# Lecture 6: Neural Networks

### Alan Ritter

(many slides from Greg Durrett)

- Neural network history
- Neural network basics
- Feedforward neural networks + backpropagation
- Applications
- Implementing neural networks (if time)

### This Lecture

# History: NN "dark ages"

### Convnets: applied to MNIST by LeCun in 1998



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LSTMs: Hochreiter and Schmidhuber (1997)

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Convnets: applied to MNIST by LeCun in 1998



LSTMs: Hochreiter and Schmidhuber (1997)

Henderson (2003): neural shift-reduce parser, not SOTA



- Collobert and Weston 2011: "NLP (almost) from scratch"
  Feedforward neural nets induce features for
  - Feedforward neural nets induce sequential CRFs ("neural CRF")
  - 2008 version was marred by bad experiments, claimed SOTA but wasn't, 2011 version tied SOTA

Input Window			/	word (	of
Text	$\operatorname{cat}$	$\mathbf{sat}$	on	the	r
Feature 1	$w_1^1$	$w_2^1$			1
Feature K	$w_1^K$	$w_2^K$	•••		ı
Lookup Table					
$LT_{W^1} \longrightarrow$					
:					
$LT_{W^K} \longrightarrow$					
	_	(	conca	t	
Linear					
$M^1 \times \odot \longrightarrow$					Π
	~		$n_{hu}^1$		
HardTanh					
$\frown$ $\sim$					
Linear					
$M^2 \times \odot \checkmark$					
		2	= #1	ags	



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- Krizhevskey et al. (2012): AlexNet for vision
- Socher 2011-2014: tree-structured RNNs working okay





(convnets work for NLP?)

Kim (2014) + Kalchbrenner et al. (2014): sentence classification / sentiment



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- for NLP?)

Kim (2014) + Kalchbrenner et al. (2014): sentence classification / sentiment

Sutskever et al. + Bahdanau et al.: seq2seq for neural MT (LSTMs work



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- Sutskever et al. + Bahdanau et al.: seq2seq for neural MT (LSTMs work for NLP?)
- Chen and Manning transition-based dependency parser (even feedforward) networks work well for NLP?)

Kim (2014) + Kalchbrenner et al. (2014): sentence classification / sentiment





- (convnets work for NLP?)
- Sutskever et al. + Bahdanau et al.: seq2seq for neural MT (LSTMs work for NLP?)
- Chen and Manning transition-based dependency parser (even feedforward) networks work well for NLP?)
- 2015: explosion of neural nets for everything under the sun

Kim (2014) + Kalchbrenner et al. (2014): sentence classification / sentiment





# sentences (and really need a lot more)

Datasets too small: for MT, not really better until you have 1M+ parallel



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- Optimization not well understood: good initialization, per-feature scaling + momentum (Adagrad / Adadelta / Adam) work best out-of-the-box



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  - Regularization: dropout is pretty helpful
  - Computers not big enough: can't run for enough iterations
- Inputs: need word representations to have the right continuous semantics





Neural Net Basics

Linear classification:  $\operatorname{argmax}_y w^\top f(x, y)$ 

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the movie was **not** all that **good** 

- Linear classification:  $\operatorname{argmax}_{y} w^{\top} f(x, y)$
- How can we do nonlinear classification? Kernels are too slow...
- Want to learn intermediate conjunctive features of the input
  - the movie was **not** all that **good**
  - [[contains *not* & contains *good*]

- Let's see how we can use neural nets to learn a simple nonlinear function
- Inputs

Output

- Let's see how we can use neural nets to learn a simple nonlinear function
- Inputs  $x_1, x_2$ (generally  $\mathbf{x} = (x_1, \ldots, x_m)$ )
- Output y (generally  $\mathbf{y} = (y_1, \ldots, y_n)$ )

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 $x_2 \quad y = x_1 \text{ XOR } x_2$  $x_1$ () $\left( \right)$ 1

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Neural Networks: XOR







 $y = a_1 x_1 + a_2 x_2$ 



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 $y = a_1 x_1 + a_2 x_2$ 



 $y = a_1 x_1 + a_2 x_2$ 



 $y = a_1 x_1 + a_2 x_2$ 





Neural Networks: XOR

 $y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$ 







Neural Networks: XOR

### $y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$ "or"







Neural Networks: XOR

 $y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$ "or"

