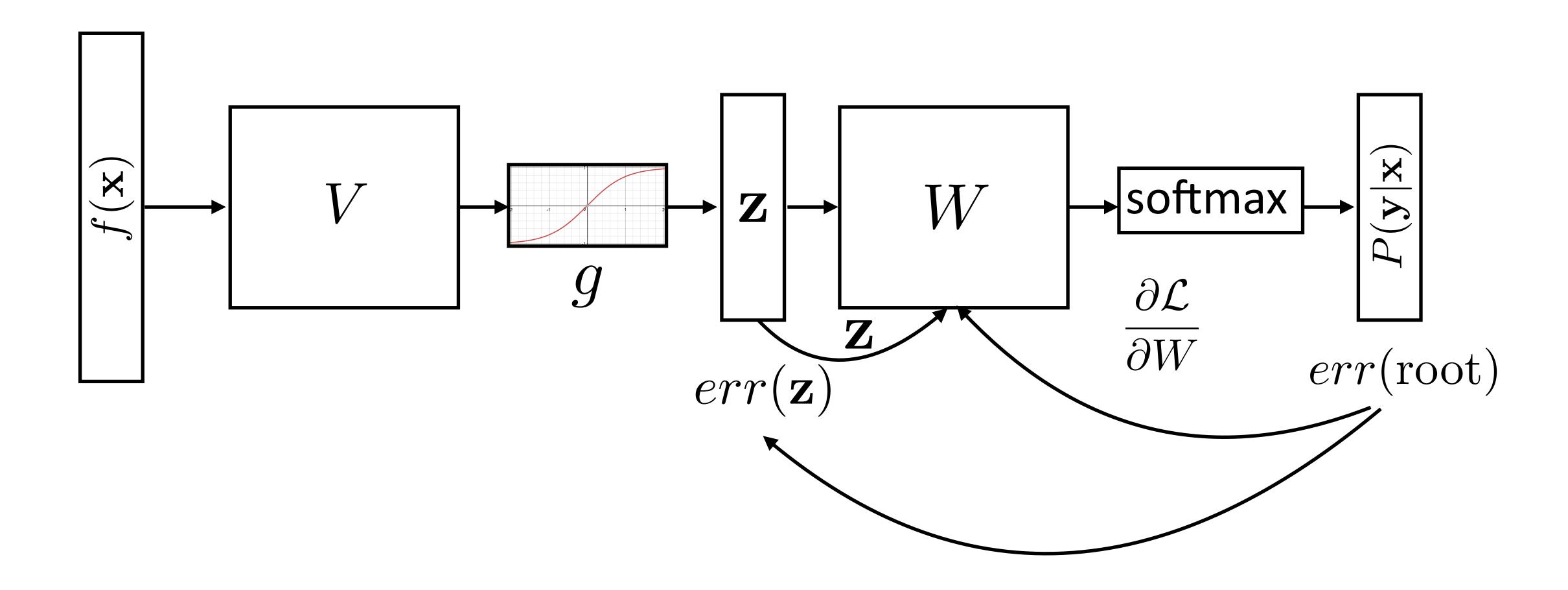
dim = num classes

$$err(\text{root}) = e_{i^*} - P(\mathbf{y}|\mathbf{x})$$
 $\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err(\mathbf{z}) = W^{\top}err(\text{root})$ dim = num_classes