### (looks like action potential in neuron)







Neural Networks: XOR

 $y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$ 







Neural Networks: XOR

 $y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$ 



 $y = -x_1 - x_2 + 2\tanh(x_1 + x_2)$ "or"





Neural Networks: XOR







the movie was not all that good



the movie was **not** all that **good** 

Neural Networks: XOR







 $y = g(\mathbf{w} \cdot \mathbf{x} + b)$  $\mathbf{y} = g(\mathbf{W}\mathbf{x} + \mathbf{b})$ Nonlinear transformation













Taken from http://colah.github.io/posts/2014-03-NN-Manifolds-Topology/





### Neural Networks

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### Linear classifier



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# Neural Networks

### Neural network



### Linear classifier





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# Neural Networks

### Neural network

### ...possible because we transformed the space!





W  $\boldsymbol{z}$ U  $\boldsymbol{x}$ ••• -

y = g(Wx + b)





y = g(Wx + b)





y = g(Wx + b) $\mathbf{z} = g(\mathbf{V}\mathbf{y} + \mathbf{c})$ 





y = g(Wx + b) $\mathbf{z} = g(\mathbf{V}\mathbf{y} + \mathbf{c})$  $\mathbf{z} = g(\mathbf{V}g(\mathbf{W}\mathbf{x} + \mathbf{b}) + \mathbf{c})$ output of first layer







"Feedforward" computation (not recurrent)





$$y = g(Wx + b)$$
  

$$z = g(Vy + c)$$
  

$$z = g(Vg(Wx + b) + c)$$
  
output of first layer

"Feedforward" computation (not recurrent)

Check: what happens if no nonlinearity? More powerful than basic linear models?

$$z = V(Wx + b) + c$$





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# Feedforward Networks, Backpropagation

 $P(y|\mathbf{x}) = \frac{\exp(w^{\top}f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top}f(\mathbf{x}, y'))}$ 

Single scalar probability

 $P(y|\mathbf{x}) = \frac{\exp(w^{\top}f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top}f(\mathbf{x}, y'))}$ 

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}\left([w \mid f(\mathbf{x}, y)]_{y \in \mathcal{Y}}\right)$ 

Single scalar probability



$$P(y|\mathbf{x}) = \frac{\exp(w^{\top}f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top}f(\mathbf{x}, y'))}$$

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}\left([w^{\top}f(\mathbf{x}, y)]\right)$ 

 $\operatorname{softmax}(p)_i = \frac{\exp(p)_i}{\sum_{i'} \exp(p)_i}$ 

Single scalar probability

$$)]_{y \in \mathcal{Y}}$$
$$(p_i)$$
$$(p_i)$$
$$P(y|\mathbf{x}) = \frac{\exp(w^{\top}f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top}f(\mathbf{x}, y'))}$$

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softmax $(p)_i = \frac{\exp(p_i)}{\sum_{i'} \exp(p_{i'})}$ 

Single scalar probability

Compute scores for all possible labels at once (returns vector)

$$P(y|\mathbf{x}) = \frac{\exp(w^{\top}f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top}f(\mathbf{x}, y'))}$$

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}\left([w^{\top}f(\mathbf{x}, y)]_{y \in \mathcal{Y}}\right)$ 

- Single scalar probability
- Compute scores for all possible labels at once (returns vector)
- softmax $(p)_i = \frac{\exp(p_i)}{\sum_{i'} \exp(p_{i'})}$  softmax: exps and normalizes a given vector

$$P(y|\mathbf{x}) = \frac{\exp(w^{\top}f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top}f(\mathbf{x}, y'))}$$

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}\left([w^{\top}f(\mathbf{x}, y)]_{y \in \mathcal{Y}}\right)$ 

$$\operatorname{softmax}(p)_i = \frac{\exp}{\sum_{i'} \exp}$$

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wf(\mathbf{x}))$ 

- Single scalar probability
- Compute scores for all possible labels at once (returns vector)
- $(p_i)$  $\operatorname{xp}(p_{i'})$
- softmax: exps and normalizes a given vector
- Weight vector per class; W is [num classes x num feats]





$$P(y|\mathbf{x}) = \frac{\exp(w^{\top} f(\mathbf{x}, y))}{\sum_{y'} \exp(w^{\top} f(\mathbf{x}, y'))}$$

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}\left([w^{\top}f(\mathbf{x}, y)]_{y \in \mathcal{Y}}\right)$ 

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 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wf(\mathbf{x}))$ 

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ 

- Single scalar probability
- Compute scores for all possible labels at once (returns vector)
- $(p_i)$  $\exp(p_{i'})$
- softmax: exps and normalizes a given vector
- Weight vector per class; W is [num classes x num feats]
- Now one hidden layer





 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ 

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ 



X





*n* features













- d hidden units
  - $\mathbf{Z}$





num\_classes x d









 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(W\mathbf{z})$   $\mathbf{z} = g(Vf(\mathbf{x}))$ 

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(W\mathbf{z})$$

Maximize log likelihood of training data

$$\mathcal{L}(\mathbf{x}, i^*) = \log P(y = i^* | \mathbf{x}) =$$

 $\mathbf{z} = g(Vf(\mathbf{x}))$ 

 $= \log (\operatorname{softmax}(W\mathbf{z}) \cdot e_{i^*})$ 

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(W\mathbf{z})$$

Maximize log likelihood of training data

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- i\*: index of the gold label
- $e_i$ : 1 in the *i*th row, zero elsewhere. Dot by this = select *i*th index

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$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i \in \mathcal{K}} V_i \mathbf{x} \cdot e_$$

 $\mathbf{z} = g(Vf(\mathbf{x}))$ 

 $= \log (\operatorname{softmax}(W\mathbf{z}) \cdot e_{i^*})$ 

 $\sum \exp(W\mathbf{z}) \cdot e_j$ 

i

 $\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum \exp(W\mathbf{z}) \cdot e_j$ 

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Gradient with respect to W

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i \in \mathcal{K}} V_{i^*} - \log \sum_{i \in \mathcal{K}} V_{i^*}$$

Gradient with respect to W

$$\frac{\partial}{\partial W_{ij}} \mathcal{L}(\mathbf{x}, i^*) = \begin{cases} \mathbf{z}_j - P(y = i | \mathbf{x}) \\ -P(y = i | \mathbf{x}) \mathbf{z}_j \end{cases}$$

 $\sum_{j} \exp(W\mathbf{z}) \cdot e_j$ 

### $\mathbf{z}_j$ if $i = i^*$

### otherwise

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i \in \mathcal{K}} V_{i^*} - \log \sum_{i \in \mathcal{K}} V_{i^*}$$

Gradient with respect to W

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 $\sum_{j} \exp(W\mathbf{z}) \cdot e_j$ W i () $\mathbf{z}_j$  if  $i = i^*$  $\mathbf{z}_j - P(y = i | \mathbf{x}) \mathbf{z}_j$ otherwise  $-P(y=i|\mathbf{x})\mathbf{z}_j$ 

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i \in \mathcal{K}} \sum_{i \in \mathcal{K}} \frac{1}{i^*} - \log \sum_{i \in \mathcal{K}} \frac{1}{i^*} \sum_{i \in \mathcal{K}} \frac{1}{i^*} \frac{1}{i^*}$$

Gradient with respect to W

$$\frac{\partial}{\partial W_{ij}} \mathcal{L}(\mathbf{x}, i^*) = \begin{cases} \mathbf{z}_j - P(y = i | \mathbf{x}) \\ -P(y = i | \mathbf{x}) \mathbf{z}_j \end{cases}$$

Looks like logistic regression with z as the features!















 $\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum \exp(W\mathbf{z}) \cdot e_j$ 

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i \in \mathcal{K}} \frac{1}{2} \sum_{i \in \mathcal{K}$$

Gradient with respect to V: apply the chain rule



- $\sum_{j} \exp(W\mathbf{z}) \cdot e_j$

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i \in \mathcal{I}} \frac{1}{2} \sum_{i \in \mathcal{I}$$

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Gradient with respect to V: apply the chain rule



 $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer

### [some math...]

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err(\mathbf{z}) = W^{\top} err(\text{root})$$
$$\dim = d$$

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ 





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 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ 