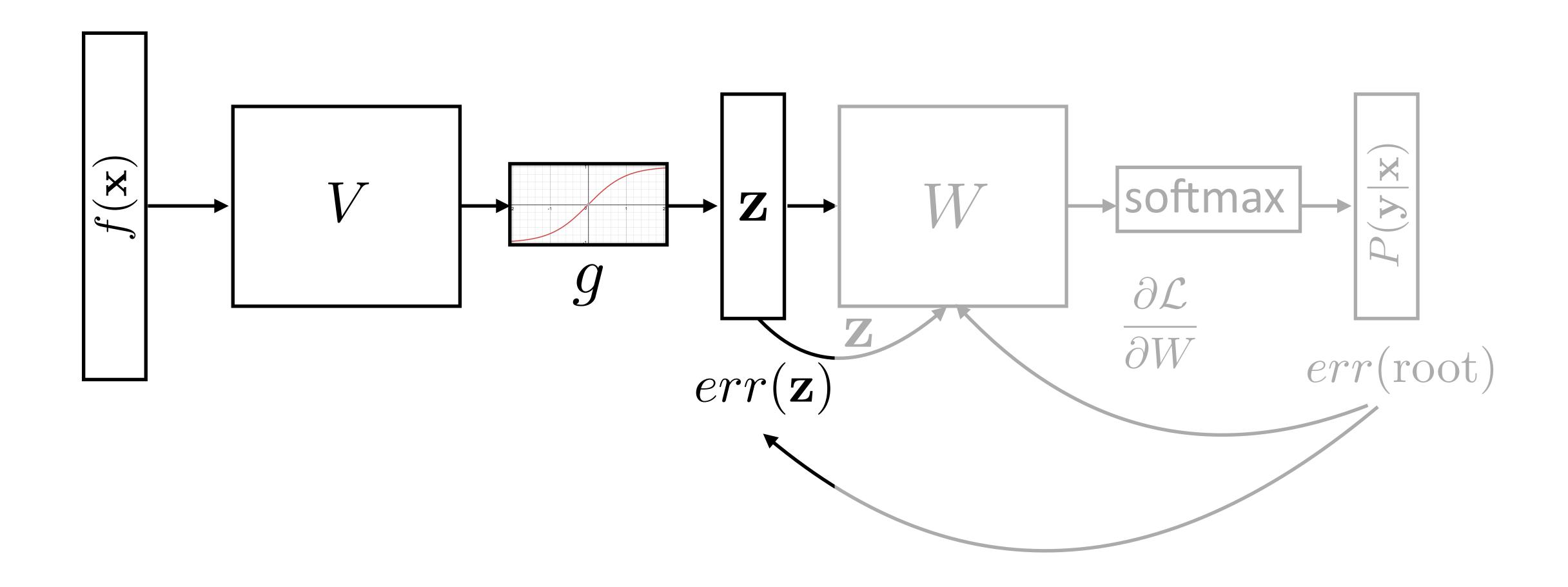
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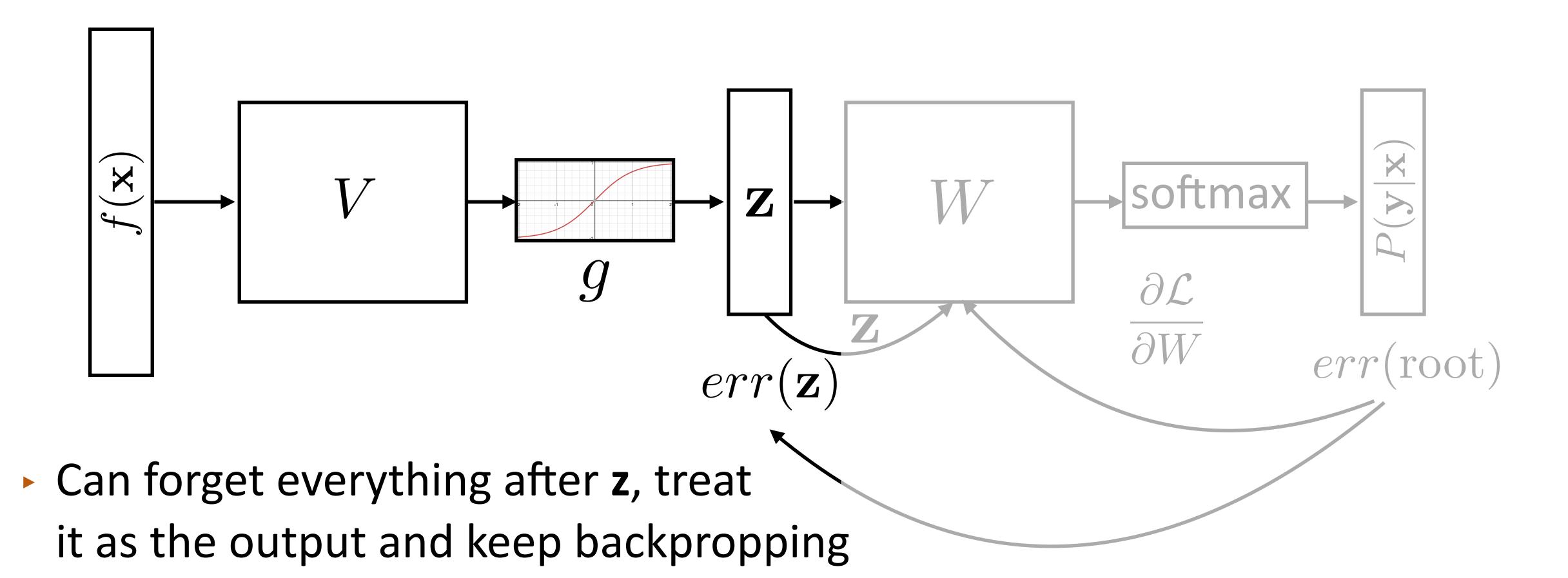
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 $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial V_{ij}} = \frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{V_{ij}}$$

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Activations at hidden layer

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Active

 $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial V_{ij}} = \frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} \boxed{\frac{\partial \mathbf{z}}{V_{ij}}} \qquad \frac{\partial \mathbf{z}}{V_{ij}} = \frac{\partial g(\mathbf{a})}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial V_{ij}} \qquad \mathbf{a} = V f(\mathbf{x})$$

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Gradient with respect to V: apply the chain rule

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First term: gradient of nonlinear activation function at *a* (depends on current value)

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Gradient with respect to V: apply the chain rule

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- First term: gradient of nonlinear activation function at *a* (depends on current value)
- Second term: gradient of linear function

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{j=1}^{m} \exp(W\mathbf{z} \cdot e_j)$$
 $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer

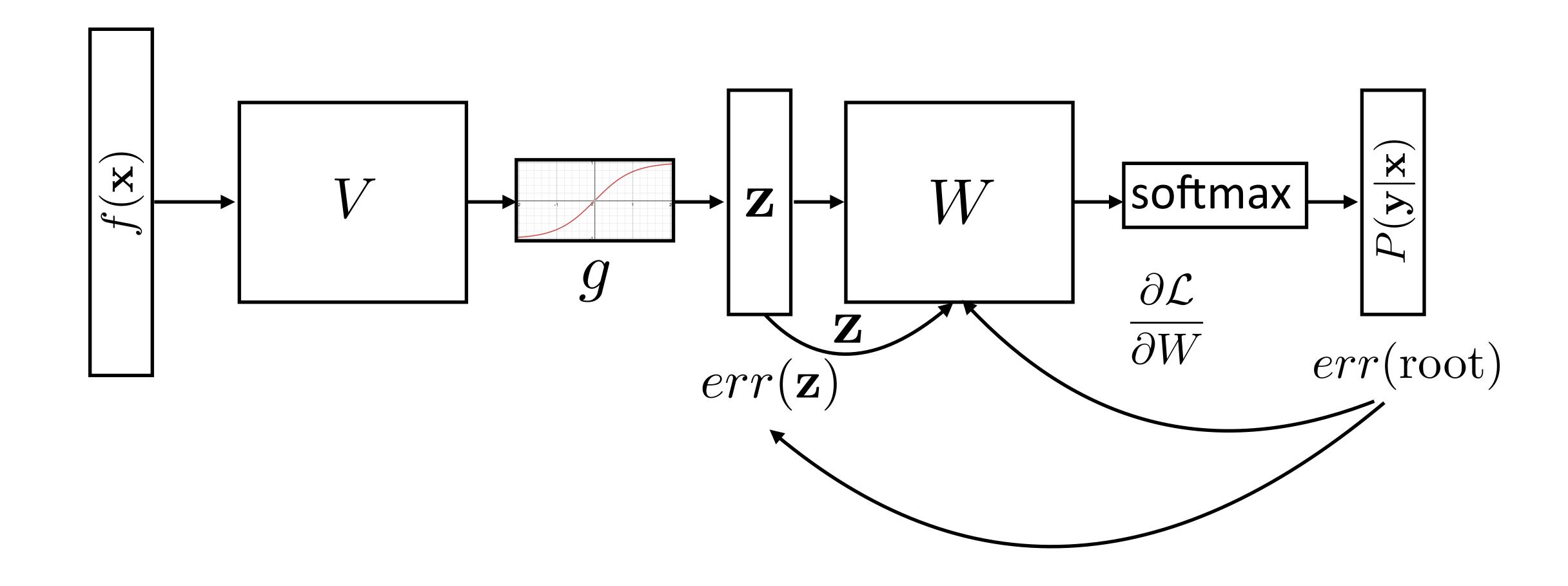
Gradient with respect to V: apply the chain rule