Can forget everything after z, treat it as the output and keep backpropping



$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{j=1}^{\infty} e_{j=1}$$

Gradient with respect to V: apply the chain rule

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial V_{ij}} = \frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{V_{ij}}$$

# $\sum_{j=1}^{m} \exp(W\mathbf{z} \cdot e_j)$

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i=1}^{\infty} \frac{1}{i^*}$$

Gradient with respect to V: apply the chain rule



# $\sum_{j=1}^{m} \exp(W\mathbf{z} \cdot e_j)$

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i=1}^{\infty} \frac{1}{i^{i+1}}$$

Gradient with respect to V: apply the chain rule



- m
- $\sum_{j=1}^{\infty} \exp(W\mathbf{z} \cdot e_j)$
- $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer


$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i=1}^{\infty} \frac{1}{i^{i+1}}$$

Gradient with respect to V: apply the chain rule



- $\mathcal{M}$ 
  - $\sum \exp(W\mathbf{z} \cdot e_j)$
- $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer



$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{i=1}^{\infty} \frac{1}{i^{i+1}}$$

Gradient with respect to V: apply the chain rule



First term: gradient of nonlinear activation function at a (depends on current value)

- m
  - $\sum \exp(W\mathbf{z} \cdot e_j)$
- $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer

$$\frac{\partial \mathbf{z}}{V_{ij}} = \frac{\partial g(\mathbf{a})}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial V_{ij}} \quad \mathbf{a} = Vf$$



$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{j=1}^{\infty} \frac{1}{j} = 1$$

Gradient with respect to V: apply the chain rule



- First term: gradient of nonlinear activation function at *a* (depends on current value)
- Second term: gradient of linear function

- ${m}$ 
  - $\sum_{j=1}^{N} \exp(W\mathbf{z} \cdot e_j)$

#### $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer

$$\frac{\partial \mathbf{z}}{V_{ij}} = \frac{\partial g(\mathbf{a})}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial V_{ij}} \quad \mathbf{a} = V$$



$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_{j=1}^{\infty} \frac{1}{j} = 1$$

Gradient with respect to V: apply the chain rule



- First term: gradient of nonlinear activation function at *a* (depends on current value)
- Second term: gradient of linear function
- Straightforward computation once we have err(z)

- ${m}$ 
  - $\sum_{j=1}^{N} \exp(W\mathbf{z} \cdot e_j)$

#### $\mathbf{z} = g(Vf(\mathbf{x}))$ Activations at hidden layer

$$\frac{\partial \mathbf{z}}{V_{ij}} = \frac{\partial g(\mathbf{a})}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial V_{ij}} \quad \mathbf{a} = V$$



### Backpropagation: Picture

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ 





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#### $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$

 $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ 

• Step 1: compute  $err(root) = e_{i^*} - P(\mathbf{y}|\mathbf{x})$  (vector)

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf$$

- Step 1: compute  $err(root) = e_{i^*} P(\mathbf{y}|\mathbf{x})$  (vector)
- Step 2: compute derivatives of W using err(root) (matrix)

 $\mathbf{f}(\mathbf{x})))$ 

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf$$

- Step 1: compute  $err(root) = e_{i^*} P(\mathbf{y}|\mathbf{x})$  (vector)
- Step 2: compute derivatives of W using err(root) (matrix)
- Step 3: compute

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err$$

 $f(\mathbf{x})))$ 

 $r(\mathbf{z}) = W^{\top} err(root)$  (vector)

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf$$

- Step 1: compute  $err(root) = e_{i^*} P(\mathbf{y}|\mathbf{x})$  (vector)
- Step 2: compute derivatives of W using err(root) (matrix)
- ► Step 3: compute  $\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err(\mathbf{z}) = W^{\top}err(\text{root})$  (vector) Step 4: compute derivatives of V using err(z) (matrix)

 $\mathbf{f}(\mathbf{X})))$ 

$$P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf$$

- Step 1: compute  $err(root) = e_{i^*} P(\mathbf{y}|\mathbf{x})$  (vector)
- Step 2: compute derivatives of W using err(root) (matrix)
- Step 3: compute  $\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err(\mathbf{z}) = W^{\top} err(root)$  (vector)
- Step 4: compute derivatives of V using err(z) (matrix)
- Step 5+: continue backpropagation (compute err(f(x)) if necessary...)