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial V_{ij}} = \frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} \boxed{\frac{\partial \mathbf{z}}{V_{ij}}} \qquad \frac{\partial \mathbf{z}}{V_{ij}} = \boxed{\frac{\partial g(\mathbf{a})}{\partial \mathbf{a}}} \boxed{\frac{\partial \mathbf{a}}{\partial V_{ij}}} \qquad \mathbf{a} = V f(\mathbf{x})$$

- First term: gradient of nonlinear activation function at *a* (depends on current value)
- Second term: gradient of linear function
- Straightforward computation once we have err(z)

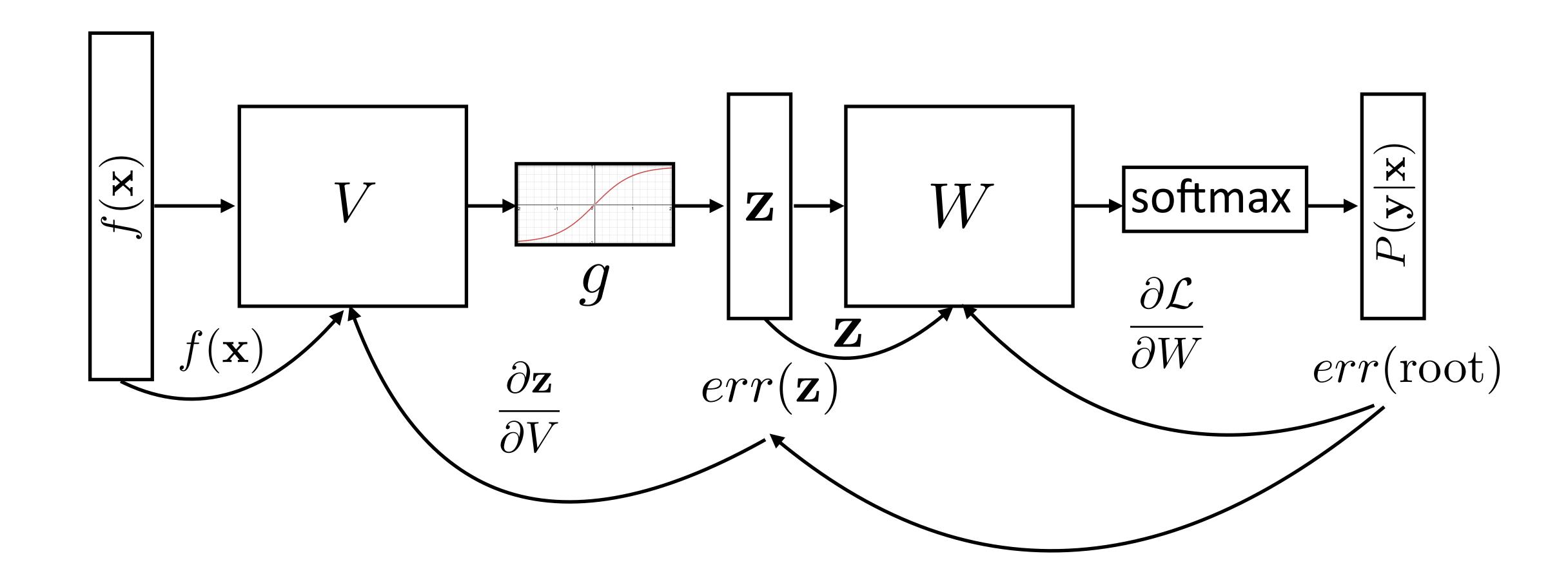
Backpropagation: Picture

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$$



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- Step 4: compute derivatives of V using err(z) (matrix)
- ► Step 5+: continue backpropagation (compute $err(f(\mathbf{x}))$) if necessary...)

 Gradients of output weights W are easy to compute — looks like logistic regression with hidden layer z as feature vector

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- Can compute derivative of loss with respect to z to form an "error signal" for backpropagation
- Easy to update parameters based on "error signal" from next layer, keep pushing error signal back as backpropagation
- Need to remember the values from the forward computation

Applications

Part-of-speech tagging with FFNNs

Part-of-speech tagging with FFNNs

55

Fed raises interest rates in order to ...

Part-of-speech tagging with FFNNs

??

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Word embeddings for each word form input

Part-of-speech tagging with FFNNs

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Fed raises interest rates in order to ...

Word embeddings for each word form input

previous word

other words, feats, etc. L...

Botha et al. (2017)

Part-of-speech tagging with FFNNs

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- Word embeddings for each word form input
- ► ~1000 features here smaller feature vector than in sparse models, but every feature fires on every example

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other words, feats, etc. L...

Botha et al. (2017)

Part-of-speech tagging with FFNNs

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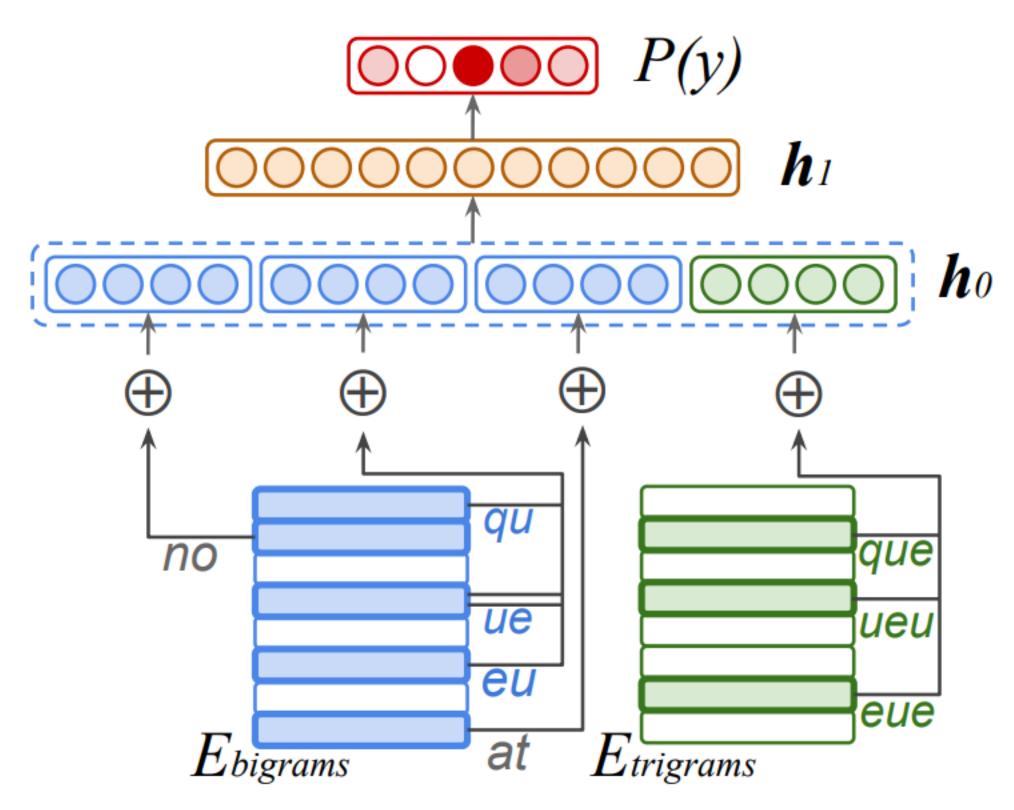
- Word embeddings for each word form input
- ~1000 features here smaller feature vector than in sparse models, but every feature fires on every example
- Weight matrix learns position-dependent processing of the words

curr word

next word

other words, feats, etc.