 $f(\mathbf{x})))$ 

Gradients of output weights W are easy to compute — looks like logistic regression with hidden layer z as feature vector

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- Easy to update parameters based on "error signal" from next layer, keep pushing error signal back as backpropagation
- Need to remember the values from the forward computation

Applications

Part-of-speech tagging with FFNNs



#### Part-of-speech tagging with FFNNs ??

#### Fed raises interest rates in order to ...



Part-of-speech tagging with FFNNs

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Word embeddings for each word form input



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Part-of-speech tagging with FFNNs

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- ~1000 features here smaller feature vector than in sparse models, but every feature fires on every example
- Weight matrix learns position-dependent processing of the words







There was no <u>queue</u> at the ...

Hidden layer mixes these different signals and learns feature conjunctions



#### Multilingual tagging results:

Model	Acc.	Wts.	MB	<b>Ops.</b>
Gillick et al. (2016)	95.06	900k	-	6.63m
Small FF	94.76	241k	0.6	0.27m
+Clusters	95.56	261k	1.0	0.31m
$\frac{1}{2}$ Dim.	95.39	143k	0.7	0.18m

Gillick used LSTMs; this is smaller, faster, and better



### Sentiment Analysis

# word embeddings from input



Deep Averaging Networks: feedforward neural network on average of

$$h_2 = f(W_2 \cdot h_1 + b_2)$$

$$h_1 = f(W_1 \cdot av + b_1)$$



### Sentiment Analysis

	Model	RT	SST fine	SST bin	IMDB	Time (s)	
	DAN-ROOT DAN-RAND DAN	77.3 80.3	46.9 45.4 47.7	85.7 83.2 86 3	 88.8 89.4	<b>31</b> 136	Ivver et al (20
Bag-of-words	NBOW-RAND NBOW BiNB NDSVM bi	76.2 79.0	42.3 43.6 41.9	81.4 83.6 83.1	88.9 89.0	91 91 	Wang and
Tree RNNs / CNNS / LSTMS	RecNN* RecNTN* DRecNN	77.7	43.2 45.7 49.8	82.4 85.4 86.6	91.2	 431	Manning (201
	TreeLSTM DCNN* PVEC* CNN-MC WRRBM*	 81.1	<b>50.6</b> 48.5 48.7 47.4	86.9 86.9 87.8 <b>88.1</b>	89.4 <b>92.6</b> 89.2	2,452	Kim (2014)





#### **Coreference Resolution**

#### Feedforward networks identify coreference arcs

Clark and Manning (2015), Wiseman et al. (2015)



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## **Coreference Resolution**

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Implementation Details

#### **Computation Graphs**

Computing gradients is hard!

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- Computing gradients is hard!

#### Automatic differentiation: instrument code to keep track of derivatives

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$$y = x * x \longrightarrow (y, dy) = codegen$$

#### **Computation Graphs**

#### Automatic differentiation: instrument code to keep track of derivatives

#### (x \* x, 2 \* x \* dx)

#### Computation is now something we need to reason about symbolically

- Computing gradients is hard!

$$y = x * x - (y, dy) = codegen$$

- Use a library like Pytorch or Tensorflow. This class: Pytorch

#### **Computation Graphs**

#### Automatic differentiation: instrument code to keep track of derivatives

#### (x \* x, 2 \* x \* dx)

#### Computation is now something we need to reason about symbolically

• Define forward pass for  $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$ 

class FFNN(nn.Module): def init (self, inp, hid, out): super(FFNN, self). init () self.V = nn.Linear(inp, hid) self.g = nn.Tanh()self.W = nn.Linear(hid, out) self.softmax = nn.Softmax(dim=0)

> def forward(self, x): return self.softmax(self.W(self.g(self.V(x))))



#### $P(\mathbf{y}|\mathbf{x}) = \operatorname{softmax}(Wg(Vf(\mathbf{x})))$

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#### Define a computation graph

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For each epoch:

#### Define a computation graph For each epoch: For each batch of data:

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### Batching

Batching data gives speedups due to more efficient matrix operations

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Need to make the computation graph process a batch at the same time

# input is [batch size, num feats] # gold label is [batch size, num classes] def make update(input, gold label)

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Batch sizes from 1-100 often work well

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