Botha et al. (2017)



There was no queue at the ...

 Hidden layer mixes these different signals and learns feature conjunctions

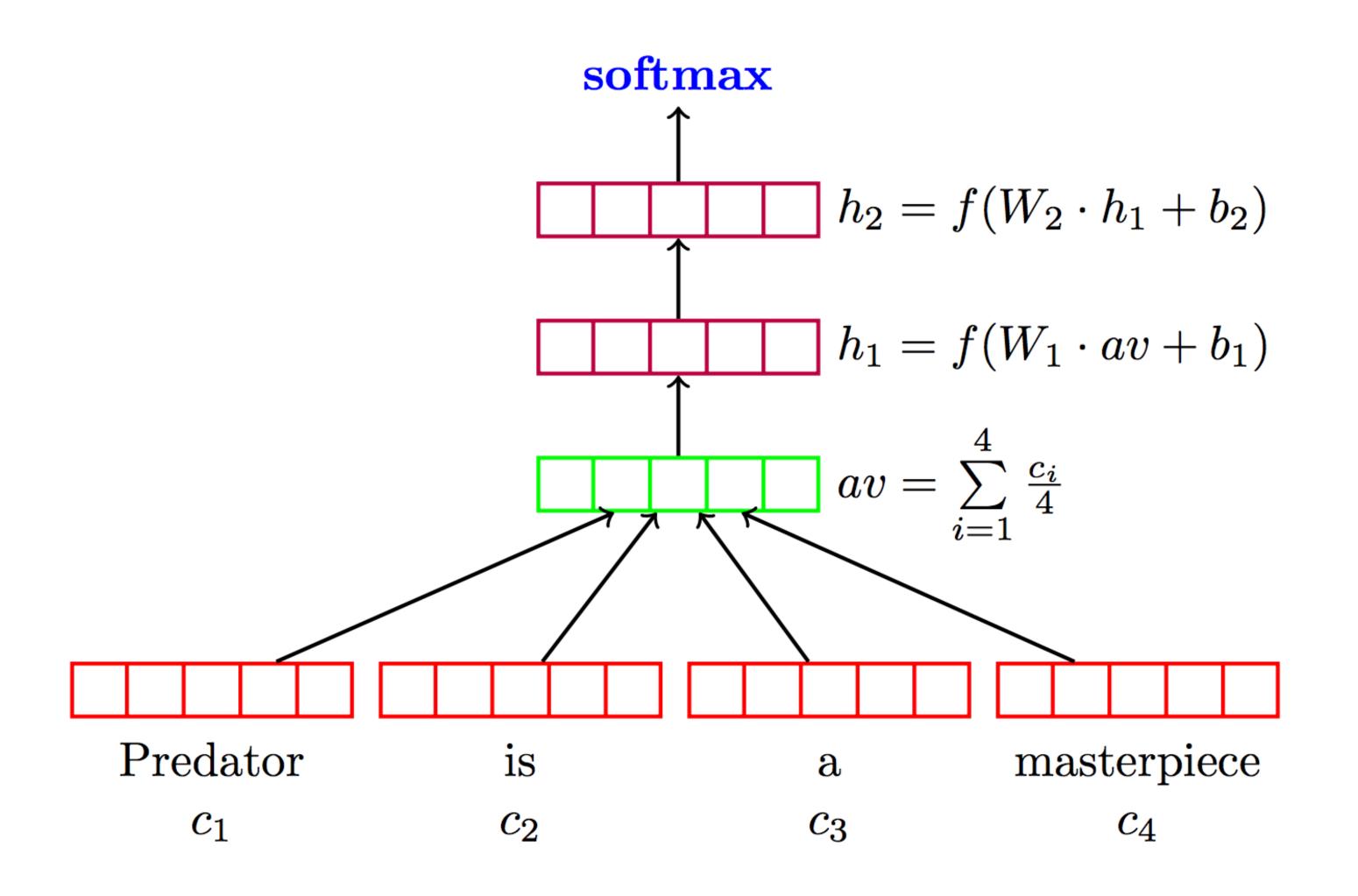
Multilingual tagging results:

Model	Acc.	Wts.	MB	Ops.
Gillick et al. (2016)	95.06	900k	_	6.63m
Small FF	94.76	241k	0.6	0.27m
+Clusters	95.56	261k	1.0	0.31m
$\frac{1}{2}$ Dim.	95.39	143k	0.7	0.27m 0.31m 0.18m

Gillick used LSTMs; this is smaller, faster, and better

Sentiment Analysis

 Deep Averaging Networks: feedforward neural network on average of word embeddings from input



lyyer et al. (2015)

Sentiment Analysis

	Model	RT	SST	SST	IMDB	Time	
			fine	bin		(s)	
	DAN-ROOT		46.9	85.7		31	
	DAN-RAND	77.3	45.4	83.2	88.8	136	
	DAN	80.3	47.7	86.3	89.4	136	lyyer et al. (2015)
vords	NBOW-RAND	76.2	42.3	81.4	88.9	91	
	NBOW	79.0	43.6	83.6	89.0	91	
	BiNB		41.9	83.1			Wang and
	NBSVM-bi	79.4			91.2		Manning (2012)
	RecNN*	77.7	43.2	82.4			iviaiiiiig (ZUIZ)
	RecNTN*		45.7	85.4			
s/	DRecNN		49.8	86.6		431	
•	TreeLSTM		50.6	86.9			
STMS 5	DCNN*		48.5	86.9	89.4		
	PVEC*		48.7	87.8	92.6		17. (2044)
	CNN-MC	81.1	47.4	88.1		2,452	Kim (2014)
	WRRBM*				89.2		

Bag-of-words

Tree RNNs /
CNNS / LSTMS

Coreference Resolution

Feedforward networks identify coreference arcs

Coreference Resolution

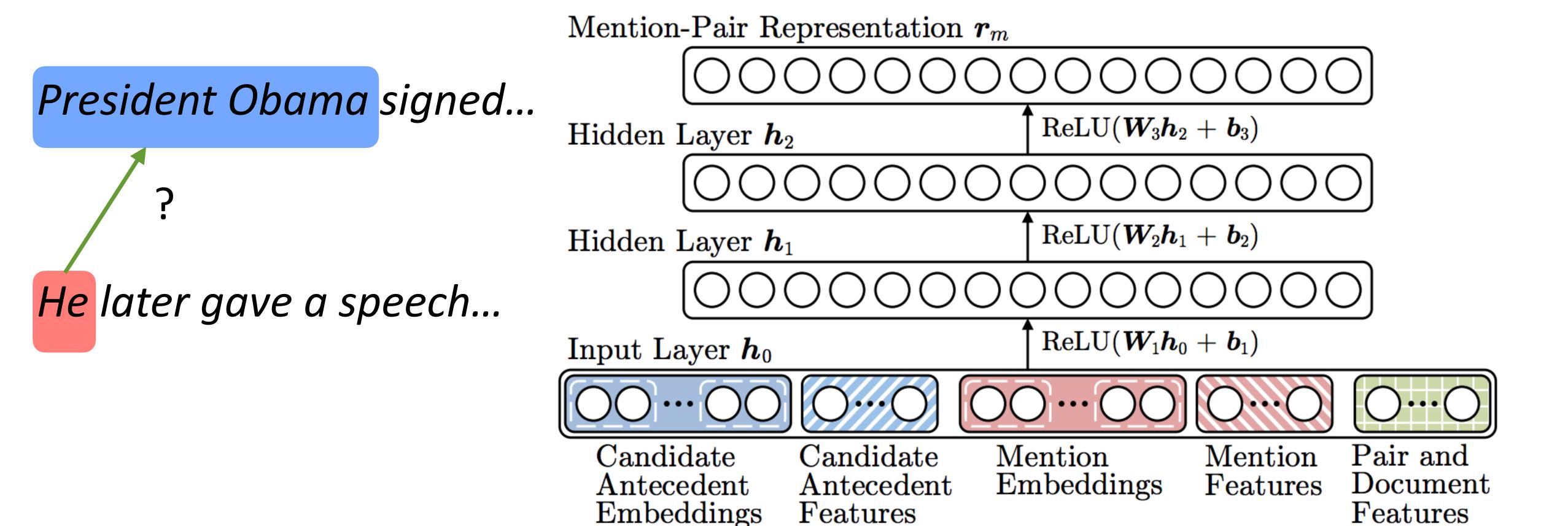
Feedforward networks identify coreference arcs

President Obama signed...
?

He later gave a speech...

Coreference Resolution

Feedforward networks identify coreference arcs



Clark and Manning (2015), Wiseman et al. (2015)

Implementation Details

Computation Graphs

Computing gradients is hard!

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Automatic differentiation: instrument code to keep track of derivatives

Computation Graphs

- Computing gradients is hard!
- Automatic differentiation: instrument code to keep track of derivatives

$$y = x * x \longrightarrow (y,dy) = (x * x, 2 * x * dx)$$
codegen

Computation Graphs

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 \longrightarrow $(y,dy) = (x * x, 2 * x * dx)$ codegen

Computation is now something we need to reason about symbolically

Computation Graphs

- Computing gradients is hard!
- Automatic differentiation: instrument code to keep track of derivatives

$$y = x * x$$
 \longrightarrow $(y,dy) = (x * x, 2 * x * dx)$ codegen

- Computation is now something we need to reason about symbolically
- Use a library like Pytorch or Tensorflow. This class: Pytorch

• Define forward pass for $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ class FFNN(nn.Module): def init (self, inp, hid, out): super(FFNN, self). init () self.V = nn.Linear(inp, hid) self.g = nn.Tanh()self.W = nn.Linear(hid, out) self.softmax = nn.Softmax(dim=0) def forward(self, x): return self.softmax(self.W(self.g(self.V(x)))

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$$

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$$

```
ffnn = FFNN()
```

```
P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x}))) ffnn = FFNN() def make update(input, gold label):
```

```
P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x}))) \quad \text{of the label}  (e.g., [0, 1, 0])  \text{def make\_update(input, gold\_label):}
```

```
P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x}))) \quad \text{of the label}  (e.g., [0, 1, 0]) \text{ffnn} = \operatorname{FFNN}() \quad \text{def make\_update(input, gold\_label):}  \text{ffnn.zero grad() # clear gradient variables}
```

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P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x}))) \quad \text{of the label} \quad \text{(e.g., [0, 1, 0])} ffnn = FFNN() def make_update(input, gold_label): ffnn.zero_grad() # clear gradient variables probs = ffnn.forward(input)
```

```
ei*: one-hot vector
P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x}))) of the label
                                     (e.g., [0, 1, 0])
ffnn = FFNN()
def make update(input, gold label):
   ffnn.zero grad() # clear gradient variables
   probs = ffnn.forward(input)
   loss = torch.neg(torch.log(probs)).dot(gold label)
```

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   loss.backward()
```

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   loss = torch.neg(torch.log(probs)).dot(gold label)
   loss.backward()
   optimizer.step()
```

Define a computation graph

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For each epoch:

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For each batch of data:

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For each batch of data:

Compute loss on batch

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Autograd to compute gradients and take step

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Decode test set

Batching data gives speedups due to more efficient matrix operations

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Need to make the computation graph process a batch at the same time

- Batching data gives speedups due to more efficient matrix operations
- Need to make the computation graph process a batch at the same time

```
# input is [batch_size, num_feats]
# gold_label is [batch_size, num_classes]
def make_update(input, gold_label)
```

- Batching data gives speedups due to more efficient matrix operations
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Batch sizes from 1-100 often work well